"Sur dix relations conséquences des équations du second ordre de Dirac," C. R. Acad. Sci. **218** (1944), 36-38.

THEORETICAL PHYSICS. – *On ten relations that are consequences of the second-order Dirac equations*. Note ([1]) by **ÉMILE DURAND.**

Translated by D. H. Delphenich

One starts with the second-order Dirac equations:

$$(1)\qquad \{\gamma_0\, B - i\varepsilon\, \gamma_{pq}\, R^{pq}\}\ \psi = 0,$$
$$(2)\qquad \psi^{\times}\, \{\gamma_0\, B^* - i\varepsilon\, \gamma_{pq}\, R^{pq}\} = 0,$$

in which one has set:

$$B = \underset{\rightarrow}{\partial}{}^p_p - 2i\varepsilon A^p \underset{\rightarrow}{\partial}_p - \varepsilon^2 A_p A^p - k_0^2, \qquad \varepsilon = \frac{2\pi e}{ch}, \qquad R^{pq} = \partial^p A^q - \partial^q A^p,$$
$$B^* = \underset{\leftarrow}{\partial}{}^p_p + 2i\varepsilon A^p \underset{\leftarrow}{\partial}_p - \varepsilon^2 A_p A^p - k_0^2, \qquad k_0 = \frac{2\pi m_0 c}{h}, \quad \psi^{\times} = i\ \psi^*\ \gamma_4\ .$$

After multiplying (1) on the left by $\psi^{\times}\gamma^A$ and (2) on the right by $\gamma^A\ \psi$, one sums over the index k of the functions ψ; γ^A denotes one of the 16 matrices in Dirac's theory.

Upon successively choosing the γ^A from the five tensorial ranks 0, 1, 2, 3, 4, one will make ten relations appear that will obviously have a tensorial character. All that one must do to obtain the desired relations is then to combine them by addition and subtraction.

Let us see what the contribution will be that is made by the terms that contain the operators B and B^*. If one sets:

$$[\gamma^A] = [\psi^{\times}\ \gamma^A\ \psi],$$
$$T^{A,\,p} = \{[\partial^{\,p}\psi^{\times}\ \gamma^A\ \psi] - [\psi^{\times}\ \gamma^A\ \partial^{\,p}\psi]) + 2i\varepsilon A^p\ [\gamma^A]$$

then one will have:

$$\psi^{\times}\ \{\gamma^A\ (B^* + B)\}\,\psi = \partial^p_p[\gamma^A] - 2\ [\partial^{\,p}\psi^{\times}\ \gamma^A\ \partial_p\psi] + 2i\varepsilon A^p\ T^{A,\,p} + 2\ (\varepsilon^2\ A_p\ A^p - k_0^2),$$
$$\psi^{\times}\{\gamma^A\ (B^* - B)\}\ \psi = \partial_p\ T^{A,\,p}.$$

The ten desired second-order relations are then written ($\bar{A}$ denotes the quantity that is dual to A):

$$(1)\qquad \psi^{\times}\ \{\gamma^0\ (B^* + B)\}\ \psi \ = \ 2i\varepsilon\ \ [\gamma_{pq}]\ R^{pq},$$
$$(2)\qquad \psi^{\times}\ \{\gamma^p\ (B^* + B)\}\ \psi \ = \ 2i\varepsilon\ \ [\bar{\gamma}_q]\,\bar{R}^{pq},$$
$$(3)\qquad \psi^{\times}\ \{\gamma^{pq}\ (B^* + B)\}\ \psi = \ 2i\varepsilon\ \ \{[\bar{\gamma}_0]\,\bar{R}^{pq} - [\gamma_0]\ R^{pq}\},$$

([1]) Session on 27 December 1943.

(4) $\psi^{\times}\{\bar{\gamma}^{p}(B^{*}+B)\}\psi = 2i\varepsilon[\gamma_q]\,\bar{R}^{pq}$,

(5) $\psi^{\times}\{\bar{\gamma}^{0}(B^{*}+B)\}\psi = -2i\varepsilon[\bar{\gamma}_{pq}]R^{pq}$,

(6) $\partial_p T^{0,p} = 0$,

(7) $\partial_q T^{p,q} = -2i\varepsilon[\gamma_q]R^{pq}$,

(8) $\partial_r T^{pq,r} = -2i\varepsilon\{[\gamma^{pr}]R^{q}{}_{r} - [\gamma^{qr}]R^{p}{}_{r}\}$,

(9) $\partial_q T^{\bar{p},q} = -2i\varepsilon[\bar{\gamma}_q]R^{pq}$,

(10) $\partial_p T^{\bar{0},p} = 0$.

In the right-hand side of equation (8), one recognizes (up to a numerical coefficient) the density of proper ponderomotor world-moment that is applied to the fictitious Dirac fluid by the field. Hence, one gets the very natural idea of choosing the physical coefficients of the abstract tensor $T^{pq,r}$ in such a manner that its divergence will be identified with the density of ponderomotor moment, by setting:

$$T^{pq,r} = -\left(\frac{4\pi}{h}\right)^2 im_0\, t^{pq,r};$$

(8) can then be written:

$$\partial_r t^{pq,r} = m^{pr} R^{q}{}_{r} - m^{qr} R^{p}{}_{r}.$$

Recall the physical interpretation that the various tensors of Dirac theory receive in the following table:

$$[\gamma^0] = -\frac{m_0}{i}\rho_0 = -i\,\omega_1, \qquad T^{0,p} = -\frac{2k_0}{ec}t^{0,p},$$

$$[\gamma^p] = -\frac{1}{ec}J^p = \frac{1}{c}f^p, \qquad T^{p,q} = \frac{4\pi i}{ch}t^{p,q},$$

$$[\gamma^{pq}] = \frac{2k_0}{e}m^{pq}, \qquad T^{pq,r} = -\left(\frac{4\pi}{ch}\right)^2 im_0\, t^{pq,r},$$

$$[\bar{\gamma}^p] = -\frac{4\pi}{h}\sigma^p, \qquad T^{\bar{p},q} = ?,$$

$$[\bar{\gamma}^0] = -\omega_2, \qquad T^{\bar{0},p} = \frac{2k_0}{iec}t^{\bar{0},p}.$$

$t^{0,p}$ is Gordon's current, $t^{p,q}$ is the asymmetric Tetrode tensor, and $t^{\bar{0},p}$ is Proca's magnetic current.

Equations (6) and (10) express the conservation of currents $t^{0,p}$ and $t^{\bar{0},p}$. Equation (7) was obtained by Tetrode in a different way that allowed him to get the necessary physical coefficients of his tensor. In the right-hand side, one recognizes the Lorentz force-power when one writes:

$$\partial_q t^{p,q} = \frac{1}{c}J_q R^{pq}.$$

Finally, O. Costa de Beauregard has calculated the two divergences of the tensor $T^{\bar{p},q}$, which remains uninterpreted. He also gave some indications regarding the calculation of its divergence relative to the differential index with the aid of the second-order Dirac equations (Thesis, pp. 85).

The first five equations have right-hand sides that are less directly interpretable. Meanwhile, one recognizes the density of potential energy in relation (1).

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