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Ordinary matter and radiating energy as different "phases" of one and the same basic material.

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Translated by D. H. Delphenich

In the present investigation, it will be shown how the difference between ordinary matter and a "light quantum gas" (cavity radiation) becomes smaller with increasing pressure. Protons, and especially electrons, increasingly take on the character of electrically-neutral light quanta. At extreme pressures, ordinary matter and cavity radiation will become identical, and in fact at any temperature, *even for an arbitrary low temperature*. One can raise the objection to the paradoxical-sounding assertion that the radiation pressure varies in proportion to T^4 ; therefore, an extremely high radiation pressure would be unthinkable at low temperature. However, a closer examination shows that this objection is based upon a misunderstanding. In conclusion, the meaning of the results obtained will be related to some questions in astrophysics.

It is known that a so-called "degeneracy" occurs for all gases (even for an ideal monoatomic gas) at sufficiently high pressure or sufficiently low temperature. For a non-degenerate ideal gas, the pressure is equal to p = nkT, where T means the absolute temperature, k is **Boltzmann**'s constant, and n is the number of free atoms in a cubic centimeter.

From Fermi's well-known theory, one has the relation:

$$p = nkT \left\{ 1 + \frac{1}{16} \cdot \frac{h^3 n}{(\pi m kT)^{3/2}} + \cdots \right\}$$
(1)

for a weakly-degenerate ideal monoatomic gas, in which *m* means the mass of an atom (*). However, this formula can be applied with sufficient accuracy only when the second term in the brackets is small in comparison to the first; i.e., $\frac{1}{16} \cdot \frac{h^3 n}{(\pi m k T)^{3/2}}$ is small in comparison to 1. In the contrary case, the degeneracy will be large, and formula (1) will

In the case of large degeneracy, from E. Fermi, one will have the relation:

be unusable then.

^{(&}lt;sup>*</sup>) **E. Fermi**, Zeit. Phys. **36** (1926), 911.

$$p = \frac{1}{20} \left(\frac{6}{\pi}\right)^{2/3} \frac{h^2 n^{5/3}}{m} + \frac{2^{4/3} \pi^{8/3}}{3^{5/3}} \cdot \frac{m n^{1/3} k^2 T^2}{h^2} + \dots$$
(2)

As far as the mean kinetic energy L of the atom is concerned (^{*}), one will always have:

$$L = \frac{3p}{2n}.$$
(3)

If the gas is found at the absolute zero point of temperature then one must set T = 0, and one will get from (2) that:

$$p = \frac{1}{20} \left(\frac{6}{\pi}\right)^{2/3} \frac{h^2 n^{5/3}}{m},$$
(4)

or more briefly:

$$p = A \frac{n^{5/3}}{m},\tag{5}$$

in which A means a constant. (3), (4), and (5) imply that:

$$L = \frac{3}{40} \left(\frac{6}{\pi}\right)^{2/3} \frac{h^2 n^{2/3}}{m} = \frac{3}{2} A \frac{n^{2/3}}{m}.$$
 (6)

We see that the degenerate gas must have a finite "zero-point pressure," as well as a finite "zero-point energy," even at absolute zero temperature. That zero-point energy poses as a temperature that the gas does not have, in reality. We would like to call this apparent temperature a "pseudo-temperature" and denote it by T_p , while the true temperature will be T_w ([†]). We shall refer to the temperature that our gas must have at the same *p* and *m* in order for it to be non-degenerate as the *pseudo-temperature*. We then define T_p by the equation:

$$p = n k T_p, \tag{7}$$

in which *p* and *n* have the same values that they have in (4). Naturally, one will have $T_p = T_w$ for a non-degenerate gas. We get from (3) and (7) that:

$$L = \frac{3}{2}k T_p \,. \tag{8}$$

One can eliminate n from (5) and (7), and that will give:

$$p = B m^{3/2} T_p^{5/2}, (9)$$

in which *B* is a constant.

^(*) **Fermi** denoted this mean kinetic energy by \overline{L} .

^{(&}lt;sup>†</sup>) Translator's note: the "w" comes from the German wahre, which means "true."

We expand our gas, which is found to be totally degenerate at absolute zero, by doing work on it adiabatically. Naturally, the true temperature T_w cannot drop any further as a result, since the gas was already found to be at absolute zero before the expansion. Doing work by the gas can happen only at the expense of its zero-point energy, which will be reduced as a result, and will thus produce a drop in the Pseudo-temperature T_p . T_w will, in turn, remain constant under adiabatic compression of the gas; namely, it will remain at 0 K. The work that is done by external forces will only be at the expense of raising the zero-point energy. The process will then be, at the same time, adiabatic and isothermal.

The zero-point velocities of the atom must get larger and larger for ever-increasing pressure. However, as long as these velocities are small in comparison to the speed of light, the mass *m* of an atom can be regarded as constant; i.e., it can be set to the rest mass m_0 . However, from (9), *p* will then be proportional to $T_p^{5/2}$. The true temperature of a non-degenerate, ideal, monoatomic gas will change by precisely the same law under adiabatic compression. We would therefore like to refer to the first phase of the total degeneracy, in which $m = m_0$ and the zero-point velocities of the atoms are small in comparison to the speed of light, as the *gaseous phase* of the total degeneracy.

At extraordinarily-high pressures, the zero-point velocities of the atoms will be comparable to the speed of light. In that case, one must set $m = m_0 / \sqrt{1 - \beta^2}$. We will then have:

$$L = c^{2} m_{0} \left(\frac{1}{\sqrt{1 - \beta^{2}}} - 1 \right) = c^{2} (m - m_{0})$$
(10)

for the mean kinetic zero-point energy.

The second term in parentheses can be neglected for extreme zero-point velocities, and that will give:

$$L = c^2 m. \tag{11}$$

One gets from (11) and (6) that:

$$c^2 m = \frac{3}{2} A \frac{n^{2/3}}{m}$$

or:

$$m = \left(\frac{3}{2}A\right)^{1/2} \frac{n^{1/3}}{c}.$$

When one introduces the value of m that was just obtained into (5) that will yield:

$$p = \left(\frac{2A}{3}\right)^{1/2} c \, n^{4/3}.$$
 (12)

We eliminate n from (12) and (7), and thus obtain:

$$p = \left(\frac{3}{2A}\right)^{3/2} \frac{k^4}{c} T_p^4 = k T_p^4,$$
(13)

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in which K is a constant. It is well-known that the pressure of cavity radiation exhibits precisely the same dependency upon temperature. Thus, we would like to refer to the second phase of the total degeneracy (in which m is much larger than m_0 , and the zeropoint velocities of the atoms approach the speed of light) as the *light phase* of the total degeneracy.

But why is the behavior of our degenerate gas so similar to the behavior of cavity radiation? The well-known theory of **Louis de Broglie** gives us an answer to this question.

According to that theory, light quanta are particles of exceptionally small rest mass m_0 . The velocities of these light particles come very close to the speed of light *c*, but do not attain it. Therefore, when one is close to the speed of light, even the most minimal change in velocity is already coupled with a large change in kinetic energy. For example, the energy quantum of violet light is considerably larger than that of red, since violet light is somewhat faster than red light. However, that difference in velocity is too small to be accessible to direct measurement. **Louie de Broglie** believed that the rest mass m_0 of a light quantum could be estimated to be around at most 10^{-50} g (*). Total degeneracy must occur only very slightly for such a small mass of the light quantum.

We get from (6) and (10) that:

$$c^{2}(m-m_{0}) = \frac{3}{40} \left(\frac{6}{\pi}\right)^{2/3} \frac{h^{2}n^{2/3}}{m}$$

or:

$$m^2 - m_0 m - \frac{3}{40} \left(\frac{6}{\pi}\right)^{2/3} \frac{h^2 n^{2/3}}{c^2} = 0.$$

We set $h = 6.55 \times 10^{-27}$ and $c = 3 \times 10^{10}$ here and get:

$$m^2 - m_0 m - 5.5035 \times 10^{-75} n^{2/3} = 0$$

in that way. The solution of this quadratic equation yields:

$$m = \frac{m_0}{2} + \left[\frac{m_0^2}{4} + 5.5035 \times 10^{-75} n^{2/3}\right]^{1/2}.$$
 (14)

In the case of protons, one must set $m_0 = 1.66 \times 10^{-24}$, for electrons, one has $m_0 = 9 \times 10^{-28}$, and for light quanta (according to **L. de Broglie**), $m_0 = 10^{-50}$.

For the density of a gas, one obviously has the equation:

$$\rho = n m \qquad \text{g cm}^{-3}. \tag{15}$$

If we introduce the value of *h* into (4) then we will get:

^(*) **Louis de Broglie**, Phil. Mag. (6) **47** (1924), 447. In another place, **de Broglie** found the upper limit for m_0 to be around 10^{-44} g, but he still thought it was possible that m_0 could be even smaller [Ann. Phys. (Leipzig) (10) **3** (1925), 79.]

$$p = 3.3021 \times 10^{-54} \cdot \frac{n^{5/3}}{m}$$
 dyn cm⁻². (16)

As far as the pseudo-temperature T_p is concerned, (7) implies that:

$$T_p = \frac{p}{kn} = \frac{p}{1.372 \times 10^{-16} n}.$$
(17)

When m_0 and n are given, one can calculate m from (14). However, one can then also calculate ρ , p, and T_p from (15), (16), and (17), resp.

<i>n</i> (number of	p (zero-	point pressure	in dyn cm ⁻²)	ρ (zero-point density in g cm ⁻³)		
particles per 3^{3}	Proton	Electron	Light quantum	Proton gas	Electron	Light quantum
cm)	gas	gas	gas		gas	gas
10 ⁻⁶⁰	1.99×10^{-150}	3.67×10^{-127}	3.30×10 ⁻¹⁰⁴	1.66×10^{-84}	9.00×10^{-88}	1.00×10^{-110}
10 ⁻⁴⁵	1.99×10^{-105}	3.67×10^{-102}	3.30×10 ⁻⁷⁹	1.66×10^{-69}	9.00×10^{-73}	1.00×10^{-95}
10^{-42}	1.99×10^{-100}	3.67×10 ⁻⁹⁷	3.28×10 ⁻⁷⁴	1.66×10^{-66}	9.00×10^{-70}	1.01×10 ⁻⁹²
10 ⁻³⁹	1.99×10 ⁻⁹⁵	3.67×10 ⁻⁹²	2.37×10 ⁻⁶⁹	1.66×10^{-63}	9.00×10 ⁻⁶⁷	1.39×10 ⁻⁸⁹
10 ⁻³⁶	1.99×10^{-90}	3.67×10 ⁻⁸⁷	4.16×10 ⁻³⁵	1.66×10^{-60}	9.00×10 ⁻⁶⁴	7.94×10 ⁻⁸⁶
10 ⁻²⁴	1.99×10^{-70}	3.67×10 ⁻⁶⁷	4.45×10 ⁻⁴⁹	1.66×10^{-48}	9.00×10 ⁻⁵²	7.42×10 ⁻⁷⁰
10 ⁻¹²	1.99×10^{-50}	3.67×10^{-47}	4.45×10 ⁻³³	1.66×10^{-36}	9.00×10^{-40}	7.42×10 ⁻⁵⁴
1	1.99×10^{-30}	3.67×10^{-27}	4.45×10^{-17}	1.66×10^{-24}	9.00×10^{-28}	7.42×10^{-38}
10 ⁶	1.99×10^{-20}	3.67×10^{-17}	4.45×10 ⁻⁹	1.66×10^{-18}	9.00×10^{-22}	7.42×10^{-30}
10^{12}	1.99×10^{-10}	3.67×10^{-7}	4.45×10 ⁻¹	1.66×10^{-12}	9.00×10^{-16}	7.42×10 ⁻²²
10^{18}	1.99	3.67×10^3	4.45×10^{7}	1.66×10^{-6}	9.00×10^{-10}	7.42×10^{-14}
10 ²⁴	1.99×10 ¹⁰	3.67×10 ¹³	4.45×10 ¹⁵	1.66	9.00×10 ⁻⁴	7.42×10 ⁻⁶
10 ²⁷	1.99×10 ¹⁵	3.64×10 ¹⁸	4.45×10 ¹⁹	1.66×10^3	9.00×10 ⁻¹	7.42×10 ⁻²
10^{30}	1.99×10^{20}	2.51×10^{23}	4.45×10 ²³	1.66×10^{6}	1.32×10^{3}	7.42×10^{2}
10 ³³	1.99×10^{25}	4.19×10 ²⁷	4.45×10 ²⁷	1.66×10^9	7.88×10^{6}	7.42×10^{6}
10 ³⁶	1.99×10^{30}	4.45×10 ³¹	4.45×10 ³¹	1.66×10^{12}	7.42×10^{10}	7.42×10^{10}
10 ³⁹	1.99×10^{35}	(4.45×10^{35})	4.45×10^{35}	1.94×10^{15}	(7.42×10^{14})	7.42×10^{14}
10 ⁴²	1.99×10 ³⁹	(4.45×10^{39})	4.45×10 ³⁹	8.29×10 ¹⁸	(7.42×10^{18})	7.42×10 ¹⁸
10 ⁴⁵	1.99×10^{43}	(4.45×10^{43})	4.45×10 ⁴³	7.50×10^{22}	(7.42×10^{22})	7.42×10^{22}
1048	(4.45×10^{47})	(4.45×10^{47})	4.45×10 ⁴⁷	(7.42×10^{26})	(7.42×10^{26})	7.42×10^{26}

Table 1.

In Table 1, I have calculated the zero-point pressure and zero-point density for the proton gas, electron gas, and light quantum gas (whose true temperatures T_w are assumed to be 0 K everywhere). That table shows that for small values of *n*, our three gases have very different pressures and densities. However, the larger *n* becomes, the smaller the differences that our gases will show in regard to *p* and ρ . The difference will vanish completely for very large *n*. Hence, e.g., 10^{36} electrons per cubic centimeter and 10^{36} light quanta per cubic centimeter will yield the same zero-point pressure (4.45×10^{31} dyn cm⁻²) and the same zero-point density (7.42×10^{10} g cm⁻³).

I have worked out the pseudo-temperatures in Table 2. We also see the same pattern here: Our three gases will exhibit ever-narrower differences with increasing n.

<i>n</i> (number of	T_p (pseudo-temperature)			n/T_p^3		
cm^{3}	Proton	Electron	Light quantum	Proton gas	Electron	Light quantum
ciii)	gas	gas	gas		gas	gas
10 ⁻⁶⁰	1.45×10^{-54}	2.67×10^{-51}	2.41×10^{-28}	3.28×10^{101}	5.23×10 ⁹¹	7.17×10^{22}
10 ⁻⁴⁵	1.45×10^{-44}	2.67×10^{-41}	2.41×10^{-18}	3.28×10^{86}	5.23×10 ⁷⁶	7.17×10^{7}
10 ⁻⁴²	1.45×10^{-42}	2.67×10^{-39}	2.39×10^{-16}	3.28×10 ⁸³	5.23×10 ⁷³	7.29×10 ⁴
10 ⁻³⁹	1.45×10^{-40}	2.67×10^{-37}	1.73×10^{-14}	3.28×10^{80}	5.23×10 ⁷⁰	1.95×10^{2}
10 ⁻³⁶	1.45×10^{-38}	2.67×10^{-35}	3.03×10 ⁻¹⁵	3.28×10 ⁷⁷	5.23×10 ⁶⁷	35.84
10 ⁻²⁴	1.45×10^{-20}	2.67×10^{-27}	3.24×10 ⁻⁹	3.28×10^{65}	5.23×10 ⁵⁵	29.29
10 ⁻¹²	1.45×10^{-22}	2.67×10^{-19}	3.24×10 ⁻⁵	3.28×10^{53}	5.23×10 ⁴³	29.29
1	1.45×10^{-14}	2.67×10^{-11}	3.24×10^{-1}	3.28×10 ⁴¹	5.23×10 ³¹	29.29
10 ⁶	1.45×10^{-10}	2.67×10^{-7}	3.24×10^{1}	3.28×10^{35}	5.23×10 ²⁵	29.29
10 ¹²	1.45×10^{-6}	2.67×10^{-3}	3.24×10^{3}	3.28×10 ²⁹	5.23×10 ¹⁹	29.29
10 ¹⁸	1.45×10^{-2}	2.67×10^{1}	3.24×10^{5}	3.28×10^{23}	5.23×10 ¹³	29.29
10 ²⁴	1.45×10^{2}	2.67×10^5	3.24×10^{7}	3.28×10^{17}	5.23×10^{7}	29.29
10 ²⁷	1.45×10^4	1.87×10^{7}	3.24×10^{8}	3.28×10^{14}	5.34×10^{4}	29.29
10 ³⁰	1.45×10^{6}	1.83×10^{9}	3.24×10 ⁹	3.28×10 ¹¹	1.64×10^{2}	29.29
10^{33}	1.45×10^{8}	3.05×10^{10}	3.24×10^{10}	3.28×10^{8}	35.13	29.29
10^{36}	1.45×10^{10}	3.24×10^{11}	3.24×10^{11}	3.30×10^{5}	29.29	29.29
10 ³⁹	1.24×10^{12}	(3.24×10^{12})	3.24×10^{12}	5.26×10^{2}	(29.29)	29.29
10 ⁴²	2.90×10^{13}	(3.24×10^{13})	3.24×10 ¹³	40.94	(29.29)	29.29
1045	3.21×10 ¹⁴	(3.24×10^{14})	3.24×10 ¹⁴	30.28	(29.29)	29.29
1048	(3.24×10^{15})	(3.24×10^{15})	3.24×10 ¹⁵	(29.38)	(29.29)	29.29

Table 2.

Some of the numbers in both tables are in parentheses. That means that one can perform the corresponding compression only while decreasing the proper volume of the particles. (Of course, I have not taken the **Lorentz** contraction into account in that.) However, I have not placed any parentheses for the light quanta, since our knowledge of the "proper volume" of the light quantum seems uncertain to me.

Finally, I have further calculated the values of n/T_p^3 . However, n/T_p^3 will be very large and variable for small values of n. Constancy will come about when m is significantly larger than m_0 ; i.e., when the "light phase" of the total degeneracy begins. We see that one will enter into the "light phase" for light quantum gases even for very small values of n. Therefore, we will be unable to observe the "gas phase" for light quanta.

The constant value that n/T_p^3 assumes for sufficiently large *n* is the same for all three of our gases – namely, 29.29. We can also write:

$$n = 29.29 \cdot T_{p}^{3} \,. \tag{18}$$

That equation will become very remarkable when we calculate the number of light quanta that are found in a cubic centimeter according to **Planck**'s formula and obtain (*):

$$n = 8\pi \left(\frac{kT}{hc}\right)^3 \int_0^\infty \frac{x^2 dx}{e^x - 1} = 20.62 \ T^3.$$
(19)

Therefore, our "zero-point theory" leads to a formula that is entirely similar to one in the usual quantum theory of radiation. Only the coefficients are different, but not by very much. In any event, they have entirely similar orders of magnitude.

Due to the similarity between formulas (18) and (19), I believe that I can draw the conclusion that: When one sets the true temperature of any cavity radiation equal to 0 K, considers the radiation pressure to be the zero-point pressure of a totally degenerate gas, the energy of the light quantum to be the zero-point energy, and the so-called temperature of the cavity radiation to be purely pseudo-temperature, one will indeed introduce an error by such assumptions, but not an especially large one.

Our three gases differ from each other by their charges: Protons have a positive charge, electrons have a negative one, and light quanta are electrically neutral. However, we will soon see that this difference will also become unnoticeable for large values of n.

We consider two electrons that move in parallel paths with equal velocities and in the same direction. Our two electrons represent two parallel and equally-directed electrical currents that attract each other, as one knows; the electrostatic repulsion will be reduced in that way. If the velocity of the electrons differs from the speed of light only slightly then the mutual electrostatic repulsion of the electrons will be almost outweighed by the aforementioned mutual attraction of the "elementary currents."

Both electrons might now move along parallel paths, but in the opposite directions. We will then have two parallel and oppositely-directed currents before us that repel each other, which will increase the effect of the electrostatic repulsion. However, one cannot forget that for a very fast-moving electron the lines of force will concentrate around the equatorial plane (i.e., the one perpendicular to the direction of motion). For extreme velocities, all lines of force will condense into a very thin "equatorial layer." Hence, it is only when the other electron passes through this thin layer that it will be subject to influence of the first electron. However, since the time of that passage is extremely short, the electron in question must experience an almost-instantaneous change in velocity, so it must radiate energy. In extreme cases, the radiated energy must be larger than the energy that is supplied by the electrostatic repulsion of the electron, which is naturally impossible. One must then infer the conclusion that our extremely fast electrons cannot affect each other at all. That is precisely the argument upon the basis of which W. F. G. **Swann** was led to suggest that extremely fast electrons could not be capable of ionizing (^{**}). Finally, we assume that the two electrons move along the line that connects them. In that case as well they will exert no noticeable effect upon each other, since their lines of force will indeed concentrate in the "equatorial plane"; i.e., the one perpendicular to their direction of motion.

^(*) One can find such a calculation in, e.g., A. S. Eddington, *The Internal Constitution of the Stars*, Cambridge, 1926, pp. 55, *et seq*.

^{(&}lt;sup>**</sup>) **W. F. G. Swann**, Phil. Mag. (6) **47** (1924), 309.

Naturally, what was just said will also relate to the mutual electrostatic influence of extremely fast protons, as well as to the interactions between protons and electrons. If we have a mixture of electrons and protons then the electrostatic forces will play no appreciable role for extreme velocities. However, should the "molecular motions" of these electrons and protons not be accompanied by a significant amount of electromagnetic radiation? That question must be answered in the negative, since we have in fact assumed a totally-degenerate gas for which the "molecular motions" are assumed to be "zero-point motions"; however, the latter are always found to take place with no electromagnetic radiation. One recalls the electron gas in metals: The electrons perform their motions even for 0 K, but it would be impossible for them to be accompanied by radiation, since otherwise the temperature of the metal in question would have to drop below 0 K. The electrons then behave in that regard as if they were electrically neutral (^{*}).

Both of our tables show that for sufficiently large n, light quantum gases will exhibit the same pressure, density, and pseudo-temperature as electron gases and proton gases, and therefore, so will a mixture of the last two. Now, it can be shown that such a mixture cannot be distinguished from a light quantum gas in regard to its electromagnetic effects. However, in the final analysis, all ordinary matter consists of electrons and protons (**). We then come to the conclusion that *at sufficiently high pressures, ordinary matter and cavity radiation (light quantum gas) will become identical in every respect, even at absolute zero.* The electrons and protons cannot be distinguished from light quanta, nor can the gas pressure cannot be distinguished from radiation pressure.

Such a concept seems highly paradoxical, since extreme radiation pressures will appear only for extremely high temperatures. How could it be possible then for cavity radiation to exert an enormous radiation pressure at absolute zero? We will soon see that we also encounter an entirely similar "paradox" for ordinary fluids and their evaporation; e.g., water. A simultaneous appearance of the fluid and vapor phase of water (or any other fluid) will be possible only when the pressure does not exceed the so-called critical pressure; i.e., when it is smaller than 217.5 atm (for water). If the pressure is also only somewhat larger - e.g., 218 atm - then a meniscus will not be obtained at any temperature. We will then always have only one phase before us that can just as well be regarded as fluid water as water vapor. From that standpoint, we will be justified in regarding water at room temperature and 218 atm as a highly compressed vapor.

We further propose that our engineering tools are so limited that we would not be in a position to achieve the "critical point" for water. In addition, all experiments concerning the evaporation of water are possible only with fluid water at present. We will then come to the conclusion that the vapor pressure in all cases is a single-valued function of temperature and increases very rapidly with it. If one then wishes to announce that water at 218 atm can be regarded as a vapor even at room temperature (i.e., $+15^{\circ}$ C) then that statement would have to seem paradoxical, since the vapor pressure of water at $+15^{\circ}$ C is

^(*) For that reason, it does not seem certain at all to me that the usual light quanta are electrically neutral in the strict sense. Perhaps there are positive and negative light quanta that combine into "molecules" under especially extreme conditions.

^(**) Whether the latter can cling together into atomic nuclei seems to be questionable to me; however, I cannot go into that matter any further here.

not 218 atm at all, but only 12.8 mm Hg. That is precisely the same "paradox" that came about in the aforementioned case of cavity radiation.

If *r* means the heat of vaporization then one will have the relation:

$$r = \frac{T}{24.29} \cdot \frac{dp}{dT} (v' - v),$$
 (20)

in which v' means the specific volume of the vapor (in liters), and v is that of the fluid (*). We see: The less that v' differs from v, the smaller will be the heat of vaporization, and the easier it will be to go from the fluid phase to the vapor phase. Now, we see from Table 1 that with increasing pressure, the density of the light quantum gas differs less and less from the density of the "material phase." Naturally, the same thing is also true for the specific volumes. We then have grounds to assume that a conversion of matter into cavity radiation can be produced much more easily and as a result of much more minor causes at high pressure that at lower pressure.

However, should it be possible for a pressure to reign in a stellar interior that is so high that no difference exists between an electron gas and a light quantum gas there? That question must be answered in the affirmative only when the mass of the star is sufficiently concentrated towards the center. Kerr Grant calculated on the grounds of the possibility that the central density of the star is 5×10^7 times than the mean density (**). From **G. I. Pokrowski**, the maximum density of matter is $\rho = 4 \times 10^{13\pm1}$ g cm⁻³ (***). When pressures of that order of magnitude prevail in the stellar interior, many question of astrophysics must take on a new character. For example, the question of the absorption coefficients of the electrons will become meaningless for radiation in the stellar interior, since the difference between electrons and light quanta will vanish there. The wellknown question of the conversion of matter into radiant energy will also be meaningful only when the pressure is not excessively large, so when matter and cavity radiation can still exist as separate phases (****).

All of our calculations up to now were based upon the assumption that the true temperature was equal to 0 K; all of the calculations actually became very simple then. In reality, the situation is much more complicated, since real stars often have a very high "true" temperature. I regard it as possible that there is also a critical temperature that

See, e.g., Landoit-Börnstein, Phys.-chem. Tabellen, 5th ed., Berlin, 1923, pp. 1580.

Kerr Grant, Nature 118 (1926), 373.

<sup>Zeit. Phys. 49 (1928), 588.
Of course, in order for the protons to also be identical with light quanta, one must assume much</sup> larger densities in the stellar interior than were possible for even **Pokrowski**. The question of whether or not such extremely high densities are nonetheless possible in stellar interiors might remain in the background for the time being. One cannot forget in that regard that such densities would require the contraction of the proper volume of the electrons (and ultimately even protons). (On the other hand, one must also consider the Lorentz contraction.) However, I cannot go further into this interesting question of the compressibility of electrons and protons here. Nevertheless, such extremely high pressures are also not at all necessary for the conversion of protons into light quanta. Even for much smaller pressures, v' - v will already have narrowed so much that, from (20), the transition from the "proton phase" to the "light quantum phase" will not require an especially violent impulse. If the electrons – or especially the protons – should split into smaller particles then the conversion of matter into radiant energy could take place for correspondingly minor pressures.

relates to the transition from matter to radiant energy, along with the critical pressure. However, I cannot go further into that question here.

It is known that from time to time a phase will happen to be unstable. For example, under some circumstances, a fluid can be heated beyond its boiling point. However, for such an overheated fluid, even the slightest perturbation would suffice to produce an explosive boiling (often with shattering of the vessel). According to G. Krebs, there exists an essential difference between that explosive boiling and a normal vigorous boiling. Even the "thin-walled piston" does not burst under normal boiling, but it might proceed as violently as it will (^{*}). From the analogy, one can suspect that similar volatile states are also possible for the conversion of matter into radiant energy. G. Krebs regarded it as having been established that "by the gradual reduction in pressure, as indeed can take place for the cooling of steam boilers when they are temporarily allowed to rest, slightly high boiling delays can exist, and as a result, explosions, and indeed with all of the other consequences." (**) An astronomical analogue for such steam boilers is defined by the red dwarfs, which in fact also cool down while the radiation pressure drops in their interiors (precisely like the vapor pressure in the interior of a cooling boiler). Should that not lead to an explosive transition from the "overheated" matter into radiant energy under certain circumstances? It would be tempting to explain the flaring up of "new stars" in that way.

As far as the cooling steam boiler of **G. Krebs** is concerned, the cooling would be accompanied by a single large explosion under very opportune conditions. Under less opportune conditions, the explosion would happen much sooner, such that with further cooling, a second, third, etc, one can take place. Instead of a single large explosion, we would have a series of smaller and less violent ones. Should it not be possible to explain the irregular changes of light from red dwarfs in an analogous way? It that were true then there would be no qualitative difference between the "new" stars and the irregularly changing ones, but only a quantitative one.

^(*) G. Krebs, Pogg. Ann. 138 (1869), 448.

^{(&}lt;sup>***</sup>) *Loc. cit.*