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The "hidden" thermodynamics of particles

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Abstract. – The author presents a glimpse of a line of research that he carried out since 1951 with the goal of obtaining a clear picture of wave-corpuscle duality. Recalling some ideas that he developed from 1926-1927 under the name of the "theory of the double solution," he first considers the corpuscle as a point of high concentration that is localized to the interior of a wave field and displaces according to a law of a certain "guiding dynamic." He then acknowledges the necessity of completing this picture by superposing a kind of random thermal agitation with the motion that is defined by that dynamic, and which will be due to the constant interaction of every particle with a kind of hidden thermostat (i.e., the sub-quantum medium of Bohm-Vigier). This has recently led him to develop a "thermodynamics of the isolated particle," whose fundamental principles he presents, along with some applications.

The interpretation of wave mechanics. – At the end of the Summer of 1963, it will be forty years ago that the necessity appeared to me of extending the double wave-like and corpuscular aspect that Einstein revealed to us for light in his theory of light quanta in 1905 to all particles. One sees quite clearly upon rereading my thesis that was submitted at the end of 1924, a republication of which has just been made, that my objective was to arrive at a synthetic picture in which the corpuscle, which is always considered to be a small localized object that generally displaces in space in the course of time, will be incorporated in a wave with a physical character in such a way that its motion is coupled to the propagation of the wave.

An essential remark that I made, which is never mentioned today in the books that present quantum mechanics, is the difference between the relativistic transformation of the frequency of a wave and that of the frequency of a clock. It is well-known that if the frequency of a clock, when considered in its proper system, is ν_0 then the frequency that is attributed to it by an observer that sees it pass by with the velocity $v = \beta c$ is $\nu_c = \nu_0 \sqrt{1-\beta^2}$; this is what one calls the phenomenon of the "slowing down of clocks" as a result of their motion. On the contrary, if a wave is a stationary wave of frequency ν_0 in a certain system of reference then that wave, when observed in a reference system that is animated with the velocity $v = \beta c$ with respect to the first one, appears to be a progressive wave that propagates in the sense of relative motion with the frequency $\nu = \frac{\nu_c}{\sqrt{1-\beta^2}}$ and the phase velocity $V = c / \beta = c^2 / \mathrm{v}$. If one attributes an internal frequency $\nu_0 = m_0 c^2 / h$ to the corpuscle, as is suggested by the fundamental quantum relation $W = h\nu$, and if one assumes that in the proper system of the corpuscle the wave that is associated with it is a stationary wave of frequency ν_0 then all of the fundamental formulas of wave mechanics – notably, the celebrated formula $\lambda = h / p$, where p is the quantity of motion of the corpuscle – are deduced immediately from the preceding remarks.

Since I have considered the corpuscle to be constantly localized in the wave, I have a good glimpse of the following consequence: The motion of the corpuscle is such that it assures the permanent agreement of the phase of the progressive wave that surrounds it with the internal phase of the corpuscle when considered as a small clock. That relation is verified immediately in the simple case of a corpuscle in uniform motion that accompanies a monochromatic plane wave that I mainly studied in my thesis. From my thesis, building upon that idea, I was led to think that when the wave has the general form:

$$\psi = a(x, y, z, t)\; e^{\frac{2\pi i}{h}\varphi(x,y,z,t)},$$

where a and φ are real, the phase agreement between the corpuscle and its wave demands that the velocity of the corpuscle at each point of its trajectory must be given by:

$$\mathbf{v} = -\frac{1}{m}\,\mathrm{grad}\,\varphi .$$

This concept links up with the one that Madelung developed in the same era, which compared the propagation of the wave ψ in wave mechanics with the flow of a fictitious fluid of density $\rho = |\psi|^2$ and velocity:

$$\mathbf{v} = -\frac{1}{m}\,\mathrm{grad}\,\varphi .$$

Using Madelung's hydrodynamic picture, my hypothesis on the motion of the corpuscle is expressed by saying that the trajectory of the corpuscle coincides with one of the streamlines of Madelung's hydrodynamical fluid. One then arrives at the idea that the wave imposes a well-defined motion on the corpuscle: This is what I have called the "theory of the pilot wave."

I was then led to go beyond that first viewpoint. In my opinion, it is not sufficient to superpose the corpuscle with a wave and impose the constraint that it be guided by the propagation of that wave; one must represent the corpuscle as being *incorporated* into the wave as if it were part of its structure. I was thus led to what I have called "the theory of the double solution": It assumes that the true wave is not homogeneous, but that it is composed of a small region of high concentration of the field that constitutes the corpuscle in the correct sense of the word, and that outside of that very small region the wave roughly coincides with the homogeneous wave that the usual wave mechanics imagines. I then presented the primary basis for that new concept in an article in the *Journal de Physique* in July, 1927.

Unfortunately, that attempt, in which I did not further introduce the idea of a nonlinear character to the wave equation for the real wave, ran into numerous difficulties. I have said several times, moreover, how, at the Solvay conference in October of 1927 I was led to abandon that attempt and rejoin, at least provisionally, the interpretation of the Copenhagen School, but in the years that followed, I had always preserved a much more "objective" manner of presentation than then ones that were being gradually employed, in order to not get too far removed from my original intuitions. In 1951, after having taught some courses on the interpretation of wave mechanics in the preceding years, in which I was led to review the objections that were raised by scholars such as Einstein, Schrödinger, and others against the usual interpretation, I found myself in a state of mind that was favorable to a return to my old concepts: The work of David Bohm that I have since learned of and my long conversations on the problem with Jean-Pierre Vigier then led me to revisit the theory of the double solution, while introducing the idea of nonlinearity into it.

I cannot detail the progress that has been made in these last years regarding the interpretation of wave mechanics by the theory of the double solution. One can follow the gradual development by referring to the two works that I published with Gauthier-Villars in 1956 and 1957, and an article that I published in the *Journal de Physique* in December of 1960. I just published a volume with Gauthier-Villars that was dedicated to a detailed critique of a series of small errors of interpretation that, it seems to me, have contributed to the orientation of quantum physics along an erroneous path. I would not wish to forget to emphasize the continuing help that Andrade e Silva has afforded me in my work during these latter years through the sharpness of his analyses, notably in the editing of my last book.

Superposition of a random motion on the regular "guiding" motion. – We have seen that in the theory of the double solution (or that of the pilot-wave) one determines the motion of the corpuscle within its wave, when written in the form $a\,e^{\frac{2\pi i}{h}\varphi}$, by assuming that its velocity is given by the guiding formula:

$$\mathbf{v} = -\frac{1}{m}\,\text{grad}\;\varphi, \tag{1}$$

m being the mass of the corpuscle. This form for the guiding formula is valid in the non-relativistic wave mechanics that corresponds to the Schrödinger equation of propagation. In the relativistic wave mechanics that corresponds to the Klein-Gordon equation of propagation, the guiding formula for a particle of electric charge ε that is placed in an electromagnetic field that is derived from a scalar potential V and a vector potential $\mathbf{A}$ takes the form:

(2) $$\mathbf{v} = -c^2 \frac{\text{grad}\,\varphi + \frac{\varepsilon}{c}\mathbf{A}}{\frac{\partial\varphi}{\partial t} - \varepsilon V},$$

which indeed reduces to the form (1) in the Newtonian approximation. In the case of particles with spin (for example, the Dirac electron), one deduces the guiding formula by always writing that the particle follows one of the streamlines of the hydrodynamical flow that corresponds to the equations of propagation.

I think that the phenomenon of the particle being guided by the ambient (wave) field results, as in the general theory of relativity, from the fact that the field equations are nonlinear, and that nonlinearity, which is manifested almost uniquely in the corpuscular region, renders the motion of the particle rigidly coupled to the propagation of the ambient wave. Upon employing a method that is analogous to the one that was employed not long ago by George Darmois in order to justify the "geodesic postulate" in general relativity, and which was developed by Lichnerowicz, one can represent the corpuscle approximately as a singularity in the wave field and show that, for reasons of continuity, the singularity in its motion must remain constantly imprisoned in the interior of an infinitely thin tube of streamlines of the external field. However, I cannot insist upon that justification for the guiding formula here.

There is, however, a consequence of the guiding formula upon which I must insist. Even when a particle is not subject to any external field, if the ambient wave is not a wave that is roughly plane and monochromatic (therefore, that wave must be represented by a superposition of monochromatic plane waves) then the motion that is imposed upon it by the guiding formula is not uniform and rectilinear. In 1927, that led me to think that the corpuscle is subject to a force by the ambient wave that bends its trajectory: That "quantum force" will be equal to the gradient – with the sign changed – of a quantum potential (Q) whose expression in non-relativistic wave mechanics is written:

(3) $$Q = -\frac{h^2}{8\pi^2 m}\frac{\Delta a}{a}.$$

However, in relativistic wave mechanics, one must consider that the quantum force is equal to the gradient – with the sign changed – of the quantity:

(4) $$M_0 c^2 = \sqrt{m_0^2 c^4 + \frac{h^2 c^2}{4\pi^2}\frac{\Box a}{a}} \qquad \left(\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right).$$

Since the quantum potential Q is defined only up to a constant, one can write:

(5) $$Q = M_0 c^2 - m_0 c^2.$$

The quantity M_0, which, in general, a function of x, y, z, t, and which reduces to the constant proper mass m_0 that is usually attributed to the particle when Q is zero, can be called the "variable proper mass" of the particle; it will play a significant role in what follows. It is, moreover, easy to verify that the definition (5), taking the value (4) of M_0 into account, reduces to the definition (3) in the Newtonian approximation.

The theory of guidance that is employed in the formulas that we just specified imposes a well-defined motion upon the particle when one is given its position in the wave at a given initial instant. This was the viewpoint that I originally adopted in 1927, and which I assumed again when I returned to the theory of the double solution a dozen years ago. However, soon it appeared to me with increasing conviction that this concept is not sufficient, and that one must superpose a type of "Brownian" motion with a random character on the regular motion in such a way that the "mean" of the particle defines the guiding formula.

In order to account for the necessity of introducing this new element into the theory, one must remember that if ρ denotes the density of the fictitious fluid in the hydrodynamical picture of wave propagation – a density that is equal to $|\psi(x, y, z, t)|^2$ in non-relativistic wave mechanics – then one is led to consider that the quantity $\rho\, d\tau$ gives the probability of the presence of the particle at the instant t in the volume element $d\tau$ in physical space. In order to justify this result in the theory of the double solution, one can start with the continuity equation for the fictitious fluid:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathbf{v}) = 0, \tag{6}$$

an equation in which $\mathbf{v}$ is precisely the guiding velocity. If one considers the motion of a volume element $d\tau$ that always contains the same fluid elements then one sees that equation (6), which, with a well-known hydrodynamical notation, can be written:

$$\frac{D}{Dt}(\rho\, d\tau) = 0, \tag{7}$$

expresses the conservation of the product $\rho\, d\tau$ in the course of time. Just as in classical statistical mechanics Liouville's theorem, which is expressed by $\frac{D}{Dt} d\tau = 0$, makes it very likely *a priori* that the probability of the presence of the representative point of a system in the element $d\tau$ of phase volume is proportional to the magnitude of that element, similarly, here equation (7) make it very likely *a priori* that the probability of the presence of the particle in the element $d\tau$ of physical space is proportional to $\rho\, d\tau$. There is a great analogy between the two problems with, however, the difference that in statistical mechanics one considers the motion of a representative point in phase space, whereas here we consider the motion of a particle in physical space.

The analogy between the two problems entails an analogy between the difficulties that are encountered when one carefully examines the proof of the desired result. In one case, as in the other one, the validity of the result demands that the element must sweep out all of the space that is accessible to it in the course of its motion (phase space in one case, physical space in the other). One knows that in order to justify this hypothesis, one is led in statistical mechanics to either try to prove a theorem of ergodicity or to introduce, with Boltzmann, a hypothesis of "molecular chaos." In the problem that we are occupied with, it is the second path that was followed by David Bohm and Jean-Pierre Vigier when they introduced the idea of a "sub-quantum medium" in 1954 in an article in the *Physical Review*, an idea whose importance is rapidly emerging as paramount to me.

According to Bohm and Vigier, there exists a hidden medium that corresponds to a level of physical reality that is more profound that the quantum or microphysical level, a medium with which all of the particles of the microphysical level will be constantly in interaction. The complexity of this medium, and it seems to me premature to specify its nature precisely, will have the consequence of implying the purely random character of the perturbations that its action imposes upon the motion of the microscopic particles.

Under the influence of these continual random perturbations that the "sub-quantum medium" communicates to it, the particle, rather than regularly following one of the trajectories that are defined by the guiding law, constantly jumps from one of these trajectories to another, and thus traverse a large number of segments of these trajectories in a very short time, and if the wave remains confined to a finite region of space then that zigzagging trajectory will soon explore that region completely. One can then justify the assertion that the probability of the presence of the particle in a volume element $d\tau$ of physical space must be taken to be equal to $|\psi|^2\,d\tau$. This is what Bohm and Vigier did in their paper: They showed that the probability distribution $|\psi|^2$ must be established very rapidly. Andrade e Silva, who has reflected upon this subject quite often, has attached the success of this proof to the properties of "Markov chains."

Without insisting upon his proofs, I would like to give a picture that makes the meaning of the foregoing more precise. Consider the flow of a fluid: Hydrodynamics defines the trajectory of a molecule of that fluid that passes through that point, and which coincides with one of the streamlines in the case of permanent motion. Now, the trajectory thus defined is only an "ideal" or "mean" trajectory that will be described in practice only if the molecule is subject to no perturbation. However, this is never true: Since the fluid is never at absolute zero, its molecules are animated with a thermal agitation that is due to their incessant mutual collisions, and for that reason each of them constantly passes through one theoretical trajectory or another. It is because the molecules thus describe a continually zigzagging trajectory that it is permissible, if ρ denotes the fluid density, to consider the quantity $\rho\,d\tau$ as measuring the probability of the presence of a given molecule in the element $d\tau$.

In order to succeed in proving the necessity of introducing the Bohm-Vigier perturbations of sub-quantum origin into the theory, I will consider the case of a hydrogen atom in a stationary energy state E, where the wave function is of the form:

$$\psi = a(x, y, z)\; e^{\frac{2\pi i}{h}Et}\;.$$

The guiding formula then gives us that $\mathbf{v} = 0$: This must say that the electron is immobile at a point in the atom and one easily verifies, moreover, that in this case the Coulomb force that the nucleus exerts on the electron is exactly equilibrated by the quantum force. However, if the electron is therefore immobile, how can one understand why the probability of the presence of the electron at any point of the atom is given by the expression:

$$|\psi|^2 = |a(x, y, z)|^2\;?$$

One can explain this if one assumes that the electron, while remaining immobile "in principle," constantly jumps from one position to the other under the influence of continual perturbations of sub-quantum origin. This example also proves, in a striking fashion, the necessity of introducing these perturbations into the theory of the double solution, i.e., of superposing a Brownian motion of random character on the regular motion of the particle that is predicted by the guiding theory.

Introduction of the idea of a hidden thermodynamics. – One knows that at the end of the last Century, thanks entirely to the work of Boltzmann and Gibbs, a statistical interpretation of thermodynamics was developed whose success has ultimately been total. However, in the same period certain authors – notably, Helmholtz and Boltzmann himself – have sought, with some success, to bring about some agreement between the quantities that are introduced by thermodynamics and the quantities of classical mechanics with no intervention of statistical considerations. These interesting attempts have been largely forgotten and have hardly been used, except in the theory of adiabatic invariants (Léon Brillouin).

During the years 1946-1948, in some notes to the *Comptes rendus de l'Academie des Sciences*, in a course that I taught at l'Institut Henri Poincaré, and in an article in *Cahiers de Physique*, I have returned to an examination of these old works, and naturally I have tried to approach them with the concepts of wave mechanics. I have thus been led to define a temperature T for a particle in motion with a velocity βc, and whose the internal cyclic frequency we have seen is given by:

$$\nu_c = \nu_0 \sqrt{1-\beta^2} = \frac{m_0 c^2}{h}\sqrt{1-\beta^2} .$$

That temperature is related to the internal cyclic frequency of the particle by:

$$kT = h\nu_c ,$$

where k is the Boltzmann constant. I was also led to define entropy by starting with the action of the particle in the sense of mechanics. These analogies did not give me complete satisfaction, but they seemed very curious to me. I also concluded by article in *Cahiers de Physique* in January of 1948 by writing down the following benchmark assertion with a certain hesitation: "There is the beginnings of a thermodynamics of material points that one can seek to develop in the context of wave mechanics: It is very difficult to say where that path can lead, and we content ourselves with having indicated the point of departure."

From whence arose the hesitation that this statement provoked? It came from the fact that, being persuaded that thermodynamics is essentially of statistical origin and is meaningful only for very complex systems, I could not comprehend how there could exist a thermodynamics that is valid for a particle that is assumed to be isolated, and that is why for several years I abandoned my attempt to develop a theory of this type.

However, in the last two years I have decided to return to the study of some ideas that have continued to hold my attention, despite everything. Now, in that era, having returned for several years to my reflection on the theory of the double solution, I admitted the existence of a sub-quantum medium. Very quickly, a light went on in my mind. If any particle that appears to us to be isolated at the microscopic level can constantly exchange energy and quantity of motion with the sub-quantum medium then that medium would play the role of a "hidden thermostat" with which the particle is constantly in energetic contact. Moreover, there is no longer *a priori* any paradox in trying to develop the thermodynamics of isolated particles. That is what I tried to do in a first note to the *Comptes rendus de l'Academie des Sciences* in August, 1962. At this moment, I have finished editing a book in which I present the principles of the "thermodynamics of the isolated particle," as I did in my last course at l'Institut Henri Poincaré during the winter of 1961-1962. Furthermore, I remark that since this thermodynamics is applicable to ensembles of interacting particles that are all in energetic contact with the hidden thermostat, the term "hidden thermodynamics of particles" will be perhaps preferable to the term that I first adopted, for obvious reasons.

However, before presenting the basis for this very attractive new theory and some applications, I must first recall some notions from relativistic thermodynamics that have been known for a long time (e.g., the work of Planck and his school around 1910), but which one rarely teaches in college courses.

Review of some notions of relativistic thermodynamics. – In relativistic thermodynamics, one proves that if a body that is imagined to be in its proper system possesses a temperature T_0 and contains a quantity of heat Q_0 for an observer that sees it pass by with a velocity βc then it possesses a temperature T and contains a quantity of heat Q that are coupled by the formulas:

$$T = T_0 \sqrt{1-\beta^2}\,, \qquad Q = Q_0\sqrt{1-\beta^2}\,. \tag{8}$$

One likewise proves, and this results almost immediately from formulas (8), that entropy is a relativistic invariant; i.e., that $S = S_0$.

Before going much further, we shall make a notational convention that will be very useful to us in what follows. If A is a quantity whose value depends upon the proper mass M_0 of a body and also some other parameters, such as the velocity or the position of this body, then we let $[\delta A]_{M_0}$ denote the small variation that A experiences when one lets the other parameters vary slightly while keeping M_0 constant, and we let $\delta_{M_0} A$ denote the small variation that A experiences when one varies M_0 slightly while the other parameters remain constant. Having said this, consider a body in its proper system: Its proper mass will be M_0 , and from the principle of the inertia of energy its internal energy will be $W_0 = M_0\, c^2$. If its proper mass is subject to a slight variation δM_0 then its internal energy will vary by $\delta W_0 = \delta M_0\, c^2$. This can happen only if the body has received or given up the quantity of energy δW_0 , and since the internal energy is an energy that is stored inside the body, one must consider the quantity $\delta M_0\, c^2$ to be a quantity of heat δQ_0. Therefore, for an observer who sees the body pass with the velocity βc, the quantity of heat that is received or given up by the body will be:

(9) $$\delta Q = \delta Q_0 \sqrt{1-\beta^2} = \delta M_0\, c^2 \sqrt{1-\beta^2} .$$

Now, as is well-known in relativistic dynamics, the Lagrange function for that observer will be:

(10) $$\mathcal{L} = - M_0\, c^2 \sqrt{1-\beta^2} + \ldots,$$

in which the unwritten terms do not depend upon M_0. We thus have:

(11) $$\delta Q = - \delta_{M_0} \mathcal{L} .$$

It is easy to recover this formula by other arguments that are more complete, which I will give in the book that I am preparing. I would like to point out one of them.

We place ourselves in the system of the observer who sees the body pass with the velocity βc. We can then write the two formulas of relativistic dynamics:

(12) $$W = \frac{M_0 c^2}{\sqrt{1-\beta^2}}, \qquad \frac{d}{dt}\left[\frac{M_0 v}{\sqrt{1-\beta^2}}\right] = f.$$

As for the second one, one derives the following expression for the work that is done on the body during the time δt:

(13) $$\delta \mathcal{T} = f\, v\, \delta t = v\, \delta \frac{M_0 v}{\sqrt{1-\beta^2}} .$$

If one assumes that the proper mass is constant then, as one habitually does in relativistic dynamics, one easily verifies that the work in question has the value:

(14) $$[\delta \mathcal{T}]_{M_0} = [\delta W]_{M_0} ,$$

which is normal. However, if the proper mass can vary then one will have for the work that is done on the body:

(15) $$\delta \mathcal{T} = [\delta \mathcal{T}]_{M_0} + \delta_{M_0} \mathcal{T} = [\delta W]_{M_0} + \frac{v^2}{\sqrt{1-\beta^2}}\, \delta M_0 .$$

Now, one thus also has:

(16) $$\delta W = [\delta W]_{M_0} + \frac{c^2}{\sqrt{1-\beta^2}}\, \delta M_0 ,$$

and from equations (15) and (16), one deduces that:

(17) $$\delta W = \delta \mathcal{T} + \delta M_0\, c^2 \sqrt{1-\beta^2} .$$

Since the principle of the conservation of energy gives $\delta W = \delta\mathcal{T} + \delta Q$, where δQ is the quantity of heat that is taken in by the body, one recovers formulas (9) and (11).

Thermodynamics of the isolated particle, or the hidden thermodynamics of particles. – In order to develop the thermodynamics of the isolated particle, we first assume that we can apply the formula:

$$\delta Q = -\delta_{M_0}\mathcal{L} \tag{11}$$

to an isolated particle at the level of microphysics. Moreover, we also assume, in accord with my old ideas of 1946-1948, that the particle, which is in permanent contact with the hidden thermostat, can be regarded as having a temperature T that is defined by the formula that has the desired relativistic covariance:

$$kT = h\nu_c = h\nu_0\sqrt{1-\beta^2} = mc_0\sqrt{1-\beta^2}\ . \tag{18}$$

We define the entropy by referring it to the hidden thermostat, which is a very complex system. We take our inspiration from the method that was formerly employed by Einstein in his work on fluctuations and write the entropy S of the hidden thermostat in the form:

$$S = S_0 + S(M_0),$$

where S_0 is the part of that entropy that is independent of the fluctuating value of the proper mass M_0 of the particle, while $S(M_0)$ is the small part of that entropy that depends upon the value of M_0 . We will then have:

$$\delta_{M_0}S = \delta S(M_0) = -\frac{\delta Q}{T} = \frac{\delta_{M_0}\mathcal{L}}{T}. \tag{19}$$

The – sign that appears before δQ comes about due to the fact that δQ is the heat *given up* by the hidden thermostat to the particle. Upon utilizing formulas (10) and (11), we get:

$$\delta S(M_0) = -k\frac{\delta M_0}{m_0}, \tag{20}$$

which finally gives:

$$S = S_0 - k\frac{M_0}{m_0}, \tag{21}$$

a fundamental formula in which the invariance of the right-hand side is indeed evident.

It is easy to infer an important first consequence from this formula.

Indeed, from the Boltzmann formula that relates the entropy to the probability, the probability of the value M_0 of the fluctuating proper mass must be proportional to $e^{S/k}$;

i.e., to e^{-M_0/m_0}. One concludes from this that the mean value of M_0 for a particle outside of any force field (classical or quantum) is:

$$\bar{M}_0 = \frac{\int_0^\infty M_0 e^{-\frac{M_0}{m_0}} dM_0}{\int_0^\infty e^{-\frac{M_0}{m_0}} dM_0} = m_0 . \tag{22}$$

Therefore, the constant proper mass m_0 that is usually attributed to the particle appears to us as the mean value of the true instantaneous proper mass M_0, which fluctuates.

Another interesting consequence of this theory is to establish a link between the principle of least action and the second law of thermodynamics. Hamilton's principle of least action tells us that if, in its natural motion that conforms to the laws of dynamics, the particle starts from a point A at the instant t_0 and arrives at a point B at the instant t_1 then the action integral taken along its motion is minimal with respect to the same integral taken along the entire "varied" motion that takes the particle at the point A at the epoch t_0 to the point B at the epoch t_1. One is then led to write:

$$\int_{t_0}^{t_1} [\delta \mathcal{L}]_{M_0} dt = 0, \qquad \int_{t_0}^{t_1} [\delta^2 \mathcal{L}]_{M_0} dt > 0, \tag{23}$$

the first and second variations of Λ being taken while keeping the proper mass M_0 constant and equal to its normal value m_0.

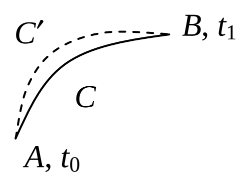

Figure 1.

Here, I have introduced a hypothesis that seems to me to have a very interesting physical significance:

If ACB is the natural trajectory then I have assumed that the varied trajectories such as $AC'B$ do not correspond to the fictitious motions that are imagined by the theoretician, as one usually assumes, but to motions that can actually occur when the proper mass M_0 of the particle is subjected to a certain series of fluctuations between t_0 and t_1. From Hamilton's principle, the fluctuating trajectory $AC'B$ must be determined by the equation:

$$\int_{t_0}^{t_1} \delta(\mathcal{L} + \delta\mathcal{L})\, dt = \int_{t_0}^{t_1} (\delta\mathcal{L} + \delta^2\mathcal{L})\, dt = 0. \tag{24}$$

Here however, since the proper mass is no longer assumed to be constant, one must write:

(25) $$\delta\mathcal{L} = [\delta\mathcal{L}]_{M_0} + \delta_{M_0}\mathcal{L}\,; \qquad \delta^2\mathcal{L} = [\delta^2\mathcal{L}]_{M_0} + \delta^2_{M_0}\mathcal{L}\,,$$

where $\delta^2_{M_0}\mathcal{L}$ denotes the set of terms in $\delta^2\mathcal{L}$ that depend upon the variation of M_0 . One thus has:

(26) $$\int_{t_0}^{t_1} \{[\delta\mathcal{L}]_{M_0} + \delta_{M_0}\mathcal{L} + [\delta^2\mathcal{L}]_{M_0} + \delta^2_{M_0}\mathcal{L}\}\, dt = 0$$

on $AC'B$. However, the integral of the first term is zero, by virtue of Hamilton's principle, and one can easily verify that the fourth term is negligible compared to the other ones. What finally remains, taking (23) into account, is:

(27) $$-\int_{t_0}^{t_1} \delta_{M_0}\mathcal{L}\, dt = -(t_1 - t_0)\, \overline{\delta_{M_0}\mathcal{L}} = \int_{t_0}^{t_1} [\delta^2\mathcal{L}]_{M_0}\, dt > 0,$$

in which $\overline{\delta_{M_0}\mathcal{L}}$ is the temporal mean between t_0 and t_1 . Then, since $t_1 - t_0$ is positive and $-\delta_{M_0}\mathcal{L}$ is the heat given up by the hidden thermostat to the particle, one sees that the temporal mean of this quantity of heat, which is zero on the natural trajectory, is positive on the fluctuated trajectory. In other words, the entropy S diminishes in the mean when one passes from ACB to $AC'B$. The entropy is therefore maximal on the natural trajectory with respect to the fluctuations, subject to the conditions of the Hamiltonian variation. The natural trajectory is therefore more probable than the fluctuated trajectories. A very curious link thus appears between the principle of least action and the second law of thermodynamics when one places it in our domain of ideas ([1]).

The situation of a particle in permanent energetic contact with a hidden thermostat presents a certain analogy with that of a granule that is visible in an optical microscope and is subject to the action of gravity, and which is found to be in suspension in a fluid whose molecules constitute a hidden thermostat. At the moment when Jean Perrin pursued his celebrated experiments on the granules of this type, some thermodynamic theories that were broadened to include fluctuations of altitude in the granules in the fluid were developed by Smoluchowsky, notably with the method of Einstein, and are very interesting to compare with our theory.

Quantum transitions and the "prerogative" of monochromatic states. – Since Bohr's theory of atom in 1913, one has attributed a character to the quantum transitions that make a quantized system in one stationary state pass to another one that one can describe as mystical. Indeed, one abandons any hope of giving it any sort of picture, and Bohr did not hesitate to assert that it "transcended" any description in terms of space and time. That is why Schrödinger was led to say, ironically, that in the present quantum theory one minutely describes the stationary states where nothing happens, but refuses to say anything about the quantum transitions where something does happen.

([1]) In figurative terms, one can say that the natural trajectory follows the path along the bottom of a valley of negentropy. For the case where there is a "kinetic focal point" with respect to A between the points A and B, see *C. R. Acad. Sc.* **257** (1963), 1822.

The idea that was introduced by the theory of the double solution – viz., that wave mechanics, in the final analysis, must rest on nonlinear equations permits one to think that if the quantum transitions are missing from the usual theory then that is because they constitute essentially nonlinear processes. They will be very brief transitory processes that are analogous to the ones that have already been encountered in several nonlinear theories of mechanics and physics when there is a brief passage from one limit cycle to another. That very attractive idea was already envisioned some years ago by Ĉap and Destouches, and it was revived recently by Fer, Lochak, and Andrade e Silva, who published some notes on the subject that were quite interesting.

Now, when Fer, Lochak, and Andrade e Silva were informed of my first note in August of 1961 on the thermodynamics of the isolated particle, after I quite justly remarked that my formulas that were deduced from the relation $dS = dQ / T$ apply only to reversible phenomena, they suggested to me that the very brief transitory states that they had considered could indeed be transformations that were accompanied by a brief augmentation of the entropy, and that the passage from one stationary state to another could indeed constitute the crossing of a valley of entropy (or mountain of negentropy).

During last winter in 1962-1963, I was led to reflect more profoundly on these interesting suggestions. In order to make the orientation of my thoughts on that subject understandable, I begin by remarking that in the usual theory, one associates a sort of prerogative to the states that one can describe as "monochromatic," where I intend that term to mean, on the one hand, the stationary states of the quantum system that are associated with a monochromatic stationary wave that is represented by a proper Hamiltonian function, and on the other hand, in the case of particles in progressive motion, the states that are associated with groups of bounded waves such that the greater part of their extension can be assimilated into a monochromatic plane wave. This prerogative consists in saying that one regards these monochromatic states as more normally realized than the states that are represented by a superposition of proper functions or monochromatic plane waves. In the beginning of Bohr's atomic theory, one considered the atom as being always necessarily found in a stationary state, and when, much later, one translated the Bohr theory into the language of wave mechanics, one assumed that the states that were represented by the superposition of proper functions had only a fleeting existence and that, by definition, the atom was always caught by the observation in a stationary state that was represented by proper functions. In quantum field theory, the same prerogative manifests itself as the fact that the "occupation numbers" are generally referred to monochromatic plane waves. In one of the very penetrating articles that were dedicated to a critique of current quantum concepts, Schrödinger emphasized, with good reason, this prerogative of monochromatic states: He thought that it was unjustified because *a priori* a superposition state has a more general character than a monochromatic state (i.e., the function $\psi = \sum_i c_i \psi_i$ is more general than the function $\psi = \psi_i$). Nevertheless, the success of the hypothesis that monochromatic states are essentially a prerogative hardly permits one to doubt, contrary to the opinion of Schrödinger, that this prerogative is unjustified.

However, everything can be explained if one assumes (as I was led to do) that the superposition states, which have a lower entropy and, in turn, a lower probability than the monochromatic states, are in some way unstable, and that the quantum transitions have an irreversible character (and perhaps even in certain cases of reversible processes) that always rapidly drives particles or systems towards monochromatic states with a higher entropy. It then becomes obvious that the superposition states will generally be of brief duration and will tend to transform into monochromatic states such that the conditions that are imposed upon the particle or system permit exchanges of energy or quantity of motion that are necessary for this to happen.

In order to establish a solid basis for the idea that I just presented, one must prove in a general fashion, starting with the hidden thermodynamics of particles, that the entropy of the superposition states is less than that of the monochromatic states. At present, I do not possess a general proof of that fact, but I have found some proofs in a number of particular cases. In order to not extend this discussion too much, I will not give these proofs here, and I will be content to point out the principle of them.

First, I once more write down the definitions (5) and (21) of the quantum potential and the entropy:

$$Q = M_0 c^2 - m_0 c^2, \qquad S = S_0 - k\,\frac{M_0}{m_0},$$

which lead us to write:

$$S = S_0 - k - k\frac{Q}{c^2}. \tag{28}$$

Now, in the case that I studied the quantum potential Q was zero in the monochromatic states, in such a way that these states have the "standard" entropy $S = S_0 - k$. In the superposition states, Q is non-zero, and one can easily prove that $\bar{Q} = \int Q\,a^2\,d\tau$ is positive. From (28), the entropy of a superposition state will then have the mean value:

$$\bar{S} = S_0 - k - k\,\frac{\bar{Q}}{c^2} < S_0 - k. \tag{29}$$

It then indeed results that in the case that I studied then entropy of the superposition states was, in the mean, less than that of the monochromatic states.

It is interesting to remark that the instability of the superposition states thus seems to be coupled to the appearance in these states of a positive quantum potential that entails an augmentation of the proper mass M_0 of the particle or system, so, from the fundamental formula (21), there is a diminishing of the entropy. This seems to show the close link that exists between our hidden thermodynamics and the new notions that were introduced by the theory of guidance and the theory of the double solution ([2]).

Conclusion. – I just presented a view of the collective evolution of my thoughts since the moment where, recalling my original ideas on the true physical significance of wave mechanics, I sought to give a reinterpretation whose most essential points, to my eyes, are that it reestablishes the clear picture of the particle as a very small object that is

([2]) On the definition of energy in the hidden thermodynamics of particles, *see* Louis de Broglie, *C. R. Acad. Sc.* **257** (1963), 1430.

constantly localized in space in the course of time and that it endows the wave with the character of a field with an objective existence that propagates in space in the course of time. In the context of the theory of the double solution, I was then led to recall the "guiding formula," which attributes a well-defined motion to the particle that is analogous to the one that classical hydrodynamics attributes to a fluid molecule, a molecule whose motion at each instant is given by one of the streamlines of the hydrodynamical flow.

I then obtained a clear picture, but I subsequently recognized that it was, without a doubt, very rigid. I then glimpsed the necessity of superposing a sort of random thermal agitation with the mean regular motion of the particle that is defined by the guiding formula, just as one superposes a random thermal agitation with the regular motion of a fluid molecule that is defined by the streamlines in the real motion, moreover. For the fluid molecule, this thermal agitation is due to its collisions with the other fluid molecules, but for an isolated particle at the microscopic level, a similar agitation can only be attributed to its permanent energetic contact with a sort of hidden thermostat that one can naturally identify with the sub-quantum medium of Bohm and Vigier.

Moreover, if one reflects on this then it seems quite natural that the present probabilistic interpretation of wave mechanics, which is often referred to as "quantum mechanics," would ultimately lead to the introduction of new thermodynamic notions, since it introduces probabilities into the behavior of a microphysical particle, even when it appears to be isolated, and that the close link that exist between statistical thermodynamics and the appearance of probabilities in physical theories almost necessarily suggests the existence of a hidden thermodynamics as a result of this behavior. Einstein has indeed sensed this For a long time now, and he thinks that the intervention of probabilities into wave mechanics must lead one to attribute a sort of Brownian motion to microphysical particles. Now, what one calls Brownian motion is also what one calls fluctuations in thermodynamics.

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