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EXPOSÉS DE PHYSIQUE THÉORIQUE

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XIII

A NEW CONCEPTION

OF

LIGHT

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INTRODUCTION

In the present pamphlet, we would like to summarize the attempts that we have been pursuing for some time with an eye towards constructing a new theory of the photon and the electromagnetic field of the light. Even if the attempt will be shown to be insufficient at the end of this account, we believe that the ideas that it suggests and the analogies that it suggests are very interesting in themselves, to the extent that they would merit some attention.

It seems useful to us to first point out the general ideas that have guided us in that attempt, which will lead us naturally to establish the plan of the present article.

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The development of our knowledge of light, notably in the last century, has permitted us to establish a certain number of facts, even though one can no longer abstract to any general theory of luminous phenomena from them. If one is to interpret interference and diffraction, then one must first confront the necessity of representing the propagation of light by the propagation of waves. One must then attribute the character of transversality to these waves in order to account for polarization. Moreover, the admirable synthesis that was realized by Maxwell obliges us to attribute an electromagnetic nature to light.

Aside from these essential characteristics of light, experiments during the last thirty or so years has revealed some other things that oblige us to return to the older corpuscular concept, at least to some degree. Above all, it is the photoelectric effect and the Compton effect that have given us the experimental proof of that granular aspect of light.

Today, one is then forced to assume that light possesses, at the same time, a corpuscular aspect and a wave-like aspect, and one must look for a theory that can represent the coexistence of these two aspects. For the moment, believe that this synthesis has been realized completely in wave mechanics. Indeed, this new mechanics has its point of departure in the idea that there is good reason to construct a general theory that intimately associates the notions of wave and corpuscle for matter, as well as for light. The successes that have been enjoyed by that idea insofar as the elements of matter are concerned – electrons, in particular – have been marvelous, as one knows. Notably, it has acquired a splendid direct experimental confirmation by the discovery of electron diffraction in crystals (by Davisson and Germer, Thomson, et al.); it has allowed us to revive and extend considerably all of our theories of atomic and molecular phenomena.

However, aside from light, the success has been much less complete. Wave mechanics has even been able to establish some relationships between wave quantities and corpuscular quantities that are classical today, and are valid for the photon as much as for material corpuscles, but the edification of that basis for a complete theory of light has encountered great difficulties. In the presence of these difficulties, certain physicists even seem inclined to doubt the existence of a true symmetry between light and matter insofar as the nature of their duality is concerned. On that point, we have a completely opposite opinion. The symmetry between matter and light that serves as the basis for the development of wave mechanics is so mentally satisfying that it seems to us to be the profound reason for the success of the new theories such that, as we see things, one will now derive no benefit from abandoning them.

Nevertheless, there is one undeniable fact: The dualistic theory of light, although it has served as the model for the dualistic theory of matter, is presently lagging behind its younger sister. What will explain this paradoxical fact?

One of its causes is certainly the form in which wave mechanics has made its rapid expansion. Indeed, that form is not relativistic. As a result, it can be applied only to corpuscles whose velocities are very small compared to c, and will thus be inappropriate to photons. Moreover, it contains no element that can permit one to define a polarization.

Another cause of the same fact is that the photon possesses certain properties that neatly distinguish it from the electron. First, photons, when they are numerous, obey Bose-Einstein statistics, and not Fermi-Dirac statistics, like electrons. As a result, in the photoelectric effect, the photon disappears – that is, it seems to be annihilated – and there exists no analogous property for material corpuscles. One then senses that in order to construct a theory of the photon, one must, in the first place, employ a relativistic form for wave mechanics that involves symmetry elements like polarization, and in the second place, introduce *something else* in order to differentiate the photon from the other corpuscles.

The first part of this program is realized immediately if one replaces the original form of wave mechanics with the more elaborate form that Dirac gave it in his theory of the magnetic electron. Dirac's theory is indeed relativistic; at least, in the sense that it is applicable to corpuscles that possess any velocity up to the limiting value of c. In addition, it introduces some symmetry elements that present a distinct lineage with those of the polarization of light, and permit one to define some electromagnetic quantities (proper magnetic moment and proper electric moment) that are attached to the corpuscle. Nevertheless, it does not suffice to assume that the photon is a corpuscle of negligible mass that obeys the equations of Dirac's theory because the model for the photon thus obtained will have – so to speak – only one-half the symmetry of a real photon; moreover, it must obey Fermi statistics, and it can be annihilated in the photoelectric effect. Something more is then needed.

One can try to introduce this "something else" in the following fashion: One can assume that the photon is composed, not of one Dirac corpuscle, but two of them. One can account for the fact that these two corpuscles – or *semi-photons* – must be complementary to each other in the same way that the positive electron is complementary to the negative electron in the Dirac theory of "holes" ([1]). Such a pair of complementary corpuscles is capable of being annihilated upon contact with matter and giving up all of its energy, and that accounts for the characteristics of the photoelectric effect perfectly. Moreover, the photon is then composed of two elementary corpuscles, so it must obey Bose-Einstein statistics, which conforms to experiment (viz., Planck's law). Finally, this model for the photon permits one to define an electromagnetic field that is linked to the

([1]) The word "complementary" is therefore employed here in a completely different sense from the one that Bohr defined in his theory of complementarity.

probability of annihilation of a photon, which is a field that has all of the character of Maxwell's light waves.

This new conception of the photon implies the existence of a type of elementary particle (viz., semi-photons) with charge and mass equal to zero – or at least, considerably smaller than that of the electron – that obeys the Dirac equations, and as a result, will be endowed with spin. We shall have to examine the possibility of identifying the semi-photon with the "neutrino" that theoreticians frequently speak of today.

It is true that an important difficulty exists in the new theory that is due to the fact that the principle of superposition is not generally satisfied by the electromagnetic fields that it defines. That difficulty is linked to the very asymmetric form in which the theory is first presented to us, and we shall point out a more symmetric conception that might possibly eliminate it.

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Having thus summarized the main aspects of our attempt, we will present it according to the following plan: We first recall the principles of Dirac's theory that will serve as the basis for our research. Conforming to the spirit of quantum theories, we are then forced to find *operators* that correspond to the potentials and electromagnetic fields that are attached to a Dirac corpuscle, and we examine the consequences of these definitions. Then, in order to give the fields the necessary physical sense of numerical quantities that measure the probabilities of interaction between matter and radiation, we shall call the *densities of matrix elements* that are defined with the aid of the corresponding operators, and represent a certain "transition" of the Dirac corpuscle from an initial energetic state to a conveniently-chosen final energetic state, the "components of the electromagnetic field that is attached to the Dirac corpuscle". Upon physically interpreting the formulas that are obtained, we will then arrive at the conception of the photon that we suggested above. We will then note that there are certain difficulties – or anomalies – in our theory, and show that they pertain to the asymmetric character of the presentation that was adopted, and we point out a more satisfying symmetric theory. Finally, after having touched upon the main problem of the interaction of matter and light, we shall make the physical significance of our hypotheses more precise in regard to certain aspects.

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SUMMARY OF DIRAC'S THEORY

We would not like to present the entirety of Dirac's theory that one finds in other books ([1]), but only recall some essential points that we will have to use later on.

Dirac developed his theory for the electron. We shall adopt a more general viewpoint by calling any corpuscle whose wave Ψ obeys the Dirac equation, with arbitrary values for the two constants "electric charge" and "proper mass," a "Dirac corpuscle."

If we take the case in which no external field acts on the corpuscle then the equation of propagation of the wave Ψ of a Dirac corpuscle will then be:

(1) $$\frac{1}{c}\frac{\partial \Psi_k}{\partial t} = \frac{\partial}{\partial x}\alpha_1\Psi_k + \frac{\partial}{\partial y}\alpha_2\Psi_k + \frac{\partial}{\partial z}\alpha_3\Psi_k + \kappa\mu_0 c\alpha_4\Psi_k,$$

where the index k takes the values 1, 2, 3, 4. The α_i are the well-known Hermitian matrices of the Dirac theory that satisfy the matrix equations:

(2) $$\alpha_i\,\alpha_j + \alpha_j\,\alpha_i = \delta_{ij},$$

where δ_{ij} represents the matrix with 0 if $i \neq j$ and 1 if $i = j$, here. The matrices α_i operate on the indices of the function Ψ_k according to the rule:

(3) $$\alpha_k\Psi_k = \sum_{j=1}^{4}(\alpha_i)_{kj}\Psi_j.$$

We specify the values of the α by setting:

(4) $$\begin{cases} \alpha_1 = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix}, \qquad \alpha_2 = \begin{vmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{vmatrix}, \\ \alpha_3 = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix}, \qquad \alpha_4 = \begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \end{cases}$$

in all of what follows ([2]).

([1]) For example, see the author's book, *L'électron magnétique*, Hermann, Paris, 1934.

([2]) As a result of the manner in which we wrote the equation of propagation here, the matrix α_4 will have the opposite sign to the one that is employed in our book on the magnetic electron.

Finally, recall that in equation (1), μ_0 represents the proper mass of the corpuscle, and that one has:

$$\kappa = \frac{2\pi i}{h}. \tag{5}$$

The states of uniform, rectilinear motion with positive energy for a Dirac corpuscle that correspond to monochromatic, plane wave solutions of equation (1) are given by the formulas:

$$\Psi_k^+ = a_k\, e^{\kappa[Wt - p(\alpha x + \beta y + \gamma z)]} \qquad (k = 1, 2, 3, 4), \tag{6}$$

where α, β, γ are the direction cosines of the direction of propagation, and the constant W and p (viz., the energy and quantity of motion, resp.) are coupled by the relativistic relation:

$$\frac{W^2}{c^2} = p^2 + \mu_0^2 c^2, \tag{7}$$

where the constant W is assumed to be positive here.

In the formulas (6), the a_k are four complex constants, only two of which are arbitrary. For example, if we arbitrarily give a_3 and a_4 the complex values A and B then we will have:

$$a_1 = -\frac{[\gamma A + (\alpha + i\beta)B]p}{W/c + \mu_0 c}, \quad a_2 = -\frac{[(\alpha - i\beta)A - \gamma B]p}{W/c + \mu_0 c}, \quad a_3 = A, \quad a_4 = B. \tag{8}$$

Equation (1) makes the z-axis play a privileged role. That is related to the fact that knowing of the function Ψ will permit one to answer questions in which the z-axis plays a special role, such as this one: What is the mean value of the component of the proper magnetic moment of the corpuscle along the z-axis?

Dirac proved that if one makes a change of reference system by a Lorentz transformation then the equation of propagation in the new system will keep the same *form* as in the old one; i.e., the form (1) with the same values for the α_i. The wave function Ψ_k will be subjected to a transformation that is expressed by the linear formula:

$$\Psi'_k = \sum_{l=1}^{4} \Lambda_{kl} \Psi_l, \tag{9}$$

where the coefficients Λ_{kl} depends naturally upon the change of coordinates that was performed. The transformation (9) that the four Ψ_k are subjected to is not that of the components of a space-time vector. In order to find the quantities that transform like the components of a tensor, it is necessary to form certain bilinear combinations of the Ψ_k, to which we shall return later on. All of this is expressed easily with the aid of the theory of spinors, which we can present here.

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The Dirac equations present a very remarkable peculiarity that first seemed to constitute a grave objection against the theory, but which seems to have turned into an advantage since the discovery of the positive electron: It admits the existence of states of motion with negative energy. Indeed, the relation (7) between W and p does not cease to be realized if one changes the sign of W of p. In particular, it results from this that any solution of type (6), where W is assumed to be positive, will correspond to an "inverse" solution of the form:

$$\Psi_k^- = b_k\, e^{-\kappa[Wt - p(\alpha x + \beta y + \gamma z)]}, \qquad (k = 1, 2, 3, 4), \tag{10}$$

with the same value of W and p. Formula (10) represents a wave that propagates in the direction α, β, γ and corresponds to a motion with negative energy $-W$ and a negative quantity of motion $-p$. Since the corpuscular velocity u must be defined to be equal to the group velocity in wave mechanics, one will have:

$$u = \frac{\partial W}{\partial p}, \tag{11}$$

and, as a result, the velocity of a corpuscle that is linked to the inverse wave (10) must be regarded as the same in magnitude and direction as that of a corpuscle that is linked to the wave (6).

Of the four complex constants b_k in formula (10), only two of them are independent, and if we give b_1 and b_2 two arbitrary, complex values C and D then we will have:

$$b_1 = C, \quad b_2 = D, \qquad b_3 = -\frac{[(\alpha + i\beta)D + \gamma C]p}{W/c + \mu_0 c}, \quad b_4 = -\frac{[(\alpha - i\beta)C - \gamma D]p}{W/c + \mu_0 c}. \tag{12}$$

In particular, if one knows the constants (8) for a solution of type (6) then one will obtains the inverse solution by setting:

$$\left\{\begin{aligned} & b_1 = -a_4^* = -B^*, \quad b_2 = a_3^* = A^*, \quad b_3 = a_2^* = -\frac{[(\alpha + i\beta)A^* - \gamma B^*]p}{W/c + \mu_0 c}, \\ & b_4 = -a_1^* = \frac{[\gamma A^* + (\alpha - i\beta)B^*]p}{W/c + \mu_0 c}, \end{aligned}\right. \tag{13}$$

where the asterisk marks the passage to the complex conjugate quantity. This is easily verified by setting $C = -B^*$ and $D = A^*$ in formulas (12). In other words, knowing a positive-energy solution Ψ_1^+, Ψ_2^+, Ψ_3^+, Ψ_4^+, one can deduce the inverse solution from the formulas:

$$\Psi_1^- = -(\Psi_4^+)^*, \qquad \Psi_2^- = -(\Psi_3^+)^*, \qquad \Psi_3^- = -(\Psi_2^+)^*, \qquad \Psi_4^- = -(\Psi_1^+)^*. \tag{14}$$

It is then easy to see that the four quantities:

$$(15) \qquad \varphi_1 = (\Psi_1^-)^* = -\Psi_4^+, \quad \varphi_2 = (\Psi_2^-) = \Psi_3^+, \quad \varphi_3 = (\Psi_3^-) = \Psi_2^+, \quad \varphi_4 = (\Psi_4^-)^* = -\Psi_1^+,$$

will obey the equations:

$$(16) \qquad \frac{1}{c}\frac{\partial \varphi_k}{\partial t} = \frac{\partial}{\partial x}(\alpha_1\varphi_k) - \frac{\partial}{\partial y}(\alpha_2\varphi_k) + \frac{\partial}{\partial z}(\alpha_3\varphi_k) - \kappa\mu_0 c(\alpha_4\varphi_k) \qquad (k = 1, 2, 3, 4),$$

which we shall call the "complementary equations" to equations (1).

If one is given the values (4) and (5) of the α_i and κ; resp., then (16) will be nothing but the conjugate equation to (1). One can also say that one obtains (16) by starting with (1) and changing the sign of the proper mass terms, and reversing the sense of the *y*-axis with respect to that of the *x* and *z* axes (i.e., passing from left-handed axes to right-handed ones).

The existence of negative-energy solutions, such as (10), first seemed to be a serious difficulty for Dirac's theory, in that it was impossible to exclude the associated motions. Indeed, these motions will correspond to properties that are completely foreign to the corpuscles that are thus animated. For example, by diminishing their energy, one increases their velocity; in other words, they accelerate upon braking. Nothing like that has been observed.

Dirac found an ingenious way of eliminating the difficulty that is defined by the existence of negative-energy states. He remarked that, according the Pauli exclusion principle, one can have no more than one electron per state, so he imagined that all of the possible negative-energy states are occupied in the normal state of the universe. It then results that there will be a uniform density of negative-energy electrons in the universe, and Dirac assumed that this uniform density would be unobservable. However, there are more electrons than would be necessary to fill all of the negative-energy states, and the surplus will occupy positive-energy states and constitute the electrons that are manifested in experiments. In the exceptional cases, a negative-energy electron can pass into a positive-energy state by a stimulated transition. There will then simultaneously appear an experimental electron and a "hole" – or "gap" – in the distribution of negative-energy electrons. Now, Dirac showed that such a gap is composed like a corpuscle that has the mass of the electron and an electric charge that is equal and opposite to it ([1]). It will be a corpuscle that is, in some way, complementary to the usual electron, namely, an anti-electron or positive electron. The theory of holes first left many physicists skeptical, because no one had ever observed positive electrons ([2]). Thanks to the beautiful experimental work of Anderson and Blackett, as well as Occhialini, we now know that under exceptional circumstances, one can now see positive electrons (i.e., positrons) manifest themselves that indeed seem to correspond to Dirac's predictions. These positive electrons are studied in laboratories almost everywhere today – notably, by Thibaud and Joliot in France. They are, in a sense, unstable, because if a gap encounters a negative electron then they can combine. There must then be the simultaneous

([1]) See, notably, P. A. M. DIRAC, *Les principes de la Mécanique quantique,* Presses Universitaires, Paris, 1931, pp. 299, *et seq.*.

([2]) It is appropriate to observe that DIRAC first sought to identify the complementary corpuscle with the proton.

disappearance of two electrons with opposite signs and the emission of radiation, and that fact has indeed been indicated by the recent experiments of Thibaud and Joliot.

Generalizing the Dirac concepts, we say that any corpuscle that obeys the Dirac equations (with an arbitrary mass and charge) must correspond to a complementary corpuscle that is to the former what the positive electron is to the negative one. In the case of the absence of a field, the wave equation of the complementary corpuscle will be precisely equation (16). A comparison of equations (1) and (16) will permit one to perceive the relation that exists between complementary corpuscles. It is just that subject that deserves to be elaborated upon.

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Wave mechanics, in its original form with just one wave function, makes observable, physical quantities correspond to operators that belong to the class of linear and Hermitian operators. If A denotes such an operator then the general principles of wave mechanics will permit one prove that the mean value of an observable, physical quantity that corresponds to A will be equal to:

$$\bar{A} = \int \Psi^* A \Psi \, d\tau \qquad (d\tau = dx\,dy\,dz) \tag{17}$$

for a corpuscle whose state is defined by the wave function Ψ.

One can also define the "mean density":

$$\bar{a} = \Psi^* A \Psi. \tag{18}$$

On the other hand, one can form some very important quantities with the aid of the operator A that were first introduced by Heisenberg that are the elements of a matrix. Indeed, let $\Psi_1, \Psi_2, \ldots, \Psi_i, \ldots$ be the sequence of wave functions that correspond to the well-defined, stable energy states that the corpuscle can assume (i.e., proper functions of the energy operator). One calls the quantities:

$$A_{ij} = \int \Psi_i^* A \Psi_j \, d\tau \tag{19}$$

the matrix elements that corresponds to the operator A.

These elements are divided into two categories. First, there will be diagonal elements for which $i = j$. They will be attached to a well-defined dynamical state of the corpuscle, and from (17), they will give the mean value of the quantity A when the corpuscle is in that state. There will then be non-diagonal elements ($i \neq j$) that involve two different states and correspond to the "transition" from the state j to the state i.

In the case of the absence of a field, the dynamical states of the corpuscle are defined by monochromatic, plane waves:

$$\Psi = a\, e^{\kappa[Wt - p(\alpha x + \beta y + \gamma z)]}, \tag{20}$$

and these are the functions that must figure in the formation of the matrix elements. Here, there are some difficulties that relate to the fact that the waves define a continuous sequence and occupy all of space. However, one can circumvent these difficulties by the use of some mathematical tricks (such as the consideration of proper differentials), upon which we shall not insist.

Naturally, instead of considering the matrix elements (19), one can also consider the corresponding densities:

(21) $$a_{ij} = \Psi_i^* A \Psi_j .$$

All of these considerations can be easily transposed into the Dirac theory, where one employs four functions Ψ, instead of just one. Here again, one will make any measurable, physical quantity correspond to a linear and Hermitian operator A, but with the essential extension that the operator can operate, not just on the variables x, y, z, but also on the index k of the wave functions. The mean value of the physical quantity that corresponds to the operator A for a corpuscle whose wave functions are Ψ_1, Ψ_2, Ψ_3, Ψ_4 will be:

(22) $$\bar{A} = \int \sum_{k=1}^{4} \Psi_k^* A \Psi_k \, d\tau .$$

One can also define the density of the mean value:

(23) $$\bar{a} = \sum_{k=1}^{4} \Psi_k^* A \Psi_k .$$

One can further define the matrix elements with the aid of the operator A. Let $\Psi_{i,1}$, $\Psi_{i,2}$, $\Psi_{i,3}$, $\Psi_{i,4}$ be the wave functions that correspond to a certain well-defined energy state of the corpuscle that is characterized by the index i; we set:

(24) $$A_{ij} = \int \sum_{k=1}^{4} \Psi_{i,k}^* A \Psi_{j,k} \, d\tau .$$

We will again have to distinguish the diagonal elements ($i = j$) that are attached to a stable state and give the mean value of the quantity A for a corpuscle that is found in that state, and then the non-diagonal element ($i \neq j$) that is attached to the transition from one state to another. Finally, in place of the matrix elements (24), one can envision the corresponding densities:

(25) $$a_{ij} = \sum_{k=1}^{4} \Psi_{i,k}^* A \Psi_{j,k} .$$

For the case of the absence of a field, the well-defined stable energy states are represented by monochromatic, plane waves (6) or (10), which define a continuous sequence. One encounters the same difficulties that one does in wave mechanics with one function Ψ, and one eliminates them by the same tricks.

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The densities of the mean values of the matrix elements that are defined by formulas (23) and (25) have great importance in the Dirac theory, because they permit one to introduce quantities that transform as tensors.

As a first example, we point out the quadri-vector density-flux that is associated with a corpuscle whose wave Ψ is known. This quadri-vector has the components:

$$\rho = \sum_{k=1}^{4} \Psi_k^* \Psi_k , \qquad \rho \mathbf{v} = -c \sum_{k=1}^{4} \Psi_k^* \boldsymbol{\alpha} \Psi_k , \tag{26}$$

in which $\boldsymbol{\alpha}$ denotes the matrix-vector whose components are $\alpha_1, \alpha_2, \alpha_3, \alpha_4$.

One will have the relation:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathbf{v}) = 0 \tag{27}$$

between the quantities (26), which expresses the permanence of the corpuscle.

Now, set:

$$\sigma_1 = i\alpha_2\alpha_3 , \qquad \sigma_2 = i\alpha_3\alpha_1, \qquad \sigma_3 = i\alpha_1\alpha_2 , \qquad \sigma_4 = i\, \alpha_1\alpha_2\alpha_3 . \tag{28}$$

Then form the quantities:

$$\left\{ \begin{aligned} S_x &= \frac{h}{4\pi} \sum_{k=1}^{4} \Psi_k^* \sigma_1 \Psi_k , \qquad S_y = \frac{h}{4\pi} \sum_{k=1}^{4} \Psi_k^* \sigma_2 \Psi_k , \\ S_z &= \frac{h}{4\pi} \sum_{k=1}^{4} \Psi_k^* \sigma_3 \Psi_k , \qquad S_4 = \frac{h}{4\pi} \sum_{k=1}^{4} \Psi_k^* \sigma_4 \Psi_k \end{aligned} \right. \tag{29}$$

with the aid of these Hermitian operators.

They define the components of a space-time quadri-vector (or, more precisely, a completely antisymmetric third-rank tensor). The first three components are the mean densities of the moment of proper rotation for the corpuscle in the state Ψ. One then obtains the components of the moment of proper rotation (i.e., spin) by integrating the quantities S_x, S_y, and S_z over space. Consider a monochromatic, plane solution of the type (6). One easily confirms that if one has $B = 0$ then the component of the spin along the z-axis will be equal to $h / 4\pi$, while if one has $A = 0$ then that same component will become $- h / 4\pi$. In the former case, one will have a *dextrogyrous* wave, while in the latter case, it will be *levogyrous*. In the general case where A and B are both different from zero, the wave Ψ will be a superposition of a dextrogyrous wave and a levogyrous one.

For a complementary corpuscle that obeys equation (16), everything happens as if α_2 were changed to $- \alpha_2$, which will change the sign of σ_2 . Moreover, it is easy to verify, with the aid of formulas (15), that the complementary corpuscle to a dextrogyrous corpuscle is dextrogyrous, while the complementary corpuscle to a levogyrous corpuscle is levogyrous.

Now, consider the six operators that are the components of $\boldsymbol{\alpha}\alpha_4$ and $\boldsymbol{\sigma}\alpha_4$. We form the six components of a second-order, antisymmetric tensor with them:

$$(30)\qquad \boldsymbol{\mu} = \sum_{k=1}^{4} \Psi_k^* \boldsymbol{\sigma}\alpha_4 \Psi_k \cdot B, \qquad \boldsymbol{\pi} = \sum_{k=1}^{4} \Psi_k^* \boldsymbol{\alpha}\alpha_4 \Psi_k \cdot B \qquad \left(B = \frac{eh}{4\pi m_0 c} \right).$$

These are the mean densities of the "proper magnetic moment" and the "proper electric moment" of the corpuscle in the state Ψ. Their integrals over space give the mean values of these moments. B is the Bohr magneton.

We finally note that the quantities:

$$(31)\qquad I_1 = \sum_{k=1}^{4} \Psi_k^* \alpha_4 \Psi_k, \qquad I_2 = \sum_{k=1}^{4} \Psi_k^* \alpha_1\alpha_2\alpha_3\alpha_4 \Psi_k$$

are invariants, the first of which undoubtedly relates to the proper mass ([1]).

We conclude with an important remark. If an operator A is such that the density of the mean value $\sum_{k=1}^{4} \Psi_k^* A \Psi_k$ possesses a certain tensorial character then the same thing will be true for all of the densities of matrix elements of the form $\sum_{k=1}^{4} \Psi_{i,k}^* A \Psi_{j,k}$. Indeed, the tensorial character will depend solely upon the manner by which the Ψ_k transform under a change of space-time coordinates. For example, the quantities $\sum_{k=1}^{4} \Psi_{i,k}^* \Psi_{j,k}$ and $\sum_{k=1}^{4} \Psi_{i,k}^* \boldsymbol{\alpha} \Psi_{j,k}$ define the components of a space-time quadri-vector for any value of i and j.

([1]) See *Electron Magnétique*, pp. 223.

FIRST VIEWPOINT

Electromagnetic operators that are attached to a Dirac corpuscle

With the goal of constructing a theory of the photon, we shall now try to attach potentials and electromagnetic fields to a corpuscle that obeys equation (1). In order to do this, in this paragraph we shall first of all place ourselves at a *first viewpoint* by seeking to define operators that correspond to these magnitudes. We commence by defining the operators that correspond to the components of an "electromagnetic potential" quadri-vector by setting:

(32) $$\boldsymbol{\mathcal{A}} = -K\,\boldsymbol{\alpha}, \qquad \mathcal{V} = K \cdot 1.$$

$\boldsymbol{\mathcal{A}}$ is the vector potential and $\mathcal{V}$ is the scalar potential. K is a constant that we leave indeterminate.

By symmetry, we introduce the operators that correspond to the components of a "potential of the second kind" by the formulas:

(33) $$\boldsymbol{\mathcal{A}}' = -K'\,\boldsymbol{\sigma}, \qquad \mathcal{V}' = K' \cdot \sigma_4 .$$

In order to define the operators that correspond to the fields, we proceed by analogy, if we remember the formulas that couple the fields to the potentials in the classical theory, and we take the six operators that are defined by:

(34) $$\left\{\begin{aligned} \boldsymbol{\mathcal{E}} &= -\operatorname{grad}\mathcal{V} - \frac{1}{c}\frac{\partial \boldsymbol{\mathcal{A}}}{\partial t} = \left(-\operatorname{grad}\cdot 1 + \frac{1}{c}\frac{\partial \boldsymbol{\sigma}}{\partial t}\right)K, \\ \boldsymbol{\mathcal{H}} &= \operatorname{rot}\boldsymbol{\mathcal{A}} = -K\operatorname{rot}\boldsymbol{\alpha}. \end{aligned}\right.$$

By symmetry, we introduce an electromagnetic field of the second kind by the formulas:

(35) $$\left\{\begin{aligned} \boldsymbol{\mathcal{H}}' &= -\operatorname{grad}\mathcal{V}' - \frac{1}{c}\frac{\partial \boldsymbol{\mathcal{A}}'}{\partial t} = K'\left(-\operatorname{grad}\sigma_4 + \frac{1}{c}\frac{\partial \boldsymbol{\sigma}}{\partial t}\right), \\ \boldsymbol{\mathcal{E}}' &= \operatorname{rot}\boldsymbol{\mathcal{A}}' = -K'\operatorname{rot}\boldsymbol{\sigma}. \end{aligned}\right.$$

The definitions (32) and (35) raise some difficulties that we must examine. A first difficulty is due to the presence of the operator $\partial / \partial t$ in these definitions, which is not an operator that possesses a well-defined Hermitian character in wave mechanics. A second difficulty consists in the fact that if the constant K is real then the potential operators (32) will be Hermitian, while the field operators (32) will not, and the opposite will be true if K is pure imaginary. The same difficulty arises for K' and the operators of the second kind. We do not consider these objections to be very grave, because we would not like to attribute the character of operators that correspond to observable quantities to the operators that we defined above, but simply the character of operators that permit us to define electromagnetic potentials and fields as densities of matrix elements that will be attached to certain transitions when we pass to a second viewpoint later on. Indeed, one knows that the obligation for an operator to be Hermitian when it corresponds to an observable, physical quantity in wave mechanics implies the following fact: The measurable values and the mean values of the observable quantity must all be real (this is physically obvious), but they will only be so if the corresponding operator is Hermitian. The hermiticity of the operator is thus coupled to the fact that quantity is measurable. It is therefore possible to consider non-Hermitian operators in wave mechanics, on the condition that one not make them correspond to measurable quantities.

We now establish some relations that are interesting from the standpoint of their correspondence with classical theory. Two obvious first relations that are analogous to the first group of Maxwell equations are the operator identities:

$$\text{div}\, \boldsymbol{\mathcal{H}} \equiv 0, \qquad \text{rot}\, \mathbf{E} \equiv -\frac{1}{c}\frac{\partial \boldsymbol{\mathcal{H}}}{\partial t}. \tag{36}$$

They signify that for *any* function Ψ with four components (which is or is not a solution (1)), one has:

$$\text{div}\, \boldsymbol{\mathcal{H}}\, \Psi_k \equiv 0 \qquad \left(\text{rot}\, \boldsymbol{\mathcal{E}} + \frac{1}{c}\frac{\partial \boldsymbol{\mathcal{H}}}{\partial t}\right)\Psi_k \equiv 0. \tag{37}$$

In order to obtain relations that have the form of the second group of Maxwell's equations, we apply the operators div $\boldsymbol{\mathcal{E}}$ and $\frac{1}{c}\frac{\partial \boldsymbol{\mathcal{E}}}{\partial t}$ – rot $\boldsymbol{\mathcal{H}}$ to a function Ψ that is a solution to equation (1). We then obtain two categories of terms, one of which contains the derivatives $\partial / \partial t$, $\partial / \partial x$, $\partial / \partial y$, $\partial / \partial z$, and the other one, in which the proper mass μ_0 appears explicitly; we call the former "purely kinetic terms" and the latter, "mass terms" ([1]). Suppose that the purely kinetic terms outweigh the mass terms by a considerable margin. This will always be realized if the proper mass μ_0 is zero, but it will also be true approximately for $\mu_0 \neq 0$ if the motion of the corpuscle is sufficiently rapid, as one easily confirms for. With that hypothesis, one will find:

$$\text{div}\, \boldsymbol{\mathcal{E}} \cdot \Psi_k \equiv 0, \qquad \left(\frac{1}{c}\frac{\partial \boldsymbol{\mathcal{E}}}{\partial t} - \text{rot}\, \boldsymbol{\mathcal{H}}\right)\Psi_k \equiv 0. \tag{38}$$

These equations are valid only for functions Ψ_k that are solutions of equation (1), and no longer identities that are valid for arbitrary Ψ_k. One has already found an analogous situation in the different attempts to do this. (See an article by Dirac, Fock, and Podolsky in the Physikalische Zeitschrift der Sowjetunion, Band 2, 1932, pp. 458.)

Under the same hypothesis (viz., the predominance of the purely kinetic terms over the mass terms), one will also find the relation:

([1]) The proper mass μ_0 does appear explicitly in the purely kinetic terms. Nevertheless, these terms can depend upon μ_0 if the constant K that figures in them depends upon it.

$$(39)\qquad \left(\frac{1}{c}\frac{\partial \mathcal{V}}{\partial t} + \operatorname{div} \boldsymbol{\mathcal{A}}\right)\Psi_k = 0,$$

which is valid for Ψ_k that are solutions of (1). This is the analogue of the Lorentz relation between the potentials, which is a relation that appears here as something that is equivalent to the Dirac equation (1) (when one can neglect the mass term), which is remarkable.

We could develop relations that are analogous to the relations (37-39) for the potentials and the fields of the second kind, but since these relations have no classical analogues, we believe that they are useless, so we stop there.

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We would like to conclude these considerations with an interesting analogy that relates to energy. In Dirac's theory, the Hamiltonian operator that corresponds to energy has the expression:

$$(40)\qquad H = \frac{c}{\kappa}\left(\alpha_1 \frac{\partial}{\partial x} + \alpha_2 \frac{\partial}{\partial y} + \alpha_3 \frac{\partial}{\partial z}\right) + \mu_0\, c^2 \alpha_4\,,$$

whose square is written:

$$(41)\qquad H^2 = \mu_0^2 c^4 + \frac{c^2}{\kappa^2}\Delta\,,$$

where Δ is the Laplacian operator. As is well-known, if one considers motions that are slow enough that the kinetic term in Δ is small with respect to the massive term in μ_0^2 then one can set, approximately:

$$(42)\qquad H = \mu_0\, c^2 + \frac{1}{2\kappa^2 \mu_0}\Delta\,.$$

This is the Newtonian approximation, upon which the old rational mechanics rests, as well as wave mechanics with one function Ψ. Indeed, the term Δ / κ^2 is the quantum analogue of the square p^2 of the quantity of motion, and in turn, $(1 / 2\kappa^2\mu_0)\,\Delta$ is the analogue of the classical expression $(1 / 2\mu_0)\,p^2$ of kinetic energy.

Now, one easily sees:

$$(43)\qquad \boldsymbol{\mathcal{E}}\,\Psi_k = K\left(-\operatorname{grad} + \frac{1}{c}\frac{\partial \boldsymbol{\alpha}}{\partial t}\right)\Psi_k = K\left(-\operatorname{rot}\frac{\boldsymbol{\sigma}}{i}\,\Psi_k + \kappa\mu_0 c\,\boldsymbol{\alpha}\,\alpha_4\Psi_k\right).$$

One then sees that upon setting:

$$(44)\qquad \boldsymbol{\mathcal{E}}_c = -K \operatorname{rot}\frac{\boldsymbol{\sigma}}{i}, \qquad\qquad \boldsymbol{\mathcal{E}}_m = -K\,\kappa\mu_0 c\,\boldsymbol{\alpha}\,\alpha_4,$$

one can decompose the "electric field" operator into purely kinetic field $\mathcal{E}_c$ and a massive field $\mathcal{E}_m$. As for the "magnetic field" operator, from its definition (34), it is entirely of the purely kinetic type. One thus has:

$$(45)\qquad |\,\mathcal{H}^2\,| = \left|\mathcal{H}_c^2\right| = K^2\,(\mathrm{rot}\,\boldsymbol{\alpha})^2 = 2K^2\,\Delta, \qquad\qquad \left|\mathcal{E}_c^2\right| = K^2\left(\mathrm{rot}\,\frac{\boldsymbol{\sigma}}{i}\right)^2 = 2K^2\,\Delta = |\,\mathcal{H}^2\,|.$$

It then results that:

$$(46)\qquad (|\,\mathcal{E}^2\,| + |\,\mathcal{H}^2\,|) = 2K^2\,\Delta.$$

Since, from (42), the purely kinetic term of H is equal to $(1\ /\ 2\kappa^2\mu_0)\ \Delta$ in the Newtonian approximation, it is easy to identify it with the expression (46) by a suitable choice of the constant K ([1]).

Without wanting to attribute too much importance to that statement, it seems to us that it might suggest the following conclusion: The classical expression for the electromagnetic energy density by one-half the sum of the squares of the fields can be only a "Newtonian approximation" that is valid only when the velocity of the electromagnetic energy is weak with respect to c, an approximation that is, consequently, unacceptable for the radiant energy.

([1]) The constant K is then found to depend upon μ_0 .

SECOND VIEWPOINT

Electromagnetic potentials that are attached to a Dirac corpuscle

We now assume a second viewpoint. In the preceding paragraph, we sought to attach *operators* to a Dirac corpuscle that would have electromagnetic significance. From now on, we shall seek to attach numerical quantities to such a corpuscle that can be identified with the electromagnetic potentials and fields.

From the physical viewpoint, the electromagnetic fields of light can be considered to be quantities that govern the exchange of energy between matter and radiation. To speak the probabilistic language of quantum mechanics, one can say that these fields determine the probabilities of the transitions that the corpuscle that takes the form of the "photon" is subject to, which are transitions that correspond to the exchanges of energy between the photon and material elements. We must then seek to define the electromagnetic quantities that are attached to the photon as quantities that are coupled to the "photoelectric" transitions that the photon experiences, such that these transitions must have, as we explained in the introduction, the very special character of being accompanied by a sort of annihilation of the photon.

Guided by these general ideas, we will try to define the electromagnetic fields of light as densities of matrix elements that are attached to a certain transition that the Dirac corpuscle experiences, with the aid of which, we will seek to construct the photon. We must employ the densities and not the matrix elements themselves, in such a fashion that the fields will depend upon the coordinates x, y, z.

In this paragraph, we suppose essentially that the initial state of the photon ([1]) is a uniform, rectilinear motion with positive energy. That initial state is therefore represented with a wave of type (6) whose constants are related by the relations (7) and (8).

But how do we choose the final state? Two remarks can guide us:

1. The wave Ψ of the final state must obviously be a solution of (1).

2. The final state must be determined when one is given the initial state, since otherwise there would be an ambiguity in the definition of the fields.

We defer an examination of the physical sense of this hypothesis until later, and choose the final state to be the state that is represented by the inverse wave of the given initial wave Ψ^+; i.e., the wave Ψ^- that is defined by formulas (14) when one starts with Ψ^+.

Having assumed that, we define the electromagnetic potentials and fields that are attached to the corpuscle in the initial state Ψ^+ as the densities of matrix elements that are generated by the operators $\mathcal{A}$, $\mathcal{V}$, $\mathcal{E}$, $\mathcal{H}$ of the preceding paragraph and relate to the transition $\Psi^+ \rightarrow \Psi^-$. In other words, we set:

([1]) Or, more precisely, the DIRAC corpuscle that speak of.

$$(47)\qquad \begin{cases} \mathbf{a} = \sum_{k=1}^{4} (\Psi_k^-)^* \mathcal{A}\, \Psi_k^+, & v = \sum_{k=1}^{4} (\Psi_k^-)^* \mathcal{V}\, \Psi_k^+, \\ \mathbf{e} = \sum_{k=1}^{4} (\Psi_k^-)^* \mathcal{E}\, \Psi_k^+, & \mathbf{h} = \sum_{k=1}^{4} (\Psi_k^-)^* \mathcal{H} \Psi_k^+, \end{cases}$$

where **a**, v, **e**, and **h** denote the electromagnetic quantities that we are introducing. By virtue of the remark that was made at the end of paragraph 2, the quantities thus defined indeed have the desired relativistic invariance. These are complex quantities, moreover. That should not be too astonishing, because there is good reason to think that in quantum theory one must employ complex expressions for electromagnetic quantities ([1]). The only transition probabilities that have any physical meaning will be proportional to the squares of the field moduli, moreover.

One can write formulas (47) in another way by introducing the functions φ_k that are defined by formulas (15).

One then finds that by henceforth writing Ψ_k, instead of Ψ_k^+, for ease of notation, one will get:

$$(48)\qquad \mathbf{a} = \sum_{k=1}^{4} \varphi_k \mathcal{A}\, \Psi_k\,, \qquad \ldots, \qquad \mathbf{h} = \sum_{k=1}^{4} \varphi_k \mathcal{H}\, \Psi_k\,,$$

which are formulas in which there are no longer any asterisks.

Upon taking the expressions (15) for the φ_k as functions of the Ψ_k into account, one will easily find that:

$$(49)\qquad \begin{cases} \sum_{k=1}^{4} \varphi_k \alpha_4 \Psi_k = 0, & \sum_{k=1}^{4} \varphi_k \alpha_1 \alpha_2 \alpha_3 \alpha_4 \Psi_k = 0, \\ \sum_{k=1}^{4} \varphi_k \boldsymbol{\sigma} \Psi_k = 0, & \sum_{k=1}^{4} \varphi_k \sigma_4 \Psi_k = 0. \end{cases}$$

Upon then introducing the explicit expressions (34) for the operators $\mathcal{E}$ and $\mathcal{H}$ into the definitions (48) of the fields, and taking the equation of propagation (1) that is satisfied by the Ψ_k into account, as well as the relations (2), (28), and (49), one will find some new expressions for the fields, namely:

$$(50)\qquad \mathbf{e} = K\,\kappa\mu_0\, c \sum_{k=1}^{4} \varphi_k \boldsymbol{\alpha}\alpha_4\, \Psi_k\,, \qquad\qquad \mathbf{h} = K\,\kappa\mu_0\, c \sum_{k=1}^{4} \varphi_k \frac{\boldsymbol{\sigma}}{i} \alpha_4\, \Psi_k\,,$$

which, when combined with the first two of relations (48), will give us the expressions that define the potentials and fields in our present theory.

([1]) Notably, see E. NÉCULCÉA, *Sur la théorie du rayonnement, d'apres M. C. G. Darwin, Actualités scientifiques*, no. 56, Hermann, Paris, 1933.

It would be easy to define the potentials and fields of the second kind with the aid of the operators $\mathcal{A}'$, …, $|\mathcal{H}'|$, of the preceding paragraph. However, by virtue of (49), these potentials and these fields would all be zero.

We perform the complete calculation of the potentials and fields upon assuming, to simplify the formulas, that one has taken the direction of propagation of the initial wave Ψ to be the z-axis, which does not restrict the generality. We therefore make $\alpha = \beta = 0$, $\gamma = 1$ in formulas (6) and (8).

By then expressing the φ_k as functions of the Ψ_k using relations (15), one will then find the following table by calculations that present no difficulties:

$$(51)\quad \begin{cases} a_x = K[\Psi_1^2 + \Psi_4^2 - \Psi_2^2 - \Psi_3^2] = 2K\dfrac{\mu_0 c}{\Delta}(B^2 - A^2)\cdot P, \\ a_y = Ki[\Psi_3^2 + \Psi_4^2 - \Psi_1^2 - \Psi_2^2] = 2K\dfrac{\mu_0 c}{\Delta} i(B^2 + A^2)\cdot P, \\ a_z = 2K[\Psi_3\Psi_4 - \Psi_1\Psi_2] = 2K\dfrac{W}{\Delta c}\cdot 2AB\cdot P, \\ v = 2K[\Psi_2\Psi_3 - \Psi_1\Psi_4] = 2K\dfrac{p}{\Delta}\cdot 2AB\cdot P, \\ e_x = K\kappa\mu_0 c[(\Psi_1^2 + \Psi_3^2) - (\Psi_2^2 + \Psi_4^2)] = -2K\kappa\mu_0\dfrac{W}{\Delta}(B^2 - A^2)\cdot P, \\ e_y = K\kappa\mu_0 ci[(\Psi_1^2 + \Psi_3^2) + (\Psi_2^2 + \Psi_4^2)] = -2K\kappa\mu_0\dfrac{W}{\Delta}(B^2 + A^2)\cdot P \\ e_z = K\kappa\mu_0 c\cdot 2[\Psi_3\Psi_4 + \Psi_1\Psi_2] = -2K\kappa\dfrac{\mu_0^2 c^2}{\Delta}\cdot 2AB\cdot P, \\ h_x = -K\kappa\mu_0 c\cdot 2i[\Psi_2\Psi_4 - \Psi_1\Psi_3] = -2K\kappa\mu_0 c\dfrac{p}{\Delta} i(B^2 + A^2)\cdot P, \\ h_y = K\kappa\mu_0 c\cdot 2[\Psi_1\Psi_3 + \Psi_2\Psi_4] = 2K\kappa\mu_0 c\dfrac{p}{\Delta}(B^2 - A^2)\cdot P, \\ h_z = K\kappa\mu_0 c\cdot 2i[\Psi_1\Psi_4 + \Psi_2\Psi_3] = 0, \end{cases}$$

a table in which we have set:

$$(52)\qquad \Delta = \frac{W}{c} + \mu_0 c, \qquad P = e^{\kappa[2wt - 2pz]}.$$

An examination of table (51) will lead to some interesting observations. First of all, the fields (51) are related to the potentials by the formulas:

$$(53)\qquad \mathbf{e} = \frac{1}{2}\left(-\operatorname{grad} v - \frac{1}{c}\frac{\partial \mathbf{a}}{\partial t}\right), \qquad \mathbf{h} = \frac{1}{2}\operatorname{rot}\mathbf{a}.$$

The factor 1/2 might be surprising. Here is its origin: If F denotes an operator that contains the differentiation symbols $\partial / \partial x, \ldots, \partial / \partial t$ linearly then one will obviously have:

(54) $$\sum_{k=1}^{4} \varphi_k F \Psi_k = \frac{1}{2} F \sum_{k=1}^{4} \varphi_k \Psi_k ,$$

because φ_k and Ψ_k depend upon the space and time coordinates through the same exponential factor. We therefore already suspect that the present theory is not very satisfying, because it does not treat the functions φ_k and Ψ_k in the same way, one of which is placed to the left of the operators $\mathcal{A}$, … in formulas (48), while the other one is placed on the right. What follows will show us the profundity of that remark.

If the proper mass μ_0 is small – or, more precisely, if one has $\mu_0 c^2 \ll W$ – then the component e_x will be negligible, and the fields (51) will be in the same position as the electromagnetic fields in Maxwell's theory of the light wave for a light wave whose frequency and wave length have the values:

(55) $$\nu = \frac{2W}{h}, \qquad \lambda = \frac{h}{2p} .$$

Here again, we find a factor of 2 that can surprise us, but one explains its presence easily by remarking that in the transition $\Psi^+ \to \Psi^-$, the Dirac corpuscle that we are considering will pass from the positive energy $+ W$ to the negative energy $- W$. Its change in energy is then $2W$, and the frequency that is associated with it must be $2W / h$, from the quantum relation. Nonetheless, here again, a more symmetric conception will permit us to render the relations (55) quite natural, later on.

For a negligible e_x, we then obtain a plane electromagnetic wave that is monochromatic and transversal, and its direction of propagation will coincide with that of the wave Ψ from which it is defined. For that wave, the two fields will be equal ([1]), perpendicular to each other, and perpendicular to the direction of propagation. Moreover, according to the values of the complex constants A and B of the wave, the electromagnetic wave can possess all of the character of physically-realizable polarization. Indeed, if one of the constants A and B is zero then one will have an electromagnetic wave that is circularly-polarized and right in one case, but left, in the other. If the constants A and B have the same modulus then one will have an electromagnetic wave with rectilinear polarization, where the azimuth of the polarization will be determined by the difference between the arguments of the complex constants A and B. Finally, in the general case where A and B are arbitrary, the electromagnetic wave will possess an elliptical polarization. We leave the verification of these results to the reader.

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([1]) Because then $W = cp$, roughly.

It is interesting to see whether our electromagnetic magnitudes (48) verify the classical relations. Starting with the definitions, one easily proves the following relations:

$$(56)\qquad \begin{cases} -\dfrac{1}{c}\dfrac{\partial \mathbf{h}}{\partial t} = \operatorname{rot}\mathbf{e}, & \operatorname{div}\mathbf{h} = 0, \\ \dfrac{1}{c}\dfrac{\partial \mathbf{e}}{\partial t} = \operatorname{rot}\mathbf{h} - 2\kappa^2\mu_0^2c^2\mathbf{a}, & \operatorname{div}\mathbf{e} = 2\kappa^2\mu_0^2c^2 v. \end{cases}$$

We indicate the proof of one of these relations; the last one, for example. We start with the equation of propagation:

$$(57)\qquad \frac{1}{c}\frac{\partial \Psi_k}{\partial t} = \frac{\partial}{\partial x}(\alpha_1\Psi_k) + \frac{\partial}{\partial y}(\alpha_2\Psi_k) + \frac{\partial}{\partial z}(\alpha_3\Psi_k) + \kappa\mu_0 c\alpha_4\Psi_k \quad (k = 1, 2, 3, 4).$$

Multiply this by α_4 on the left. By taking (2) into account, one will obtain:

$$(58)\qquad \frac{1}{c}\alpha_4\frac{\partial \Psi_k}{\partial t} + \frac{\partial}{\partial x}(\alpha_1\alpha_4\Psi_k) + \frac{\partial}{\partial y}(\alpha_2\alpha_4\Psi_k) + \frac{\partial}{\partial z}(\alpha_3\alpha_4\Psi_k) = \kappa\mu_0 c\Psi_k .$$

Multiply (58) by φ_k on the left and sum over the index k. The first term that is obtained in the left-hand side will be zero, by virtue of (49). What remains is then:

$$(59)\qquad \sum_{k=1}^{4}\varphi_k\left[\frac{\partial}{\partial x}(\alpha_1\alpha_4\Psi_k) + \frac{\partial}{\partial y}(\alpha_2\alpha_4\Psi_k) + \frac{\partial}{\partial z}(\alpha_3\alpha_4\Psi_k)\right] = \kappa\mu_0 c\sum_{k=1}^{4}\varphi_k\Psi_k .$$

Multiply this by the constant $K\kappa\mu_0 c$ and take (54) into account. We will indeed find the last relation in (56). The other relations in (56) are proved in an analogous fashion.

From (56), we conclude that if the terms in μ_0^2 are negligible (i.e., if one again has $\mu_0 c^2 \ll W$) then Maxwell equations will be verified.

Moreover, it will result from the last two equations in (56) that the Lorentz relation between the potentials:

$$(60)\qquad \frac{1}{c}\frac{\partial v}{\partial t} + \operatorname{div}\mathbf{a} = 0$$

will be verified. It can, moreover, be deduced from (57) directly upon multiplying by $K\varphi_k$ and summing over k, since $\sum_{k=1}^{4}\varphi_k\alpha_4\Psi_4$ is zero.

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PHYSICAL INTERPRETATION OF THE PRECEDING RESULTS

HYPOTHESIS ON THE NATURE OF THE PHOTON

We must now reflect a bit on the physical significance that the preceding considerations might have.

We have implicitly assumed that a Dirac corpuscle that obeys equation (1) exists in the photon. The photoelectric interaction between radiation and matter corresponds to a transition $\Psi^+ \to \Psi^-$, to which one attaches electromagnetic quantities (48) whose role must be that of measuring the probability of that interaction. But what does the wave Ψ^- represent? It represents a negative energy state of the Dirac corpuscle, and that state must be considered to be unoccupied before the transition $\Psi^+ \to \Psi^-$, since that transition is possible. We are then led to consider the photon as being composed of the set of a Dirac corpuscle and a "hole" whose motions are coupled, because they are animated with the same corpuscular velocity in the same direction (refer to the considerations that accompanied equation (11)).

The transition $\Psi^+ \to \Psi^-$ corresponds to the annihilation of the photon, so the corpuscle with energy W and quantity of motion p must fill in the energy gap $-W$ and the quantity of motion $-p$, while giving up matter with energy $2W$ and quantity of motion $2p$, which would correspond to values of frequency ν and wave length λ that are given by the relations (55). If the constant B in Ψ^+ is zero then Ψ^+ will be a monogyrous wave, and the corpuscle in the initial state will have spin $h / 4\pi$. Under the transition $\Psi^+ \to \Psi^-$, its spin will pass from $h / 4\pi$ to $-h / 4\pi$, and it will, in turn, give up a moment of rotation $h / 2\pi$ to the matter. On the contrary, if the constant A is zero then Ψ^+ will be a monogyrous wave with the opposite sense to the one n the preceding case, and the corpuscle in its initial state will have spin equal to $-h / 4\pi$. Under the transition, $\Psi^+ \to \Psi^-$, its spin will pass from $-h / 4\pi$ to $h / 4\pi$, and in turn, it will give up a moment of rotation of $-h / 2\pi$ to the matter. Now, in the two special cases that we just envisioned (viz., $A = 0$ or $B = 0$), the electromagnetic wave that is associated with the transition $\Psi^+ \to \Psi^-$ will have, as we have seen from our definitions, a circular polarization, which will be right in one case and left in the other one. One then sees that the two circular polarizations of the electromagnetic waves will correspond to the transfer of a moment of rotation that is equal to $h / 2\pi$ or $-h / 2\pi$, respectively, from the light to the matter. This result is in accord with the analyses that were presented by other some authors ([1]).

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One can present our hypothesis on the nature of the photon in a more symmetric form whose importance we will understand better later on by transforming the preceding image in a manner that will correspond to the passage from formulas (47) to formulas (48),

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([1]) See DARWIN, *loc. cit.*

precisely. Dirac's argument has indeed shown us that the absence of the corpuscle from the wave Ψ^- of energy – W and quantity of motion – p is equivalent to the existence of a complementary of energy W and quantity of motion p whose wave is given by (15). We can thus say that under our hypotheses the photon will be composed of a set of two complementary corpuscles waves Ψ and φ are linked by the relations (15) that satisfy the equations of propagation (15) and (16), respectively. That new manner of presenting things, which corresponds to the expression (48) for electromagnetic quantities, allows us to express the situation that was pointed out above in a very different fashion. The two complementary constituents of the photon have the same energy W, the same quantity of motion p, and the same proper mass μ_0, so the photon will collectively have an energy of $2W$, a quantity of motion $2p$, and a proper mass $2\mu_0$ that are coupled by the formula:

$$\frac{(2W)^2}{c^2} = (2p)^2 + (2\mu_0)^2\, c^2. \tag{61}$$

The photon is capable of passing from that initial state to a final state where the energy, quantity of motion, proper mass, and spin are zero (state of annihilation); that photoelectric transition will correspond to the mutual annihilation of two complementary constituents. The photon then gives up all of its energy $2W$ and quantity of motion $2p$ to the matter, which makes the relations (55) entirely natural.

As far as spin is concerned, first suppose that $B = 0$ in the expression for Ψ. One then easily accounts for the fact that the two complementary corpuscles each have spin $h / 4\pi$ in the initial state. The collective photon will then have spin $h / 2\pi$, and upon passing through the photoelectric transition to the state of zero spin, it will give up a moment of rotation $h / 2\pi$ to the matter. One will likewise find – $h / 2\pi$ in the case where $A = 0$.

We finally note that the fact that we are considering the photon to be composed of two corpuscles of spin $h / 4\pi$ raises difficulties that relate to statistics, as we pointed out in the introduction. Indeed, photons certainly obey Bose-Einstein statistics, which only leads to Planck's law of black-body radiation. Now, one knows that these statistics must be followed by physical entities that are composed of an *even* number of Dirac corpuscles, since any entity that is composed of an odd number such corpuscles must follow Fermi statistics. There is then a very strong objection against any attempt to assimilate the photon to a Dirac corpuscle; it will vanish at the moment when we regard the photon as a set of two complementary Dirac corpuscles.

NECESSITY OF A MORE SYMMETRIC THEORY
SKETCH OF THAT THEORY

We now arrive at some remarks that are of paramount importance. If we assume the idea of a photon that is composed of two complementary corpuscles, and if we reflect upon all of the theory that was developed in the preceding paragraphs then we will arrive at the conclusion that there is something inexact in the presentation of that theory. Indeed, we regard the photon as a set of two complementary corpuscles whose waves Ψ and φ are coupled by the relation 15. It then results that the initial state of the photon must be characterized by the set of two wave functions Ψ and φ. Now, cast your eyes upon the definitions (48) of the electromagnetic quantities. We see that the function φ figures *on the left* of the operator there, as if it were characterizing the *final* state of the photon. It then seems obvious to us that the function φ must be *to the right* of the operator, since, in reality, it belongs to the initial state of the photon, and the place to the left of the operator must be reserved for the wave function that characterizes the finial state of the photon that is "annihilated" after the photoelectric effect.

It is easy to account for the fact that this defect in the theory is closely linked to the abnormal factors of 1/2 in formula (53). Indeed, these factors arise from the fact that the electromagnetic operators in definitions (48) operate upon only the function Ψ, and not on the function φ. That is already one important reason for us to attempt to ameliorate the theory in the sense that was indicated above.

However, there is another reason that obliges us to make that attempt that is much more important. That main reason is the following: The theory of fields that we developed in the last paragraph does not satisfy the superposition principle. In order to be able to recover the interpretation of classical interference and diffraction phenomena, it is indeed necessary that the following superposition principle be satisfied: "If the wave Ψ is the superposition of several monochromatic, plane waves then the electromagnetic fields that are attached to the wave Ψ must be obtained by the superposition of the electromagnetic fields that correspond to each monochromatic component of Ψ taken separately." Now, it is easy to see that this is not true with our present definitions. (This is also the reason for limiting ourselves to the case where Ψ is monochromatic, up to now.) For example, consider the simple case of a wave Ψ that is composed of the superposition of two monochromatic, plane waves Ψ_1 and Ψ_2 . These waves Ψ_1 and Ψ_2 correspond to the waves φ_1 and φ_2 under formulas (15), and if we then construct the expressions for the electromagnetic quantities with the aid of definition (48) then we will obtain, in addition to terms of the form:

$$\sum_{k=1}^{4} \varphi_{1,k} F \Psi_{1,k} \qquad \text{and} \qquad \sum_{k=1}^{4} \varphi_{2,k} F \Psi_{2,k} \ ,$$

some "rectangular" terms such as:

$$\sum_{k=1}^{4} \varphi_{1,k} F \Psi_{2,k} \;.$$

It is these rectangular terms that prevent the superposition principle from being valid for the fields, and we even see that the difficulty comes from the fact that the functions φ and Ψ are separate from each other in the definition (48).

It is then necessary for us to develop a more symmetric theory that will eliminate these difficulties. We shall try to sketch out that theory here.

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We start with the fact that the functions Ψ and φ of two complementary corpuscles obey equations (1) and (16), which we transcribe explicitly here in the form:

(62)
$$\begin{cases} \dfrac{1}{c}\dfrac{\partial \Psi_i}{\partial t} = \dfrac{\partial}{\partial x}\sum_{l=1}^{4}(\alpha_1)_{il}\Psi_l + \dfrac{\partial}{\partial y}\sum_{l=1}^{4}(\alpha_2)_{il}\Psi_l + \dfrac{\partial}{\partial z}\sum_{l=1}^{4}(\alpha_3)_{il}\Psi_l + \kappa\mu_0 c\sum_{l=1}^{4}(\alpha_4)_{il}\Psi_l, \\[2ex] \dfrac{1}{c}\dfrac{\partial \varphi_i}{\partial t} = \dfrac{\partial}{\partial x}\sum_{m=1}^{4}(\alpha_1)_{im}\varphi_m - \dfrac{\partial}{\partial y}\sum_{m=1}^{4}(\alpha_2)_{im}\varphi_m + \dfrac{\partial}{\partial z}\sum_{m=1}^{4}(\alpha_3)_{im}\Psi_m - \kappa\mu_0 c\sum_{m=1}^{4}(\alpha_4)_{im}\varphi_m, \end{cases}$$

where the indices i and k can take the values 1, 2, 3, 4.

Multiply the first equation in (62) by φ_k and the second one by Ψ_i, and the add them, while remarking that the Ψ and the φ both have derivatives in x, y, z. One then gets:

(63)
$$\begin{aligned} \frac{1}{c}\frac{\partial}{\partial t}(\Psi_i\varphi_k) = {} & \frac{1}{2}\frac{\partial}{\partial x}\left[\sum_{l=1}^{4}(\alpha_1)_{il}\Psi_l\varphi_k + \sum_{m=1}^{4}(\alpha_1)_{km}\Psi_i\varphi_m\right] \\ & + \frac{1}{2}\frac{\partial}{\partial y}\left[\sum_{l=1}^{4}(\alpha_2)_{il}\Psi_l\varphi_k - \sum_{m=1}^{4}(\alpha_2)_{km}\Psi_i\varphi_m\right] \\ & + \frac{1}{2}\frac{\partial}{\partial z}\left[\sum_{l=1}^{4}(\alpha_3)_{il}\Psi_l\varphi_k + \sum_{m=1}^{4}(\alpha_3)_{km}\Psi_i\varphi_m\right] \\ & + \kappa\mu_0 c\left[\sum_{l=1}^{4}(\alpha_4)_{il}\Psi_l\varphi_k - \sum_{m=1}^{4}(\alpha_4)_{km}\Psi_i\varphi_m\right]. \end{aligned}$$

Now, we obviously have:

$$\varphi_k = \sum_{m=1}^{4}\delta_{km}\varphi_m \qquad \text{and} \qquad \Psi_i = \sum_{m=1}^{4}\delta_{il}\Psi_l \;.$$

We then set:

(64)
$$\Phi_{ik} = \Psi_i\,\varphi_k\,,$$

and define four matrices with 16 rows and 16 columns by the formulas:

(65)
$$\begin{cases} (A_1)_{ik,lm} = \frac{1}{2}[(\alpha_1)_{il}\,\delta_{km} + (\alpha_1)_{km}\,\delta_{il}], \\ (A_2)_{ik,lm} = \frac{1}{2}[(\alpha_2)_{il}\,\delta_{km} - (\alpha_2)_{km}\,\delta_{il}], \\ (A_3)_{ik,lm} = \frac{1}{2}[(\alpha_3)_{il}\,\delta_{km} + (\alpha_3)_{km}\,\delta_{il}], \\ (A_4)_{ik,lm} = \frac{1}{2}[(\alpha_4)_{il}\,\delta_{km} - (\alpha_4)_{km}\,\delta_{il}], \end{cases}$$

where the alternation of signs in the right-hand side is essential. One can then write equation (63) in the form:

(66)
$$\frac{1}{c}\frac{\partial \Phi_{ik}}{\partial t} = \frac{\partial}{\partial x}\sum_{l,m=1}^{4}(A_1)_{ik,lm}\Phi_{lm} + \frac{\partial}{\partial y}\sum_{l,m=1}^{4}(A_2)_{ik,lm}\Phi_{lm} + \frac{\partial}{\partial z}\sum_{l,m=1}^{4}(A_3)_{ik,lm}\Phi_{lm} + \kappa(2\mu_0)\,c\sum_{l,m=1}^{4}(A_4)_{ik,lm}\Phi_{lm},$$

or also in the symbolic form:

(67)
$$\frac{1}{c}\frac{\partial \Phi_{ik}}{\partial t} = \left[\frac{\partial}{\partial x}A_1 + \frac{\partial}{\partial y}A_2 + \frac{\partial}{\partial z}A_3 + 2\kappa\mu_0 c\,A_4\right]\Phi_{ik} \qquad (i, k = 1, 2, 3, 4).$$

To our eyes, equation (67), which represents a system of 16 simultaneous equations that relate to 16 components of the wave function Φ, constitutes the true wave equation of the photon. From now on, we must study it for its own sake, with no concern for the manner in which it was obtained. Indeed, we have reason to again consider Ψ and φ separately, while we must now consider these two wave functions as both being based upon the function Φ.

The matrices A are Hermitian, but it is essential to note that *one does not have:*

(68)
$$A_i\,A_j + A_j\,A_i = 2\delta_{ij}\,;$$

i.e., the relations (2) are not valid for the A. This complicates the study of equation (67) somewhat.

We immediately note a fact that seems to be essential: Equation (67) admits a solution Φ^0 that is capable of representing the state of the annihilated photon. That solution is the following one:

(69)
$$\Phi_{ik}^{0} = \delta_{ik}\,.$$

In order to see this, it will suffice to remark that the derivatives of the constants δ_{ik} are all zero, so the only thing to be verified is that:

(70) $$A_4 \Phi^0_{ik} = \sum_{e,m=1}^{4} [(\alpha_4)_{ie} \delta_{km} - (\alpha_4)_{km} \delta_{ie}] \delta_{em} = (\alpha_4)_{km} - (\alpha_4)_{ki}$$

is zero for all values of i and k. Now, this is obvious, since α_4 is a diagonal matrix.

That remarkable solution Φ^0 obviously corresponds to a state of zero energy and quantity of motion ([1]). It is thus appropriate for the representation of the annihilation state of the photon.

Starting from that last remark, we shall quite naturally seek to define the electromagnetic quantities that are attached to the photon as being linked to the transition $\Phi \to \Phi^0$. We take the following operators to be the electromagnetic operators, which are an obvious transposition of the ones that we previously made use of:

(71) $$\begin{cases} \mathcal{A} = -K\mathbf{A}, \qquad \mathcal{V} = K \cdot 1, \\ \mathcal{E} = -\operatorname{grad} \mathcal{V} - \dfrac{1}{c}\dfrac{\partial \mathbf{A}}{\partial t} = K\left(-\operatorname{grad} + \dfrac{1}{c}\dfrac{\partial \mathbf{A}}{\partial t}\right), \\ \boldsymbol{\mathcal{H}} = \operatorname{rot} \boldsymbol{\mathcal{A}} = -K \operatorname{rot} \mathbf{A}, \end{cases}$$

where $\mathbf{A}$ is the vector-operator with components A_1, A_2, A_3 . We then define the electromagnetic quantities by the formulas:

(72) $$\begin{cases} \mathbf{a} = \sum_{i,k=1}^{4} \Phi^0_{ik} \mathcal{A} \Phi_{ik}, \qquad v = \sum_{i,k=1}^{4} \Phi^0_{ik} \mathcal{V} \Phi_{ik}, \\ \mathbf{e} = \sum_{i,k=1}^{4} \Phi^0_{ik} \mathcal{E} \Phi_{ik}, \qquad \mathbf{h} = \sum_{i,k=1}^{4} \Phi^0_{lk} \mathcal{H} \Phi_{ik}. \end{cases}$$

Upon appealing to (67) and (65), one shows by a somewhat detailed calculation that one can also write:

(73) $$\begin{cases} \mathbf{e} = K\kappa(2\mu_0)c \sum_{i,k=1}^{4} \Phi^0_{ik} \mathbf{A}\, A_4 \Phi_{ik}, \\ \mathbf{h} = K\kappa(2\mu_0)c \sum_{i,k=1}^{4} \Phi^0_{ik} \dfrac{\mathbf{S}}{i} A_4 \Phi_{ik}, \end{cases}$$

where $\mathbf{S}$ is the vector-operator whose components are $iA_2 A_3$, $iA_3 A_1$, $iA_1 A_2$, which is then analogous to the vector-operator $\boldsymbol{\sigma}$ in formula (28).

We can verify that formulas (73) are the generalizations of formulas (50). For example, one first has:

(74) $$(A_1 A_4)_{ik,lm} = \sum_{n,p=1}^{4} (A_1)_{ik,np} (A_4)_{np,lm}$$

([1]) It will result from what follows that the spin of the photon is likewise zero in that state.

$$= \frac{1}{4}[(\alpha_1\alpha_4)_{il,}\delta_{km} - (\alpha_1\alpha_4)_{km,}\ \delta_{il} + (\alpha_1)_{km}(\alpha_4)_{il} - (\alpha_1)_{il}(\alpha_4)_{km}] .$$

Hence, upon taking into account the Hermitian character of the *a*, along with their reality, one will get:

$$(75)\qquad \sum_{i,k=1}^{4}\Phi_{ik}^{0}A_1A_4\Phi_{ik} = \frac{1}{4}\sum_{i,k,l,m=1}^{4}\delta_{ik}[(\alpha_1\alpha_4)_{il}\,\delta_{km} - (\alpha_1\alpha_4)_{km}\,\delta_{il} + \cdots]\Phi_{lm} = \sum_{l,m=1}^{4}\alpha_1\alpha_4)_{lm}\,\Phi_{lm} .$$

If one then refers to formula (64) then one will see that the definition (73) of e_x is indeed the prolongation of the definition (50), and one will easily make that same statement for the other components of the fields. However, our new definitions have the great advantage over the old ones of combining the initial solitary states of the two complementary constituents of the photon into just one function Φ, and thus escaping the objection of non-superposition. Indeed, it is obvious that our new definitions (72-73) will satisfy the superposition principle, because if the initial state of the photon is the superposition of two states whose wave functions are $\Phi^{(1)}$ and $\Phi^{(2)}$ then we will have:

$$(76)\qquad e_x = K\kappa(2\mu_0)\ c\ \sum_{i,k=1}^{4}\Phi_{ik}^{0}A_1A_4(\Phi_{ik}^{(1)} + \Phi_{ik}^{(2)})$$

$$= K\kappa(2\mu_0)\ c\left[\sum_{i,k=1}^{4}\Phi_{ik}^{0}A_1A_4\Phi_{ik}^{(1)} + \sum_{i,k=1}^{4}\Phi_{ik}^{0}A_1A_4\Phi_{ik}^{(2)}\right], \text{ etc.}$$

and there is indeed the superposition of fields.

On the other hand, since the functions that depend upon space and time coordinates here are all on the right of the operator in the definition (72), we are certain to have:

$$(77)\qquad \mathbf{e} = -\,\text{grad}\ v - \frac{1}{c}\frac{\partial \mathbf{a}}{\partial t}, \qquad\qquad \mathbf{h} = \text{rot}\ \mathbf{a},$$

with the abnormal factor of 1/2 in formulas (53).

*

* *

We would not like to develop the theory of equation (67) for the photon completely here; we would only like to point out the broad strokes.

We have four wave functions in the Dirac equation, two of which – viz.,Ψ_1 and Ψ_2 – correspond to the spin $h / 4\pi$ and the other two – viz., Ψ_2 and Ψ_4 – correspond to spin – $h / 4\pi$. Hence, for any monochromatic solution there will exist two constants *A* and *B* that indicate the proportions in which the dextrogyrous wave and the levogyrous waves are superposed in order to define the wave in question. Here, we have 16 functions Φ, and we can see that they are divided into three groups: First, there are the eight functions whose indices are 11, 22, 33, 44, 13, 24, 31, and 42 (i.e., indices of the same parity), which correspond to spin 0. Then, there are four functions whose indices are 21, 43, 23, and 41 (i.e., indices with different parity, the first of which is even), which correspond to spin $h / 2\pi$. In general, *three* constants will enter into a monochromatic solution of equation (67) that will indicate the proportions with which that monochromatic wave is composed of the superposition of the three "pure cases" that correspond to the values 0, $h / 2\pi$, and $-h / 2\pi$ of spin.

For example, envision a wave Φ that is plane and monochromatic and propagates in a direction that we shall take to be the z-axis. Set:

$$\Delta = \frac{W}{c} + 2\mu_0 c, \qquad P = e^{\kappa[Wt - pz]}. \tag{78}$$

We easily obtain the following expressions for the Φ_{ik} ([1]):

$$\tag{79} \begin{cases} \left.\begin{array}{l} \Phi_{11} = \Phi_{22} = \Phi_{33} = \Phi_{44} = \dfrac{pC_0}{\Delta} \cdot P, \\ \Phi_{24} = -\Phi_{13} = +\dfrac{p^2 C_0}{\Delta^2} \cdot P, \quad \Phi_{42} = -\Phi_{31} = C_0 \cdot P, \end{array}\right\} \text{spin } 0 \\ \left.\Phi_{41} = -C_2 \cdot P, \quad \Phi_{23} = +\dfrac{p^2 C_2}{\Delta^2} \cdot P, \quad \Phi_{43} = -\Phi_{21} = \dfrac{pC_2}{\Delta} \cdot P \right\} \text{spin } -\dfrac{h}{2\pi} \\ \left.\Phi_{32} = C_1 \cdot P, \quad \Phi_{14} = -\dfrac{p^2 C_1}{\Delta^2} \cdot P, \quad \Phi_{34} = -\Phi_{12} = \dfrac{pC_1}{\Delta} \cdot P \right\} \text{spin } +\dfrac{h}{2\pi} \end{cases}$$

One then easily finds the electromagnetic quantities that are attached to the corpuscle in the state (79) from the definitions (72-73):

([1]) The constants W and p are coupled by the relation: $W^2 / c^2 = p^2 + (2\mu_0)^2 c^2$.

$$
(80) \quad \begin{cases}
a_x = \dfrac{4K\mu_0 c}{\Delta}(C_1 - C_2)\cdot P, \\
a_y = \dfrac{4K\mu_0 c}{\Delta} i(C_1 + C_2)\cdot P, \\
a_z = \dfrac{4KW}{c\Delta} C_0 \cdot P, \\
v = \dfrac{4Kp}{\Delta} C_0 \cdot P, \\
e_x = -\dfrac{4K\kappa\mu_0 W}{\Delta}(C_1 - C_2)\cdot P, \\
e_y = -\dfrac{4K\kappa\mu_0 W}{\Delta} i(C_1 + C_2)\cdot P, \\
e_z = -\dfrac{16K\kappa\mu_0^2 c^2}{\Delta} C_0 \cdot P, \\
h_x = -\dfrac{4K\kappa\mu_0 cp}{\Delta} i(C_1 + C_2)\cdot P, \\
h_y = -\dfrac{4K\kappa\mu_0 cp}{\Delta}(C_1 - C_2)\cdot P, \\
h_x = 0.
\end{cases}
$$

It is useful to compare table (80) with table (51), and one confirms that they correspond to each other by replacing C_0, C_1, C_2 with AB, $A^2 - B^2$, $A^2 + B^2$, respectively. However, here we have three independent constants, instead of two. Moreover, the total mass $2\mu_0$ of the photon replaces the mass μ_0 of the unique Dirac corpuscle that we considered previously. Relations (77) are obviously satisfied by the expressions (80).

The components of the spin zero corpuscle that is characterized by the constant C_0 give rise to a longitudinal electric wave, which is, moreover, negligible if $\mu_0 c^2 \ll W$. The components of the spin $+ h / 2\pi$ corpuscle give rise to a transverse wave, and the polarization character of that wave will depend upon the values of the constants C_1 and C_2. If C_1 is zero then one will have circular polarization; on the contrary, if C_2 is zero then one will have circular polarization with the opposite sense. If C_1 and C_2 have the same modulus then one will find rectilinear polarization whose azimuth is determined by the difference between the complex constants C_1 and C_2. Finally, for arbitrary C_1 and C_2, one will have the general case of elliptic polarization.

From these few glimpses, one sees that the new form of the theory preserves all of the character of the old theory that is satisfying and eliminates the things that are not. A more complete study of equation (67) would be very interesting.

SOME FINAL REMARKS

We cannot conclude this pamphlet without saying a few words about the manner by which the question of the interaction between matter and radiation presents itself from the standpoint of our theory. Indeed, that question is essential because radiation is manifested only by its action on matter, in such a way that the touchstone of a theory of radiation is, by definition, the way that it permits one to predict the exchanges of energy between the material elements and radiation. Unfortunately, up to the present no one has made a sufficient study of that problem, so here we must confine ourselves to some very brief observations.

In classical theory, as well as in quantum theory, which is modeled on the classical theory by "correspondence," the problem of the interaction between matter and radiation is, in some way, decomposed: On the one hand, there are certain terms in the equations that are utilized for the prediction of the motion of material elements that translate into the existence of the Lorentz force, which are terms that permit one to find out how that motion is modified by the presence of radiation. On the other hand, the right-hand sides of the Maxwell-Lorentz equations, in which the quantities ρ and $\rho\mathbf{v}$ appear, translate into the action of the presence and motion of electric charges on the electromagnetic field and permit one to calculate how a charged, material corpuscle is capable of giving rise to radiation by its motion. That decomposition of the problem of interaction is seen very neatly, for example, in the classical interpretation of dispersion and diffusion, where one begins by calculating the motion that an electron takes on under the influence of incident radiation, and the radiation that is created by the motion of that electron. If one reflects on this then such a decomposition will seem artificial. The interaction between matter and radiation must be described with the aid of just one homogenous system of equations that permits one to just as well predict the action of radiation on matter as the opposite action of matter on radiation. In particular, in the case of elementary phenomena, it must be possible to write an equation that will express the interaction of a photon with an electron. That equation will be a wave equation that involves the function Ψ of the photon-electron system, and in which interaction terms will appear that translate into the Lorentz force. Thanks to that equation, one can calculate the probability for that photon to give up its energy to the electron by its annihilation (i.e., the photoelectric effect), and one must find that probability to be proportional to the square of a matrix element that contains the previously-defined electromagnetic quantities. However, the *same* equation must further serve to calculate the probability of the inverse process, in which an electron gives up a fraction of its energy to an annihilated photon by making it leave the annihilation state, which is a process that constitutes the emission of radiation by matter. One must also be able to treat all of the questions that are concerned with dispersion, diffusion, the Compton effect, etc., with the aid of that very equation.

Here, we will content ourselves with some observations that are entirely perfunctory, without making any attempt to write the photon-electron equation, since we are not certain that we have found a satisfactory form for it.

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Our new conception of the photon implies the existence of a new kind of elementary corpuscle (viz., the semi-photon), whose charge and mass are enormously smaller than that of the electron, which is a corpuscle that is endowed with spin and obeys the Dirac equation (1). Now, certain physicists – such as Pauli and Fermi – have already been led to postulate the existence of corpuscles of that kind in order to interpret the apparent non-conservation of energy during the emission of a continuous spectrum of β rays from certain radioactive bodies. These hypothetical corpuscles have been called "neutrinos" by Fermi. Frances Perrin ([1]) has shown that the neutrino – if it exists – must have a mass that is negligible with respect to that of the electron, but nonetheless, it is not identifiable with the photon, because one does not have that the electromagnetic field is capable of acting on matter. This would be explained quite well if the photon were composed of a neutrino and a complementary corpuscle (i.e., an anti-neutrino), in such a way that the neutrino will be the semi-photon. Indeed, the isolated neutrino obviously cannot be annihilated, and in turn, will not possess an electromagnetic field. On the contrary, when it is associated with an anti-neutrino, it will form a photon that is capable of annihilation, and will consequently have an electromagnetic field.

Once again, the neutrino exists only in the imagination of theoreticians. If experimental research someday succeeds in exhibiting evidence of a corpuscle of that nature then there will be good reason to examine whether there are not situations in which two of these corpuscles are capable of forming a photon, as well as other situations in which a photon can disassociate into two of these corpuscles.

If the neutrino truly has no electric charge then one can hardly see how one can distinguish it from the anti-neutrino. The distinction between corpuscle and complementary corpuscle becomes rather theoretical here. However, one cannot exclude the possibility that the semi-photon can have a small charge ε, because the complementary corpuscle would then have a charge $-\varepsilon$, so the photon would indeed be neutral, which seems to be required by the impossibility of deflecting light by even the most powerful electric fields. In that case, it would be possible to distinguish the two constituents of the photon – at least, in principle.

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Let us add one more word on the proper mass of the photon: In our theory, it seems difficult to suppose that the mass μ_0 of the semi-photon is rigorously zero. It is important to remark that if μ_0 were not zero then *that would be one of the most important constants in physics.* The non-vanishing of the proper mass of the photon that we have already envisioned in previous work ([2]) implies certain difficulties. In particular, there will exist a critical frequency, above which, the electromagnetic field can no longer propagate, and in the neighborhood of which, its velocity will be roughly less than c. We do not see this as an insurmountable objection, but as we have recently shown ([3]), it would imply the existence of an upper limit for μ_0 on the order of 10^{-44} grams, which is 10^{16} times smaller than the mass of the electron.

([1]) *Comptes Rendus* **197** (1933), 1625.
([2]) Doctoral thesis, Masson, 1924.
([3]) *loc. cit.*