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On the quantum mechanics of collision processes.

[Preliminary announcement ([1])]

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By examining collision processes, the viewpoint will be developed that quantum mechanics, in its *Schrödinger* form, not only allows one to describe stationary states, but also quantum jumps.

Up to now, the quantum mechanics that was founded by *Heisenberg* has been applied exclusively to the calculation of stationary states, and the vibration amplitudes that are associated with transitions. (I deliberately avoid the phrase "transition probabilities.") In that regard, the formalism that has been developing quite far since then is well-established. However, this way of posing the question addresses only *one aspect* of the quantum-theoretic problem; in addition, it raises the just-as-important question of the essence of the "transition" itself. The opinions seem to be split in regard to that issue; many assume that the problem of transitions in quantum mechanics cannot be posed in the foregoing form, but that it would be necessary to introduce some new concepts. As a result of a general impression that the logical structure of quantum mechanics is closed, I myself came to suspect that this theory is complete, and that it must then contain the transition problem. I believe that I have now succeeded in proving that.

Bohr has already directed attention to the fact that all of the main difficulties in the quantum picture that we encounter in the emission and absorption of light by atoms also appear in the interaction of atoms at short separations, and thus, for collision processes. For them, one deals with systems of material particles that are subordinate to the formalism of quantum mechanics, instead of with the very obscure wave fields. I have therefore taken the problem in hand of examining more closely the interaction of a free particle (α-ray or electron) and an arbitrary atom and establishing whether a description of collision processes is not possible within the context of the foregoing.

Of the various forms of the theory, only the *Schrödinger* form proves to be suitable, and on that basis I would like to regard it as the most fundamental formulation of quantum laws. The general train of thought in my argument is now the following:

If one would like to calculate the interaction of two systems quantum-mechanically then, as is known, one cannot, as in classical mechanics, single out one state of one system and establish how it would be influenced, since all of the states of both systems

([1]) This announcement was originally prepared for "Naturwissenschaften," but due to space limitations, it could not be included there. I hope that publishing it here will not seem superfluous.

are coupled in a complicated way. That is also true for an aperiodic process, such as a collision, in which a particle (say, an electron) comes in from infinity and again vanishes to infinity. However, here the picture confronts the fact that before, as well as after, the collision, when the electron is sufficiently distant and the coupling is small, one must be able to specify a well-defined state of the atom and a well-defined rectilinear motion of the electron. We must deal mathematically with the asymptotic behavior of the coupled particles. I did not succeed in doing that with the matrix formulation of quantum mechanics, but with the *Schrödinger* formulation.

According to *Schrödinger*, an atom in its n^{th} quantum state is an oscillatory process of a state quantity in all of space with constant frequency $\frac{1}{h}W_n^0$. In particular, an electron that moves rectilinearly is such an oscillatory process, and it corresponds to a plane wave. If two of them interact with each other then a complicated oscillation will result. However, one sees directly that one can stipulate its behavior at infinity. Indeed, one has nothing but a "deflection problem," in which a plane wave that is incident upon the atom is deflected or scattered; in place of the boundary conditions that one employs in optics for the description of screens, here one has the potential energy of the interaction of the atom and the electron.

The problem is then: Solve the *Schrödinger* wave equation for the atom-electron combination under the boundary condition that, for a certain direction in electron space, the solution goes asymptotically to a plane wave with just that direction of advance (viz., the incoming electron). With the solution, thus-characterized, we shall now be interested chiefly in the behavior of the "deflected" wave at infinity; that will then describe the behavior of the system after the collision. We then look into this a bit more closely. Let $\psi_1^0(q_k)$, $\psi_2^0(q_k)$, … be the eigenfunctions of the unperturbed atom (we assume that there is only a discrete series of them). The unperturbed (rectilinearly) moving electron corresponds to the eigenfunction $\sin\frac{2\pi}{\lambda}(\alpha x + \beta y + \gamma z + \delta)$, which defines a continuous manifold of plane waves whose wave length (according to *de Broglie*) is linked to the energy τ of the translational motion by the relation $\tau = \frac{h^2}{2\mu\lambda^2}$. The eigenfunction of the unperturbed state in which the electron comes out of the + z direction will then be:

$$\psi_{n\tau}^0(q_k, z) = \psi_n^0(q_k)\sin\frac{2\pi}{\lambda}z .$$

Now, let $V(x, y, z; q_k)$ be the potential energy of the interaction between the atom and the electron. With the help of simple perturbative calculations, one can show that there is a uniquely-defined solution of the *Schrödinger* differential equation when one considers the interaction V that goes asymptotically to the function above for $z \to +\infty$.

We now come to the problem of how that solution function will behave "after the collision."

We can now calculate: The deflected wave that is produced by the perturbation has the asymptotic expression:

$$\psi^{(1)}_{n\tau}(x, y, z; q_k) = \sum_m \iint_{\alpha x+\beta y+\gamma z+\delta>0} d\varpi\, \Phi_{\substack{nm\\ \tau}}(\alpha, \beta, \gamma) \sin k_{\substack{nm\\ \tau}}(\alpha x+\beta y+\gamma z+\delta)\, \psi^0_m(q_k)$$

at infinity. That means: The perturbation can be regarded as a superposition of solutions of the unperturbed process at infinity. If one calculates the energy that belongs to the wave length $\lambda_{\substack{nm\\ \tau}}$ according to the *de Broglie* formula that was given above then one will find that:

$$W_{\substack{nm\\ \tau}} = h\nu^0_{nm} + \tau,$$

in which ν^0_{nm} are the frequencies of the unperturbed atom.

If one would now like to reinterpret this result as a corpuscular one then only one interpretation will be possible: $\Phi_{\substack{nm\\ \tau}}(\alpha, \beta, \gamma)$ determines the probability ([1]) that the electron that comes out of the *z*-direction will be deflected into the direction that is determined by α, β, γ (and with a change of phase δ), in which its energy τ has increased by a quantum $h\nu^0_{nm}$ at the cost of the energy of the atom (collision of the first kind for $W^0_n < W^0_m$, $h\nu^0_{nm} < 0$; collision of the second kind for $W^0_n > W^0_m$, $h\nu^0_{nm} < 0$).

Schrödinger's quantum mechanics then gives a completely definite answer to the question of the effect of a collision; however, one is not dealing with any causal relationship. One gets *no* answer to the question "what is the state after the collision," but only to the question "how probable is a prescribed effect of the collision" (in which, one must naturally verify the quantum-mechanical law of energy).

This raises the whole problem of determinism. From the standpoint of our quantum mechanics, there is no quantity that could establish the effect of a collision causally *in the individual cases*; however, up to now, we have no clue regarding the fact that there are internal properties of the atom that require a definite collision effect, even from experiments. Should we hope to discover such properties (perhaps phases of the internal atomic motions) and to determine the individual cases? Or should we believe that the agreement between theory and experiment regarding our inability to give conditions for the causal evolution is in pre-stabilized harmony with the fact that such conditions do not exist? I myself tend to abandon determinism in the atomic world.

In practice, there exists indeterminism, in any case, for experiment physicists, as well as for theoretical ones. The "profit function" Φ that has been much-studied by experimenters is now also rigorous theoretically. One can find it from the potential energy of the interaction $V(x, y, z; q_k)$; thus, the computational processes that are necessary for this must be developed in order to communicate them at this point. Here, I

([1]) Editor's remark: A more precise argument will show that the probability is proportional to the square of the quantity $\Phi_{\substack{nm\\ \tau}}$.

would only like to say a few words about the meaning of the function $\underset{\tau}{\Phi_{nm}}$. For example, if the atom is in the normal state $n = 1$ before the collision then it will follow from:

$$\tau + h\nu_{1m}^{0} = \tau - h\nu_{m1}^{0} = \underset{\tau}{W_{1m}} > 0$$

that one must necessarily have $m = 1$for an electron with less energy than the lowest excitation level, and thus, $\underset{\tau}{W_{11}}$; that will then result in the “elastic reflection” of the electron with the profit function $\underset{\tau}{\Phi_{11}}$. If τexceeds the first excitation level then there will also be excitation with the profit $\underset{\tau}{\Phi_{12}}$, in addition to the reflection, etc. If the atom in question is in the excited state $n = 2$ and $\tau < h\nu_{21}^{0}$ then there will be reflection with profit $\underset{\tau}{\Phi_{22}}$ and collisions of the second kind with the profit $\underset{\tau}{\Phi_{21}}$. If $\tau > h\nu_{21}^{0}$ then further excitations will enter in, etc.

The formulas then give the qualitative behavior under collisions completely. The quantitative exhaustion of the formulas for special cases must remain deferred to a more thorough examination.

It does not seem excluded to me that the close connection of mechanics and statistics, as it comes to the fore here, will require a revision of the basic concepts of statistical thermodynamics.

I further believe that the problem of the irradiation and radiation of light must also be treated in a completely analogous way to the “boundary-value problem” of the wave equation and would lead to a rational theory of damping and line width that is in harmony with the picture of light quanta.

A thorough presentation will appear in this Zeitschrift next.