"Sulle dinami in involuzione," Rend. dell'Accad. scienze fisiche Napoli **8** (1869), 166-167.

On dynames in involution

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The action of many forces on a system of invariable form is, as is known, defined by six quantities, namely, the sum of the components of the given forces parallel to three orthogonal axes and the sum of their moments with respect to the same axes: These six quantities constitute what PLÜCKER called the *coordinates of a dyname*. One can replace them with the six forces that act along the edges of a tetrahedron and form a system that is equivalent to the given force system. Now, in this letter we propose to examine the properties of dynames that take the form of their coordinates verifying one or more homogeneous equations of first degree.

The fundamental concept in this study is the consideration of a bilinear form in the coordinates of two dynames that represents the sum of all the tetrahedra that can be formed by taking two of their opposite edges to be the lines that represent an arbitrary force of a system that is equivalent to the first dyname and an arbitrary force of a system that is equivalent to the second dyname, respectively. When such an expression is annulled, one calls the two dynames *mutually harmonic*. Now, all of the dynames whose coordinates verify 1, 2, 3, 4, or 5 given homogeneous equations of first degree will be harmonic with respect to any assigned dynames whatsoever.

The coordinates of the dynames, when subjected to the conditions above, will be expressed *syzygetically* with the coordinates of the given dyname, and will be 5, 4, 3, 2, or 1 in number. When one varies the multipliers in these syzygetic relations, all of the dynames that one obtains will be called quintuples, quadruples, triples, pairs, or simply *in involution*. To each such set of dynames in involution there corresponds another *complementary* multiplicity, namely, a simple, pair, triple, quadruple, or quintuple. The two *associated* involutions are such that whereas any dyname of the first involution is harmonic with respect to all the dynames of the second one, vice versa, any dyname of the second involution will be harmonic with respect to all of the dynames of the first.

Consider some dynames in a series of dynames of two associated involutions that admit a resultant. The lines along which the resultants act belong to the first or second series, and are the zero-moment axes with respect to the dynames of the second or first series, resp., and the lines of action of the first and second resultants intersect each other.

Examining separately the various cases of involution, the peculiarities that relate to complexes with zero-moment axes and lines that are *conjugate* with respect to the various dynames of the involution are discussed in the memoir, as well as the special properties of the lines of action of the resultants.

Finally, among the dynames of an involution, one finds ones that annul the coordinates – that is to say, dynames in equilibrium. One finds that this cannot take place unless one annuls all of the determinants that appear in a certain rectangular matrix. Depending upon the order of the determinant that one annuls, an involution of dynames in equilibrium that has a *lower* multiplicity will be *contained* in the given involution, and that order is the amount by which the multiplicity of the given involution must be *reduced* in order to get that multiplicity. If one supposes that there are dynames in equilibrium in an involution then the lines of action of the resultants that are contained in the series of given dynames will be subject to the laws that pertain to the involutions of lower multiplicity.