"Über die Quantentheorie der Strahlung," Zeit. Phys. **24** (1924), 69-87.

On the quantum theory of radiation

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Without deviating from the classical law of the propagation of radiation in empty space, the goal of this article is to arrive at a reasonable description of optical phenomena that is closely related to the quantum-theoretic interpretation of spectra. In that way, the continuous radiative phenomena will be linked with the discontinuous atomic processes by the laws of probability using **Einstein**'s process. However, by appealing to virtual oscillators, which can be associated with the discontinuous processes according to the correspondence principle, those laws will be interpreted in a somewhat-different way from what one usually does.

Introduction. – When one tries to interpret the processes of interaction between radiation and matter theoretically, one will be led to consider two different, apparently contradictory, pictures for them. On the one hand, the interference phenomena upon which the action of all optical instruments depend essentially require a continuous picture of the same type as the one that is included in the wave theory of light, especially in the form in which that theory was developed on the basis of classical electrodynamics. On the other hand, the phenomena of the exchange of energy and quantities of motion between radiation and matter to which the observation of optical phenomena will ultimately lead back require a picture that includes essentially discontinuous processes. Thus, the aforementioned phenomena have led to the development of the theory of light quanta, whose most paradoxical form even contradicts the wave-like constitution of light. With the current state of science, it does not seem likely that it would be possible to free oneself from the formal character of the interpretation of atomic processes. That is especially expressed by the fact that for the time being, one skips over the detailed description of the mechanics of the discontinuous processes that the quantum theory of spectra refers to as transitions between stationary states. Nonetheless, as we will show in the present treatise, it seems possible that in conjunction with the correspondence principle, a reasonable picture of optical phenomena can be devised when the discontinuous processes in the atom are coupled with the continuous radiation field in a way that deviates somewhat from the conventional one. The essentially-new assumption that is introduced in **§ 2** that the atom is already in a position to communicate with distant atoms, even before the appearance of a transition process, by a virtual radiation field goes back to **Slater** ([1]). Originally, he was of the opinion that one could achieve a better harmony between the physical picture of the electrodynamical theory of light and the quantum theory of light in which the

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([1]) **J. C. Slater**, Nature **113** (1924), pp. 307.

emission and absorption transitions in communicating atoms would seem to be pair-wise coupled together. However, it was emphasized by **Kramers** that the aforementioned assumption, rather than leading to the formulation of an intimate coupling between those processes, would compel one to imagine that the transition processes in distant atoms would be independent of each other to a greater degree than had been assumed up to now. The present work represents the result of a mutual discussion with the author on the meaning that these assumptions might possibly have for the further development of quantum theory. It can be regarded, in some respects, as a supplement to the recently-appeared first installment of a paper by **Bohr** on the principles of quantum theory, in which several of the problems that are touched upon here are discussed more thoroughly ([1]).

§ 1. The principles of quantum theory. – The electromagnetic theory of light gives not only a wonderfully-suited picture of the propagation of radiation through free space, but it has also proved to be well-suited to the interpretation of phenomena that are connected with the interaction of radiation and matter at the macroscopic scale. One can then achieve a general description of the phenomena of emission, absorption, refraction, scattering, and dispersion on the basis of the assumption that the atom includes electrically-charged particles that can perform harmonic oscillations about stable equilibria, and which can exchange energy and quantity of motion with the radiation field according to the laws of classical electrodynamics. On the other hand, the aforementioned phenomena are known to point to a number of approaches that contradict the consequences of classical electrodynamics. One such contradiction was first posed beyond any doubt in the case of the law of heat radiation. Starting from the classical picture of the emission and absorption of radiation by a harmonic oscillator, **Planck** found that agreement between the experiments on heat radiation could be achieved only by the introduction of a novel assumption that started from the fact that one should include only certain states of the oscillating particles in the equilibrium statistical distribution. The energy in those states would be found to be equal to an integer multiple of the quantum $h\omega$, in which ω is the natural frequency of the oscillator, and h is a universal constant. As **Einstein** could show, an immediate support for the experiments on the specific heat of solid bodies would be achieved independently of the radiative phenomena. At the same time, that author proposed his well-known "theory of light quanta," according to which radiation should not propagate like the continuous wave-trains of the classical theory of light, but rather as discrete units that should include an energy of $h\nu$ in a small spatial region, where h means **Planck**'s constant, and ν means the quantity that is interpreted as the number of waves that pass per unit time in the classical picture. Although the great heuristic value of that hypothesis emerged clearly in the confirmation of **Einstein**'s predictions in regard to the photoelectric effect, the theory of light quanta cannot be regarded as a satisfactory solution to the problem of the propagation of light, which is already brought to light by the fact that the "frequency" ν of the radiation that appears in that theory is defined by experiments with interference phenomena. However, those phenomena obviously require a wave-like constitution of light in order to interpret them.

In defiance of that, the fundamental difficulties in the ideas of quantum theory have shown that it is possible to employ those ideas in conjunction with results that are derived elsewhere

([1]) **N. Bohr**, "Über die Anwendung der Quantentheorie auf den Atombau. I. Die Grundpostulate der Quantentheorie," Zeit. Phys. **13** (1923), pp. 117. That paper, which also includes more detailed references to the literature, will always be cited as G. d. Q. in what follows.

concerning atomic structure in order to interpret the investigations into the emission and absorption spectra of the elements. That interpretation is based upon the fundamental postulate that an atom can exist in a number of distinguished states, namely, the so-called "stationary states," to which one ascribes a curious degree of stability that the concepts of classical electrodynamics are not in a position to account for. That stability emerges from the fact that a change in the state of the atom must always consist of a process of complete transition from one stationary state to another. For optical phenomena, that postulate is coupled with the further assumption that in the event that a transition between two stationary states is accompanied by the emission of radiation, that radiation will consist of a train of harmonic waves whose frequency will be determined by the relation:

$$h\,\nu = E_1 - E_2\,, \tag{1}$$

in which E_1 and E_2 mean the values of energy for the atom in its initial (final, resp.) state. It is further assumed that the inverse transition process can take place as a consequence of being illuminated by light of just the same frequency. The applicability of those assumptions for the interpretation of the spectra of the elements is essentially due to the fact that in many cases, it has proved to be possible to calculate the energy values for the stationary states of an isolated atom with the help of simple rules on the basis of motions that are described by the ordinary laws of electrodynamics to a high degree of approximation (G. d. Q., Chap. I, **§ 1**). By contrast, the concepts of electrodynamics do not allow us to describe the details of the mechanism of the transitions.

As far as the occurrence of transition processes are concerned, with the current state of science, it seems necessary to content oneself with probabilistic considerations. Such considerations were introduced by **Einstein** ([1]), who succeeded in giving an especially-simple derivation of **Planck**'s law of blackbody radiation under the assumption that an atom in a given stationary state would possess a certain probability for it to go "spontaneously" to a stationary state of lower total energy in a unit of time, and that an atom under the influence of external radiation of a suitable frequency would have a certain probability of making an "induced" transition to another stationary state with greater or lesser total energy. Combined with the demand of thermal equilibrium between the radiation field and matter, **Einstein** further reached the conclusion that the exchange of energy under a transition process would always be coupled with an exchange of quantity of motion of magnitude $h\nu / c$, just as would be the case if the transition were accompanied by the emission or stopping of a small unit that possessed the speed of light c and the total energy $h\,\nu$. He could conclude that the direction of that quantity of motion for the induced transition would be the same as the direction of propagation of the emitted light waves, but that for the spontaneous transition, the direction of the quantity of motion would be distributed according to the laws of probability. That result, which was regarded as an argument for the physical reality of light quanta, has recently found an important application to the explanation for the remarkable appearance of a change in the wavelength of radiation that is scattered by free electrons, which was brought to light by the investigation of **A. H. Compton** ([2]) into the scattering of Röntgen radiation into light. The application of probabilistic considerations to the problem of the equilibrium between free electrons

([1]) **A. Einstein**, Phys. Zeit. **18** (1917), pp. 121.
([2]) **A. H. Compton**, Phys. Rev. **21** (1923), pp. 207. See also **P. Debye**, Phys. Zeit. **24** (1923), pp. 161.

and radiation, which led to that discovery, was treated successfully in recent times by **Pauli** ([1]), and the formal analogy between his results and the laws that the transitions between stationary states of atoms obey was emphasized by **Ehrenfest** and **Einstein** ([2]).

If one overlooks the fundamental difference between the quantum-theoretic picture of atomic processes and the picture that is based upon the ordinary concepts of electrodynamics then the former must ultimately appear to be a natural generalization of the latter. That is especially brought to light by the demand that the former should lead to agreement with the observations in the limit of the classical theory where we consider phenomena that depend upon the total statistical effect of a large number of atoms, and in which we have to deal with stationary states where the difference between neighboring states is relatively small. For the case of emission and absorption of spectral lines, that connection between the two theories has led to the postulation of the "correspondence principle," which demands the general association of each of the possible transitions between two stationary states with a certain harmonic oscillatory component in the electric moment of the atom (G. d. Q., Chap., **§ 2**). That principle has made it possible to lay the foundation for the evaluation of transition probabilities, and in that way, to bring the problem of the intensity and polarization of spectral lines into a close relationship with the motion of electrons in the atom.

The correspondence principle has given rise to the comparison of the reaction of an atom to a radiation field with the reaction to such a field as one would expect from classical electrodynamics for a number of "virtual" harmonic oscillators whose frequencies are equal to the ones for the different possible transitions to other stationary states of well-defined frequencies according to equation (1) (G. d. Q., Chap. III, **§ 3**). One such picture was used by **Ladenburg** in his attempt to numerically connect the experimental results on dispersion with the considerations of transition probabilities. Even in the case of the interaction between free electrons and radiation, the possibility of applying such considerations is suggested by the analogy between the change in wavelength of scattered radiation and the classical Doppler effect of radiation from a moving source that was emphasized by **Compton**.

Although the correspondence principle makes it possible to draw conclusions about the mean lifetime of an atom in a given stationary state by evaluating the transition probabilities, on the other hand, the problem of the time interval inside of which the radiation that is coupled with a transition takes place has given rise to great difficulties. That difficulty, together with other known paradoxes in quantum theory has even strengthened the doubt that has been raised from various directions ([3]) about whether the interaction between matter and radiation can be interpreted at all by means of a causal space-time description of the kind that has been used to interpret natural phenomena up to now (G. d. Q, Chap. III, **§ 1**). However, as was mentioned in the introduction, it seems possible that a marked advance in the interpretation of the observed radiative phenomena could be achieved when those phenomena are coupled with the stationary states and the transitions between them in a way that differs somewhat from the conventional one without abandoning the formal character of the theory.

([1]) **W. Pauli**, Zeit. Phys. **18** (1923), pp. 272.

([2]) **P. Ehrenfest** and **A. Einstein**, Zeit. Phys. **19** (1924), pp. 301.

([3]) Cf., **O. W. Richardson**, *The Electron Theory of Matter*, 2nd ed., Cambridge, 1916, pp. 507, which was perhaps where such a conception of things was expressed clearly for the first time.

§ 2. Radiation and transition processes. – We will assume that a given atom in a certain stationary state can continually communicate with other atoms, and indeed by means of a space-time mechanism that is virtually equivalent to a radiation field and associates the presence of the virtual harmonic oscillators with the various possible transitions to other stationary states that would correspond to the classical theory of radiation. Moreover, we assume that the presence of transition processes is coupled with that mechanism by the laws of probability for the given atom itself, as well as for the other atoms that it can communicate with, and which is analogous to the laws of **Einstein**'s theory for the transitions between stationary state that are induced by external radiation. From our perspective, we shall consider the transitions in that theory that are referred to as spontaneous to be induced by virtual radiation fields that are coupled to the virtual oscillators that are associated with the motion of the atom itself. On the other hand, the induced transitions in **Einstein**'s theory take place as a result of the virtual radiation in the surrounding space that originates in other atoms.

Whereas, on the one hand, those assumptions will imply no change in the connection between atomic structure and the frequency, as well as intensity and polarization of spectral lines, that originates in the relation (1) and the correspondence principle, on the other hand, it will lead to a novel picture of the space-time occurrence of the various transition processes that the observation of optical phenomena ultimately leads back to. Thus, the occurrence of a given transition in a given atom will depend upon the original state of the atom, as well as the states of any atoms with which it can communicate by means of the virtual radiation field, but not on the occurrence of transition processes in the latter atoms.

On the one hand, we will see that in the limiting case where successive stationary states differ only slightly from each other our viewpoint will lead to a connection between the virtual radiation and the motion of particles in atoms that will gradually go to the ones that are required by the classical theory of radiation. In fact, neither the motion nor the constitution of the radiation fields will experience any essential changes in that limiting case due to the transitions between the stationary states. As far as the appearance of transition processes is concerned, which is the essential path of quantum theory, we shall, on the other hand, dispense with any possible causal connection between transitions in distant atoms, and in particular, the direct application of the principles of the conservation of energy and quantity of motion that are so characteristic of the classical theory. In our way of looking at things, the applicability of those principles to the interaction between individual systems of atoms is restricted to those interactions in which the atoms are close enough to each other that the forces that connected with the radiation field according to the classical theory will be small in comparison to the conservative parts of the forces that originate in the electrical charges of the atoms. Interactions of that kind, which we can refer to as "collisions," are known to offer a typical example of the postulated stability of the stationary states since the experimental results, when interpreted on the basis of the conservation laws for energy and quantity of motion, do indeed agree with the picture in which the colliding atoms are found in stationary states before, as well as after, the process [G. d. Q., Chap. I, **§ 4**] ([1]). By contrast,

([1]) Obviously, those considerations are valid only to the extent that one can ignore the radiation that is coupled with the collision. Although the energy of that radiation is very slight in many cases, its appearance will might have a fundamental significance. That was emphasized by **Franck** in connection with his explanation of **Ramsauer**'s important results [Ann. Phys. (Leipzig) **64** (1922), pp. 513; *ibid.*, **66** (1922), pp. 546] concerning the collisions between for interactions between atoms at great distances from each other, for which one could not speak of a simultaneous reciprocal action according to the classical theory, we will assume that the individual transition processes are independent, which would definitely contrast with the classical requirement of the conservation of energy and quantity of motion. We then assume that an induced transition would not have its direct cause in a transition in a distant atom for which the energy difference between the initial and final state is the same. In fact, when an atom has contributed to the induction of a transition in a distant atom, and indeed by means of the virtual radiation field that originates in the virtual oscillator that is associated with a certain possible transition to another stationary state, nonetheless, the atom can ultimately very well complete any other transition. Indeed, the experiments that have been done do not allow one to test that assumption, for the moment. However, it might emerge that the degree of independence of the transition processes that is assumed here could yield the single possibility of obtaining a consistent description of the interaction between radiation and atoms into which the laws of probability enter essentially. That independence reduces not only the conservation of energy to a statistical law, but also the conservation of quantity of motion, because just as we assume that every transition process that is induced by radiation is accompanied by a change in the energy of the atom by an amount $h\nu$, following **Einstein**, we will assume that every such process is accompanied by a change in the quantity of motion of the atom by an amount $h\nu / c$. If the transition is induced by the virtual radiation fields of distant atoms then the direction of the quantity of motion will coincide with the direction of propagation of the wave in the field. By contrast, when the transition is induced by the proper virtual radiation, we will naturally assume that the change in the quantity of motion is distributed according to the laws of probability, and indeed in such a way that the changes in the quantity of motion that accompanies the transitions that is induced in other atoms by that radiation will be compensated statistically for every direction in space.

Hence, we shall not seek the basis for the observed statistical conservation of energy and quantity of motion in a possible deviation from the electrodynamical theory of light relative to the laws of propagation of radiation in free space, but in the special properties of the interaction between the virtual radiation field and the irradiated atoms. We will then assume that those atoms act as the sources of secondary virtual radiation waves that possess the same frequency as the incident radiation and interfere with the original wave. In the case in which the frequency of the incident radiation coincides closely with the frequency of one of the virtual oscillators that are associated with the different possible transitions, the amplitudes of the secondary spherical waves will be especially large and those wave will exhibit phase relationships with the incident wave such that the intensity of the virtual radiation field will be increased or decreased by the

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atoms and slow electrons, from which it seemed to emerge that in certain cases, the electron could fly freely through the structure of the atom without being influenced by its presence. Namely, in those cases, when a change in the motion of the electron actually takes place due to a "collision," according to the classical theory, the appearance of radiation that is large enough that a perceptible association of radiation with the possible transition processes that are required by the correspondence principle could hardly be achieved [cf., **F. Hund**, Zeit. Phys. **13** (1923), pp. 241]. With the conception of things that is in this article, such a connection would be regarded as, on the one hand, just as natural as looking for the origin of radiation, and not mainly in the occurrence of transition processes. On the other hand, it must be emphasized that here we are dealing with a case in which a strict distinction between stationary motion and transition processes cannot be drawn with the present state of the theory as a result of the considerable magnitude of the classical radiation damping.

interference, and in that way, the power of that field to induce transitions in other atoms will be strengthened or weakened. Whether a weakening or a strengthening will result depends upon whether the corresponding virtual oscillator is associated with a transition in the atom to a stationary state of greater total energy or one with lower total energy, respectively. Obviously, that conception of things is closely related to the thoughts that **Einstein** voiced in regard to the introduction of the probabilities for induced transitions of both types, namely, ones in which the energy of the atom experiences an increase and ones in which it experiences a decrease. Despite the space-time separation of absorption and emission processes that is so characteristic of quantum theory then we can, however, expect a far-reaching formal analogy with classical electrodynamics in our picture as far the interaction between the virtual radiation field and the virtual harmonic oscillators that are associated with the motion in the atom is concerned. It actually seems possible that, guided by that analogy, we can arrive at a connected and seemingly-complete description of optical phenomena that accompany the propagation of light through a material medium in which, at the same time, the close coupling of those phenomena with the spectra of the atoms in the medium will emerge clearly.

§ 3. Capacity of spectral lines to interfere. – Before we go further into the general problem of the interaction between atoms and a virtual radiation field, in the section, we will briefly consider the properties of the field that originates from a single atom to the extent that it is coupled with the capacity of the light that emanates from one and the same source to interfere. Obviously, the constitution of that field has nothing to do with the details of the transition processes themselves, whose durations we will assume are not long in comparison to a period of the radiation or the motion of the particles in the atom. From our way of looking at things, those processes pertain to just the conclusion of the time interval inside of which the atom is in a position to communicate with other atoms by means of the corresponding virtual oscillators.

By contrast, an upper limit for the capacity to interfere will obviously be given by the mean time during which the atom remains in the initial state that is associated with the transition in question. The evaluation of that mean lifetime of the stationary state that is based upon the correspondence principle has obtained a general confirmation by the well-known experiment regarding the duration of the canal rays that escape from a high vacuum (cf., G. d. Q., Chap. II, **§ 4**). The interpretation of that experiment will take a very simple form in light of our new conception of things. Namely, one sees that from our perspective, the evolution of the illumination is not due to the details of the transitions, but only the relative number of atoms in different stationary states in the various parts of the ray. For example, when all of the escaping atoms possess the same velocity and are originally found in the same state, we might expect that the illumination power along the ray will decrease exponentially to the same degree for all spectral lines that are associated with the same state. The experimental equipment that is currently available is hardly sufficient to test those considerations at present.

If we ask what the capacity of spectra lines to interfere might be, as well as how to measure that with optical instruments, then the mean lifetime of the stationary states would obviously determine an upper limit to that capacity. However, we must keep in mind that the observable sharpness of a given spectral line that originates from the statistical result of the effects of a large number of atoms depend upon not only the lengths of the individual wave-trains that conclude with transitions, but also on any possible uncertainty in the definition of the frequency of those waves. If we recall the way in which the frequency of the spectral lines is coupled with the energy of the stationary states by the relation (1) then it is interesting to observe that the upper limit for the sharpness of the spectral lines in question can be closely connected with the limits of precision for the definition of motion and the energy in the stationary states. In fact, the postulate of the stability of the stationary states sets the precision with which the motion in those states can be described by classical electrodynamics, which is an *a priori* limit that also emerges directly in our picture due to the fact that the job of the virtual radiation field does not consist of continuously changing the motion of the atom, but of inducing transitions under which the energy and quantity of motion of the atom will experience finite changes (G. d. Q., Chap. II, **§ 4**). In the limiting domain in which the motions in the stationary states will differ from each other only relatively little, the upper limit for the capacity of the individual wave-trains to interfere will coincide with the limits of precision with which the frequency of radiation can be determined by means of (1) when the influence of the imprecision in the definition of the two states is measured by the type of independent errors. In the general case where the motions in the two states might differ from each other considerably, the upper limit on the capacity for the wave trains to interfere is intimately linked with the definition of the motion in each stationary state that defines the initial state of the transition. However, here as well, we might expect that the observable sharpness of the spectral lines can be determined by means of equation (1) when we combine the influence of the flaw in the definition of the final state with the influence of the flaw in the definition of the final state of the transition process in question in a manner that is similar to how we combine independent errors.

It is just that influence of the flaw in the definition of both stationary states on the sharpness of a spectral line that makes it possible for a reciprocity to exist between the constitution of a line when it is emitting, on the one hand, and when it is absorbing, on the other, just as would be required for thermal equilibrium by **Kirchhoff**'s law. In connection with that, recall that the apparent deviation from that law, which emerges in the context of the number and behavior of the lines in the oft-observed striking difference between the emission and absorption spectrum of an element, finds it direct explanation in quantum theory when one includes the differences in the statistical distributions of the atoms in their stationary states under various external influences.

Closely connected with the aforementioned problem of the sharpness of spectral lines that originates in atoms under constant external conditions is the question of the spectrum that originates in atoms where the external forces change considerably within a time interval with the same order of magnitude as the mean lifetime of the stationary states. One encounters such a problem in certain experiments by **Stark** on the influence of electric fields on spectral lines. In those experiments, the radiating atoms move with high velocities, and the periods of time in which they go from one point to another, in which the intensity of the field is very different, are only a small fraction of the lifetime of the stationary states that are coupled with the lines under investigation. Nonetheless, **Stark** found that if one ignores a Doppler effect of the usual kind then the radiation that is emitted by moving atoms is influenced by electric fields at each point in the same way that the radiation of atoms at rest would be influenced by the constant effect of the field strengths at that point. Whereas the interpretation of those results, as has been stressed by various authors [1], would give rise to complications when one bases it upon the current quantum-theoretical description of the connection between radiation and transition processes, **Stark**'s result obviously agrees with the picture that has been established in this treatise. In fact, the motion in the stationary states will vary continuously while the atom moves through the field, and therefore the same thing will be true for the virtual harmonic oscillators that are associated with the possible transitions. For that reason, the virtual radiation field that originates in a moving atom will be the same as when the atom moves in a field of constant intensity during its entire trajectory, and in any case when (as was the case in **Stark**'s experiments) the radiation that originates in other parts of the trajectory is prevented from arriving at those parts of the apparatus where the observation of the phenomena takes place. One will see that in a problem of this type, a far-reaching reciprocity between the observable emission and absorption phenomena is also ensured, and indeed it is due to the intrinsic symmetry in our picture relative to the coupling between transition processes in the one direction, on the one hand, and the radiation field, on the other.

§ 4. Quantum theory of spectra and optical phenomena. – Although according to quantum theory, the observation of optical phenomena is ultimately determined by transition processes, as was emphasized in the introduction, the logical interpretation of those phenomena must include those continuous trajectories that are so characteristic of the classical electrodynamical theory of the propagation of light in material media. According to that theory, the phenomena of reflection, refraction, and dispersion are ascribed to a scattering of light that takes place as a result of the forced oscillations of the electrical particles in the individual atoms that are due to the electromagnetic forces of the radiation field. As far as that is concerned, at first glance, the postulate of the stability of the stationary states brings with it a fundamental difficulty. However, as is suggested to some extent by the correspondence principle, the objection can be raised that in that way one would be led to compare the reaction of the atom to a radiation field with the scattering that would originate in a number of virtual harmonic oscillators according to the classical theory that would be associated with the various possible transitions. However, in that way, one might imagine that the analogy between the classical theory and the quantum theory, as it is formulated by means of the correspondence principle, is of an essentially formal nature, which is especially underscored by the fact that according to quantum theory, the absorption and emission of radiation is coupled with the various transition processes, and therefore the various virtual oscillators. However, for the interpretation of the experimental results concerning the emission and absorption spectra, it is just that essential point that seems to indicate how the scattering phenomena are coupled with the effect of the virtual oscillators relative to the emission and absorption of radiation. In a later treatise, it is our intention to show how a quantitative theory of dispersion that is similar to **Ladenburg**'s can be constructed using the current conception of things [2]. Here, we will be content to emphasize once more the continuous character of optical phenomena, which seems to admit no interpretation in the spirit of a causal coupling with transition processes in propagating media.

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[1] Cf., **K. Försterling**, Zeit. Phys. **10** (1922), pp. 387, and **A. J. Dempster**, Astrophys. Journ. **57** (1923), pp. 193.

[2] **Remark by the editor:** The main points of such a theory are described briefly by **Kramers** in a communication to "Nature" that will appear shortly.

We encounter an instructive example of those considerations in the experiments concerned with absorption spectra. Namely, strictly speaking, one cannot assert, as is often done for the sake of brevity, that the absorption of light in monoatomic vapors when the frequencies of the light coincide with certain lines in the emission spectra of the atoms has its origin in the transition processes that appear in the atoms of the vapor and which are induced by those wave-trains in the incident radiation that possess the frequency of the absorption lines. The fact that those lines appear in the spectroscope is due to the decrease in the intensity of the incident radiation that takes place as a result of the peculiarities of the secondary spherical waves that emanate from each of the illuminated atoms. Therefore, the induced transitions play the role of only an accompanying effect by which the statistical conservation of energy will be ensured. The presence of secondary coherent wave-trains is, at the same time, answerable to the anomalous dispersion that is coupled with the absorption lines and emerges quite clearly in the phenomenon that **Wood** ([1]) discovered, namely, the selective reflection from the wall of the container of a metallic vapor at sufficiently-high pressure. At the same time, the appearance of induced transitions between stationary states under selective absorption will be observed immediately in the fluorescent radiation that originates, to a large part, in the presence of a small number of atoms that would be raised to a stationary state of higher energy by the radiation. As is known, fluorescent radiation can be suppressed by the addition of foreign gases. As far as the part of the radiation that originates in the atoms in higher stationary states is concerned, that appearance can be explained by collisions that act in such a way as to reverse the considerable increase in the probability of the atom being in its normal state. Likewise, the part of the fluorescent radiation that consists of coherent scattered radiation, just like the phenomena of absorption, dispersion, and reflection, will experience changes under the addition of foreign gases that can be linked with the broadening of the spectral lines that is produced by the collisions ([2]). One will see that any conception of the absorption phenomena that deviates essentially from the one that was just described can hardly be justified, in any event, when it can be show that the absorption of spectral lines is qualitatively independent of the source of the radiation, which is similar to what was already proved for the ordinary phenomena of reflection and refraction in which the transitions in the medium do not occur in the same way (cf., G. d. Q., Chap. III, **§ 3**).

The problem of the scattering of light by free electrons provides another interesting example. As **Compton** proved by means of the reflection of Röntgen rays in crystals, that scattering is accompanied by a change in frequency that is different in different directions and agrees with the constitution of the radiation that is emitted by a hypothetical moving source according to the classical theory. As mentioned before, **Compton** arrived at a formal interpretation of that phenomenon on the basis of the quantum theory of light by assuming that an electron receives a quantum of the incident light, and at the same time can once more emit a light quantum in a different direction. As a result of that process, the electron will take on a certain velocity in a certain direction that is determined by the laws of conservation of energy and quantity of motion, just like the frequency of the re-emitted light, which would ascribe an energy of $h\nu$ and a quantity of motion $h\nu / c$ to each light quantum. In contrast to that picture, we regard the scattering of radiation by

([1]) **R. W. Wood**, Phil. Mag. **23** (1915), pp. 639.
([2]) Cf., e.g., **Chr. Fürchtfauer** and **G. Joos**, Phys. Zeit. **23** (1922), pp. 73.

electrons as a continuous phenomenon under which each electron is associated with the emission of secondary coherent waves. In that way, the incident virtual radiation gives rise to a reaction by each electron that is similar to the scattering that one would expect from the classical theory of an electron that possesses the velocity of the aforementioned hypothetical radiation source and exhibits forced oscillations under the influence of the radiation field. The fact that in this case the virtual oscillator moves with a velocity that is different from that of the radiating electron itself obviously implies a picture of things that is especially foreign to the classical way of thinking. With respect to the fundamental deviations from the classical space-time description, which is completely rooted in the idea of virtual oscillators, with the current state of the theory, it nonetheless seems hardly justified for one to wish to condemn a formal interpretation like the one considered. On the contrary, such an interpretation even seems necessary when one would like to account for the observed phenomena in whose description that wave conception of radiation indeed plays an essential role. Precisely as in **Compton**'s theory, we assume, at the same time, that the radiating electron possesses a certain probability for its quantity of motion to experience a certain finite change in any given direction. The statistical conservation of the quantity of motion would be ensured by that effect, which enters into quantum theory in place of the continuous adaptation of the quantity of motion that would accompany scattering of the type described according to the classical theory, analogous to the aforementioned statistical conservation of energy in the phenomenon of absorption spectra. In fact, the laws of probability that **Pauli** derived for the exchange of quantity of motion under the interaction of free electrons and radiation points to an essential analogy with **Einstein**'s laws, which are valid for the transitions between well-defined stationary states in an atomic system. The considerations of **Einstein** and **Ehrenfest** that were mentioned in **§ 2** are especially suited to the task of highlighting that analogy.

We will encounter a problem that is similar to the scattering of light by free electrons in the scattering of light by an atom, independently of whether the frequency of radiation is high enough to induce transitions in which an electron would leave the atom completely. Namely, as **Pauli**, and more recently **Smekal** ([1]) have emphasized, in order to ensure the statistical conservation of the quantity of motion, we must assume that transition processes can occur under which the quantity of motion of the scattering atom experiences a finite change without the relative motion of the particles in the atom changing as a result, as in the transition processes that are usually considered in spectral theory. We will also see that our picture of transition processes of the type that were discussed in closely related to optical scattering phenomena in a way that is analogous to the coupling of spectral phenomena with transition processes in which the internal motions in the atom change. However, based upon the large mass of the atomic nucleus, the change in velocity of the atom under such transitions is small enough that it would have no discernible influence on the energy of the atom and the frequency of the scattered radiation. Nonetheless, it is highly significant that the conversion of the quantity of motion is a discontinuous process, while the scattering itself is an essentially continuous phenomenon in which all scattered atoms participate independently of the intensity of the incident radiation. The discontinuous changes in the quantity of motion of the atoms are the causes of the observed effects on the atoms that are described by radiation pressure. Obviously, that conception of things satisfies the requirements for temperature equilibrium

([1]) **A. Smekal**, Naturwissenschaften **11** (1923), pp. 875.

between a (virtual) radiation field and a reflecting surface that **Einstein** ([1]) derived, and in which he glimpsed a support for the quantum theory of light. At the same time, it hardly needs to be emphasized that it also agrees with the apparent continuity of the actual observations of radiation pressure. Namely, if we consider a solid body then a change of $h\ \nu / c$ in its total quantity of motion would be completely unnoticeable, and even vanishingly small for visible light, in comparison to the irregular changes in the quantity of motion of a body that is in thermal equilibrium with its environment. However, in any discussion of the actual experiments, we must observe, at the same time, that we will encounter the question of whether we can neglect that time duration of the transition itself, or in other words, whether the limit inside of which the formulation of the principles of quantum theory are valid has already been exceeded (cf., G. d. Q., Chap. II, **§ 5**).

The latter considerations give an example of how our conception of optical phenomena allows a natural coupling with the usual continuous description of the macroscopic phenomena, to whose interpretation the **Maxwell**ian theory is so wonderfully suited. The advantage that our formulation of the principles of quantum theory has over the usual representation of that theory in this context can perhaps be illustrated more clearly by the phenomenon of the emission of electromagnetic waves, e.g., by an antenna, as in radiotelegraphy. In this case, a reasonable description of the phenomena in the spirit of an emission of radiation under separate successive transition processes between hypothetical stationary state of the antenna would not be possible. Namely, if one focusses on the smallness of the changes in energy under that transitions, as well as the quantity of energy radiated from the antenna per unit time, then one will see that the duration of the individual transition processes can be only an exceptionally small fraction of a period of oscillation for the electricity in the antenna, and that as a result, it would not be justified for one to describe the result of that process as the emission of a train of waves of that period. However, with our present conception of things, we describe the job of the electrical oscillation in the antenna as the production of a (virtual) radiation field that again induces changes in the motion of the electrons by means of the laws of probability. In that case, we can regard those changes as practically continuous because even if it were possible to justify making a distinction between individual energy jumps of $h\ \nu$, the magnitude of that step would be entirely negligible in comparison to the energy of the antenna. In connection with that, one observes that, given the present state of science, the emphasis on the "virtual" character of the radiation field, which seems so necessary for any meaningful description of atomic phenomena, will automatically lose all meaning in a case like the one considered here, in which the field exhibits all of those properties that classical electrodynamics ascribes to an electromagnetic field, to the extent that one considers its observable interaction with matter.

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([1]) **A. Einstein**, Phys. Zeit. **10** (1909), pp. 817.