"La teoria delle distorsioni e le deformazioni finite dei solidi elastici," Rend. R. Acc. dei Lincei (5) **25** (1916), 191-192.

Mechanics – *The theory of distorsions and the finite deformations of elastic solids.* Note by corresp. E. ALMANSI ([1]).

Translated by D. H. Delphenich

The March-April issue of Nuovo Cimento contained some observations that were made by prof. Somigliana on a paper by Dr. G. Armanni, "Sulle deformazioni finite dei solidi elastici isotropi."

Without aligning myself with the opinion that was expressed by prof. Somigliana, permit me to dedicate a few words to that argument.

As is known, it follows from the postulates of the ordinary theory of elasticity that a simply-connected body upon which no volume forces or surface tensions act cannot be found in a state of *regular* deformation ([2]).

According to the note of prof. Somigliana: "It is upon the basis of this non-existence of deformations in the absence external forces in simply-connected bodies that the study of deformations of this nature can lead one to abandon the usual theory of elasticity and study deformations that are not quite small – or infinitesimal, as they say."

"Research of this type is, in fact, suggested by the more common observation that says that one can produce a state of elastic stress in a body no matter how the body is connected and without the aid of external forces. For example, wedge a thin layer with sharp edges into it. It will then be necessary to bring the theory into agreement with the facts."

"However, the explanation for that discord, which seems to be the primary objective of the paper in question, was not actually given at the time, so there is no point in abandoning the classical theory of elasticity, which is found to be perfectly adapted to the complete study of this new class of phenomena, as well…"

Now, the special deformations that defined the subject of the paper that was being addressed do not, in fact, enter into the class of phenomena to which prof. Somigliana alluded (distortions of either simply or multiply-connected bodies). They are deformations with *continuous displacements*, while distorsions always have discontinuity surfaces in their displacements.

The usual theory does not contemplate the case of an elastic body that is found in a state of regular deformation with continuous displacements while it is also not subject to external forces, and that observations will lead us to admit that such deformations can exist.

For example, consider an elastic plate whose middle surface is a spherical cap. Imagine that the plate is deformed while the edge is held fixed, and exert a pressure on

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([1]) Received by the academy on 26 September 1916.

([2]) V. Volterra, "Un teorema sulla teoria dell'elasticità," Rend. R. Acc. dei Lincei **14** (1905), February.

the convex surface in such a way that the convexity is required to switch to the opposite side. After the deformation has occurred, suppose that all of the external forces are eliminated. The plate will assume a state of equilibrium that is different from the original one. The displacements that relate to this state of deformation are obviously continuous; one sees no reason to believe that the six characteristics of the deformation are not regular.

We are then in the presence of one of the cases to which we alluded above.

In the paper in question, the inversion of an *entire* spherical shell, which is an operation that, in practice, cannot be performed without making a slit in the shell, making all of the shell pass through the slit (so to speak), and finally welding its two edges in such a manner as to make the points that coincided initially agree. However, the displacements will still be represented as continuous functions.

Another example of this is defined by the inversion of a cylindrical ring: Namely, the operation that consists in forcing the internal cylindrical surface to pass to the external one, and vice versa. An infinitude of other examples can be adduced from that.

One has not, therefore, presented a new aspect of the theory of distortions, but the study of a new class of phenomena that have no relationship to distortions.

The fact that it is necessary to abandon the concept of *infinitesimal deformations* in these phenomena certainly appears in the cited examples, and it is obvious that deformations of that nature cannot be equated to *infinitesimal* deformations in any way.