"Ueber die Bewegungsformen, welche den electromagnetischen Erscheinungen zu Grunde gelegt werden können," Ann. Phys. Chim. **52** (1894), 417-431.

# On the forms of motion that can serve as the basis for electromagnetic phenomena

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The attempt to reduce magnetic and electric phenomena to phenomena of motion without the assumption of an electric fluid that exerts any action at a distance has always pointed to the fact (more or less directly in the various cases) that it is the "cyclic motions" (i.e., speaking in full generality, systems of motions in which recurring motions are present) that play the most-important role in those phenomena.

In a previous article, I attempted to show that one could derive all of the fundamental equations of electrodynamics for media at rest, as well as in motion, from the assumption that *magnetic phenomena* were based in such forms of motion ([1]). In what follows, I would like to substantiate the assumption that was made there that the phenomenon of galvanic current would be based upon motions of the (mono-) cyclic type in yet another way and work through some of the consequences that would then be implied by that to a greater extent than was found in *loc. cit.*

**1. The electrodynamical law of induction is characteristic of systems of cyclic motions.** – It can be shown that a law that corresponds to the law of induction is true for all systems of motions with one (monocycle) or more (polycycles) recurring motions. (Some examples of polycyclic motions are given by several coupled top motions, different bodies with different temperatures in contact with each other, the field of several current loops). The aforementioned law can be expressed in its most-general form as:

*When increasing the rate of change $q_r$ of a general parameter $p_r$ ($q_r = dp_r / dt$) in a problem of motion makes a force $P_i$ increase that strives to increase the parameter $p_r$, a corresponding increase in $q_i$ will decrease the force $P_r$ that is applied to $p_r$, i.e.:*

$$\frac{\partial P_i}{\partial q_r} = -\frac{\partial P_r}{\partial q_i}.$$

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([1]) **H. Ebert**, Wied. Ann. **51** (1894), pp. 268.

The relationship between the electrodynamical effects and induction phenomena has that form. That is because if we understand $q_r$ to mean the rate of change of a parameter $p_r$ that determines the position or form of any current loop in a magnetic field, so $P_r$ is a "ponderomotive" force, $q_i$ is a quantity that corresponds to the current motion, and therefore $P_i$ will be an "electromotive" force then the theorem says: If a certain change in the current motion ($q_i$) seeks to produce a certain change in the position or force of the current-carrying body ($P_r$) then an electromotive force $- P_i$ will be induced in the conductor that acts to oppose the aforementioned current motion ($q_i$) when the change that is sought is actually effected (by an external force). (**Lenz**'s rule)

That opposition in the effects finds its most intuitive expression for a current in an electromagnetic field in the opposition of the so-called left and right-hand rules. If one points one's index finger in the direction of the magnetic field lines (which always start from the South pole of the field magnet and converge at the North pole), the middle finger in the direction of the electric current, and the thumb in the direction of motion of the conductor in the field then the relationship between the three fingers of the *left* hand will give the direction of the driving motion that a current-carrying conductor will experience in the field, while the *right* hand will give the direction of the current that is induced by a motion that is performed on the conductor.

**H. von Helmholtz** has given the general condition for that law to be true for a system of motions, and as further examples, he cited a similar opposition in the forces of mechanics in the example of the top in a **Cardan** suspension, the relationship between pressure and temperature in thermodynamics (when increasing the temperature increases the pressure in a system of bodies and increasing compression does the same thing to the temperature), as well as the Peltier effect and the thermal relationships in strings ([1]). We would like to show that this theorem is true for all systems of recurring motion, so the validity condition that **Helmholtz** posed in *loc. cit.* is always fulfilled for it. One finds that it is in the nature of "cyclic systems" that this theorem is true.

The fact that the electromagnetic, electrodynamical, and induction phenomena are governed by a law that is, in turn, characteristic of cyclic systems of motion seems to me to be a compelling argument for the opinion that those phenomena are also actually based in recurring motions. It is known that **Maxwell** sought to experimentally verify the properties of the parameters that determine the current motion that are true for such systems. (*Treatise II*, **§ 574** and **575**). However, one will have every right to emphasize that his experimental equipment did not possess the degree of precision that would allow him to resolve that question with certainty ([2]).

By contrast, there are many more well-established and long-proven precise applications of the law of induction with regard to the cited theorem in much-better-defined ways that gave information about the fundamental motions, just like the power of distance in **Coulomb**'s law was deduced with no objection by means of the theorem that "free electricity" at rest would only be found on the outer surface of the conductor, which is a law that is characteristic of the second negative power. In regard to that more-indirect meaning, it seems to me that the following proof might be important:

*Proof of the theorem above for polycyclic systems of motion:* If the state of an arbitrary system at time $t$ is determined by the parameter $p$ and the velocity $q = dp / dt$ then $P$ will be the force

([1]) **H. von Helmholtz**, Crelle's Journal **100** (1886), pp. 163.
([2]) **G. Wiedemann**, *Electricitätslehre*, 3rd ed., Bd. **4**, 1885, pp. 1164. **§ 1586**.

(positive when it strives to increase its coordinate), and if $H$ is the kinetic potential (which is equal to the potential $\Phi$ minus the kinetic energy $L$) then:

$$\frac{\partial P_i}{\partial q_r} = \frac{\partial(-P_r)}{\partial q_i} \tag{1}$$

when:

$$\frac{d}{dt}\left[\frac{\partial^2 H}{\partial q_i \cdot \partial q_r}\right] \tag{2}$$

vanishes, as one will see when one defines equation (1) by using the **Lagrange** equations of motion in the second form ([1]). We would like to call the expression (2) whose value dictates whether equation (1) is or is not true the "characteristic" and denote $\partial^2 H / \partial q_i \cdot \partial q_r$ more briefly by $A_{ir}$. Since the momenta $s_i = \partial L / \partial q_i = -\partial H / \partial q_i$, we will have $A_{ir} = -\partial s_i / \partial q_r$. The $A_{ir}$ are then negatives of the coefficients in the expression for the kinetic energy $L$, which is generally a homogeneous function of degree two in the $q$ ([2]).

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([1]) Cf., **H. von Helmholtz**, Crelle's Journal **100** (1886), pp. 163. – For the derivation, instead of (7) on pp. 162, line 3 from below, one should probably consult the formula on the previous page on line 8 from below that was provided with the number (9) [cf., also pp. 165, where the same equation was referred to in the derivation of equations (9.h)]. Moreover, the two sums over $\mathfrak{a}$ in that formula should probably be deleted.

([2]) *Remark:* From a known theorem from the theory of invariants, in all problems of motion, there are three, and indeed *only three* invariants. One of them is *energy*, the second is the *potential energy* (*Ergal*) of the forces. The third one is the functional determinant of the momenta with respect to the velocities:

$$J = \begin{pmatrix} s_1 & s_2 & \cdots \\ q_1 & q_2 & \cdots \end{pmatrix}$$

which unfortunately still lacks a special name, despite its great significance in all problems of motion. (Perhaps the term "the *momental*" would not be inapt.) The fact that one actually has an invariant is easy to show in the simple case of coordinate transformations. That is because if one transforms the general coordinates $p$ into the coordinates $a$ by means of $p_i = f_i(a_1, a_2, a_3, \ldots)$, such that:

$$q_i = \partial f_i / \partial a_1 \cdot b_1 + \partial f_i / \partial a_2 \cdot b_2 + \ldots, \qquad \text{in which } b_k = da_k / dt$$

are the new velocities, then one will have:

$$\begin{pmatrix} q_1 & q_2 & \cdots \\ b_1 & b_2 & \cdots \end{pmatrix} = \begin{pmatrix} p_1 & p_2 & \cdots \\ a_1 & a_2 & \cdots \end{pmatrix} = D,$$

where $D$ is the modulus of the transformation. However, from the multiplication theorem of functional determinants:

$$\begin{pmatrix} s_1 & s_2 & \cdots \\ b_1 & b_2 & \cdots \end{pmatrix} = \begin{pmatrix} s_1 & s_2 & \cdots \\ q_1 & q_2 & \cdots \end{pmatrix}\begin{pmatrix} q_1 & q_2 & \cdots \\ b_1 & b_2 & \cdots \end{pmatrix} = \Delta \cdot D,$$

so $\Delta$ is an invariant.

The determinant of our $A_{ir}$, multiplied by $(-1)^z$ [$z$ = number of parameters], will therefore change under linear transformations of the velocities by only a factor that is equal to the modulus of the transformation.

Therefore, in general, one will have:

$$\frac{\partial^2 H}{\partial q_i \cdot \partial q_r} = A_{ir}\,(p_1, p_2, \ldots, p_k, \ldots, p_i, \ldots, p_r, \ldots)\,, \tag{3}$$

i.e., it is a function of only the parameters.

However, in special cases, $L$ can also include first powers of the $q$, or even powers higher than two, *inter alia*, when the presence of the so-called "hidden motions" allows one to eliminate certain parameters ([1]). **H. von Helmholtz** has shown that a function $H'$ will always exist then from which the forces can be derived in completely the same way using the principle of least action as $H$, but in this case, the $A$ can still include the $q$. We then have two cases to distinguish: The cases in which no parameters are eliminated ("complete systems") and the cases in which certain parameters are eliminated ("incomplete systems").

What values would the $A_{ir}$ have in both cases for polycyclic systems of motions?

According to **Helmholtz**, a polycyclic system of motions is characterized by the fact that the parameters that determine the motion can be divided into two classes: viz., the rapidly-varying ones and the slowly-varying ones. The rapidly-varying ones represent the recurring motions. The ratios of the rate of change of the parameter to those of the other ones will decide whether they are to be regarded as slowly-varying or likewise fast-varying. Thus, along with the recurring motion of Earth's rotation around its axis, which is regarded as the cyclic variable, the parameters that determine the nutation and precession of the Earth's axis are to be regarded as slowly-varying. In the same way, compared to the repetitive motions of gas molecules, the rotation of the Earth, just like the motion of a compressed piston, can be regarded slowly-varying. For a galvanic current, the "slowly-varying parameters" will determine, e.g., the form and position of the conduction path, while the cyclic variables will characterize the rapid, perhaps vorticial, recurring motions, which we suggest by the field of force lines for the current.

Further properties of those variables and their derivatives can be inferred from that definition immediately.

$\alpha$) *The slowly-varying parameters:*

$$p_{\mathfrak{a}}^{(1)}, \quad p_{\mathfrak{a}}^{(2)}, \quad \ldots,\ p_{\mathfrak{a}}^{(m)}, \ldots,\ p_{\mathfrak{a}}^{(n)}, \ldots$$

Since they characterize defining data of the system that vary slowly, their derivatives with respect to time, namely, the $q_{\mathfrak{a}}$, will be small, and the $q'_{\mathfrak{a}} = dq_{\mathfrak{a}} / dt$ will be especially small.

$\beta$) *The rapidly-varying cyclic parameters:*

$$p_{\mathfrak{b}}^{(1)}, \quad p_{\mathfrak{b}}^{(2)}, \quad \ldots,\ p_{\mathfrak{b}}^{(m)}, \ldots,\ p_{\mathfrak{b}}^{(n)}, \ldots$$

([1]) **H. von Helmholtz**, *loc. cit.*, pp. 147.

Since they represent the recurring motions, their main property is that they enter only in the form of their time derivatives $q_\mathfrak{b}$, but they themselves do not play a role in the quantities that characterize the motion (e.g., the expressions for the energies). If each particle that leaves its place is immediately followed by another one that is equal to it and moves the same way, e.g., as for a homogeneous body of rotation that rotates around its axis with a uniform velocity or a fluid that moves uniformly in an annular channel, then the individual values of the parameters $p_\mathfrak{b}$ that determine the position of a particular particle must be meaningless for the collective motion. Hence, for (true) cyclic motions, the $p_\mathfrak{b}$ will not appear at all, but only their derivatives $q_\mathfrak{b}$ with respect to time. If the state of the system drifts very little from a certain mean state of motion in the course of time then all of the $q'_\mathfrak{b} = dq_\mathfrak{b} / dt$ will be small, in addition (therefore, they are *static*, mono or polycyclic systems, resp.)

For polycyclic systems, the phenomena are determined by essentially the $p_\mathfrak{a}$ and the $q_\mathfrak{b}$, while the $p_\mathfrak{b}$ do not appear ([1]).

If one considers that fact, as well as the orders of magnitude of the individual variables for the case of polycycles, then that will give the following:

A. *Complete systems.* – $H$ includes the $q$ only to degree two, while the $A_{ir}$ are functions of only the $p_\mathfrak{a}$.

If we then construct the "characteristic" (2) on pp. 3 then we will get:

$$\frac{d}{dt}\left[\frac{\partial^2 H}{\partial q_i \cdot \partial q_r}\right] = \sum_{(m)} \frac{\partial A_{ir}}{\partial p_\mathfrak{a}^{(m)}} \cdot q_\mathfrak{a}^{(m)}$$

here; however, all of the $q_\mathfrak{a}$ are small. Since the $\partial A / \partial p$ are finite quantities, we can set the right-hand side equal to zero when we neglect first-order infinitesimals, so the validity of the theorem (1) is proved for this case.

B. *Incomplete systems*. – $H$ includes the $q$ powers that are even higher than two, so the $A_{ir}$ will include some of the $q$ ($q_\mathfrak{a}$ or $q_\mathfrak{b}$, or both classes), in addition to the $p_\mathfrak{a}$. In this case, the characteristic will be:

$$\frac{d}{dt}\left[\frac{\partial^2 H}{\partial q_i \cdot \partial q_r}\right] = \sum_{(m)} \frac{\partial A_{ir}}{\partial p_\mathfrak{a}^{(m)}} \cdot q_\mathfrak{a}^{(m)} + \sum_{(m)} \frac{\partial A_{ir}}{\partial q_\mathfrak{a}^{(m)}} \cdot q_\mathfrak{a}'^{(m)} + \sum_{(n)} \frac{\partial A_{ir}}{\partial q_\mathfrak{b}^{(n)}} \cdot q_\mathfrak{b}'^{(n)} .$$

According to what was above, each term in this is multiplied by a quantity that is considered to be infinitely-small of first order for polycyclic systems. The characteristic will not be appreciably different from zero then, and theorem (1) will be proved for this case.

([1]) Cf., on this also: **L. Boltzmann**, *Vorlesungen über* ***Maxwell****'s Theorie*, **1**, Leipzig, 1891, pp. 16.

We can then write, in full generality, for all polycyclic systems:

(6) $$\frac{\partial P_{\mathfrak{a}}^{(m)}}{\partial q_{\mathfrak{a}}^{(n)}} = \frac{\partial(-P_{\mathfrak{a}}^{(n)})}{\partial q_{\mathfrak{a}}^{(m)}}, \qquad \frac{\partial P_{\mathfrak{b}}^{(m)}}{\partial q_{\mathfrak{b}}^{(n)}} = \frac{\partial(-P_{\mathfrak{b}}^{(n)})}{\partial q_{\mathfrak{b}}^{(m)}}, \qquad \frac{\partial P_{\mathfrak{a}}}{\partial q_{\mathfrak{b}}} = \frac{\partial(-P_{\mathfrak{b}})}{\partial q_{\mathfrak{a}}},$$

i.e., the relation (1) is true for the slowly-varying parameters, as well as for the forces that are applied to the cyclically-varying parameters, and ultimately for the relationships between the forces of the two kinds.

If only a cyclic motion is present (top, simple current, thermal motion in a body with a uniform temperature) then the first and third relations will be true. The law of induction is a special case of the third of equations (6).

**2. Relationships between magnetic and electric forces and the elementary motions and deformations of a continuous medium.** – In the previous article, it was shown that when we assume that magnetic phenomena are based in the cyclic nature of fundamental processes, the expressions for the electric forces are determined uniquely by the facts of experiments, which find their most concise expression in **Hertz**'s system of equations (pp. 275). We will then require *only one* hypothesis. The electric forces are then represented as something that is caused by spatial differences in the rotations that take place at the individual points of space. In that form, the expressions for those forces can initially give a somewhat strange impression. One often finds that it is advantageous to imagine the processes in the electromagnetic field by means of the picture of a *continuous medium* that is deformed in a certain way and to which one ascribes *elastic* properties of one kind or another. It can be shown that the attempt that I made to interpret electric forces can also be easily made consistent with that picture since it only requires the natural continuation of the conceptual constructions that are introduced into the theory of continuous media. That is because in the same way that we arrived rotations from simple deformations, we will arrive at the ones that we have set equal to electric forces from the former. In the language of the aforementioned theory, they are only deformations of a higher order then, and there is no obstacle to giving them an intuitive elastic interpretation.

Namely, if:

(1.a) $$m, n, o$$

are the components of a linear, infinitely-small displacement, so:

(1.b) $$u = \frac{dm}{dt}, \qquad v = \frac{dn}{dt}, \qquad w = \frac{do}{dt}$$

are the components of the rate of deformation, then we will get the components of the rotations:

(2.a) $$\xi = \frac{\partial o}{\partial y} - \frac{\partial n}{\partial z}, \qquad \eta = \frac{\partial m}{\partial z} - \frac{\partial o}{\partial x}, \qquad \zeta = \frac{\partial n}{\partial x} - \frac{\partial m}{\partial y},$$

and the rates of rotation:

(2.b) $$\xi' = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \qquad \eta' = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \qquad \zeta' = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y},$$

from them in a known way by an operation that we had called "taking the curl," to borrow from **O. Heaviside** (*loc. cit.*, pp. 276). If we apply the curl operation to the latter once more then we will get:

(3.a) $$\mathfrak{x} = \frac{\partial \zeta}{\partial y} - \frac{\partial \eta}{\partial z} = \nabla^2 m + \frac{\partial}{\partial x}\left(\frac{\partial m}{\partial x} + \frac{\partial n}{\partial y} + \frac{\partial o}{\partial z}\right), \quad \text{etc.,}$$

(3.b) $$\mathfrak{x}' = \nabla^2 u + \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right), \qquad \text{etc.,}$$

in which $\nabla^2$ is the operator $-(\partial^2 / \partial x^2 + \partial^2 / \partial y^2 + \partial^2 / \partial z^2)$. The $\mathfrak{x}$, $\mathfrak{y}$, $\mathfrak{z}$ are then deformations of a higher order than the simple rotations at a point.

In the cited article, it was shown that one carry all of the facts of observation in one's calculations by making the magnetic forces proportional to the $\xi'$, $\eta'$, $\zeta'$, so to the curl of the simple rates of displacement (1.b), and making the electric forces proportional to the $\mathfrak{x}$, $\mathfrak{y}$, $\mathfrak{z}$, so to the curl of the curl of the displacements themselves (1.a) ([1]). That might be the simplest expression for the processes that are at the basis for magnetic and electric phenomena in language of the theory of continuous medium.

If we use rectangular coordinates as our basis, as is done here, then the functional determinant:

$$D = \begin{pmatrix} x & y & z \\ a & b & c \end{pmatrix}$$

(*loc. cit.*, pp. 286) will have the value 1. For those special values, the general equations (19) and (20), pp. 290, of the electromagnetic field will go to the **Hertzian** equations that are valid for rectangular coordinates. They are then characterized by the value $D = 1$. However, in the theory of continuous media, the equation $D$ = constant implies incompressibility ([2]). If one imagines that the electric phenomena are produced by processes in a continuous medium then it must be considered

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([1]) In formula (4), pp. 276, only the factor $l$ stood in front of the square bracket, and not $l^2$. Similarly, $l^2$ must be replaced by $l$ in the corresponding expressions, pp. 279, and the third formula from the top on pp. 278 is missing the factor $\varepsilon$.

([2]) **H. Hankel**, *Zur allgemeinen Theorie der Bewegungen der Flüssigkeiten*, **Kirchhoff**, *Vorlesungen*, **1**, 1883, pp. 163.

to be "incompressible," or when we appeal to **Maxwell**'s terminology, its "divergence" $\partial m / \partial x + \partial n / \partial y + \partial o / \partial z$ must vanish, so (1):

$$(4) \qquad \varepsilon X = l\,\mathfrak{x} = l \cdot \nabla^2 m\,, \quad \varepsilon Y = l\,\mathfrak{y} = l \cdot \nabla^2 n\,, \quad \varepsilon Z = l\,\mathfrak{z} = l \cdot \nabla^2 o\,.$$

One sees that even in this case, relations will exist that correspond to **Poisson**'s theorem in the theories of action-at-a-distance: The displacements are the vector potentials of the electric forces (up to constant factors). However, the main difference lies in the fact that hypothetical fluid elements that characterize phenomena of *motion* will enter in place of the masses.

**3. Relationships between the cyclic theory of electromagnetic phenomena and the theory of vortices.** – In neither the previous article nor in the meantime has it been necessary for us to introduce a more definite representation of the nature of the recurring motions that we have based electromagnetic phenomena upon. In fact, all of the equations can be obtained from merely the most general equations of cyclic motion, and with no special assumptions besides. However, have already occasionally referred to the close connection with the theory of vortices, namely, in our reference to **Poynting**'s theory of the migration of electromagnetic energy. A fluid that is filled with vortex strings is an example of a polycyclic system of motions, and when all vortex velocities can be expressed rationally in terms of one variable $q$, it will be a monocyclic system of motions. The considerations will take on great simplicity when we appeal to fluid vortices as an explanation.

First of all, the example of hydrodynamics shows that cyclic recurring motions can exist next to each other in a medium without affecting each other reciprocally, and without it becoming necessary for us to call upon mediating mechanisms between them, as **Maxwell** believed that he would have to do in his early work. Here, we shall distinguish vortex surfaces, which are composed of vortex lines that lie next to each other continuously, from vortex strings whose space is filled completely with vortex lines like the way that the space of a magnetic field is filled up with magnetic lines of force. A vortex cylinder, i.e., a cylindrical space that is filled with vortices and enveloped by them, corresponds to the tube of forces here. The moment of the vortex cylinder (mean vorticity $q$ times the cross-section $Q$) corresponds to the force flux $\mathfrak{M}\,Q$ since we have made the force $\mathfrak{M}$ proportional to the velocity $q$ (cf., *loc. cit.*, pp. 273). If no conversion of field energy into heat takes place then the moment will be constant along the entire vortex cylinder in that case, and here it is the force flux that will be equal at each location along the force tube (theorem of the conservation of force flux).

It might seem conspicuous that we have made the magnetic force proportional to the angular velocity itself and not to its square, as when one makes the centrifugal force of a free rotating body proportional to the latter. However, **Maxwell** had already shown that we must do that in continuous

(1) If one writes the expressions for the electric polarization $\varepsilon X$, etc., in that form, then the similarity between them and the expressions $f = 1/8\pi\ \nabla^2\ m$, etc., that **R. T. Glazebrook** gave in a paper in the molecular theory of electromagnetic phenomena [Phil. Mag. (5) **11** (1881), pp. 397] will emerge [Cf., also **J. H. Poynting**, Phil. Trans. **175**, Pt. II (1884), pp. 361]. The aforementioned work was unknown to me when I was writing up my own.

media ([1]). The tensions and compressions that appear along the lines of force are then proportional to the squares of the vorticities.

If one imagines that the recurring cyclic motions that are at the basis of the current motion are vortex motions along the current conductor enclosing magnetic lines of force then one can further derive **Poynting**'s formula for the migration of the field energy towards the conductor, where it is transformed into heat, from the theory of vortices. In **Maxwell**'s first theory of 1861, in which the direct relationship between the lateral pressure of the magnetic force tube and its energy content was shown, the expansion and contraction of the force tube around the conductor was implied immediately (cf., *loc. cit.*, pps. 270 and 297): If the energy of the force tubes that arrive at the conducting material were extracted then to some extent that might give it the ability to resist the pressure of the surrounding tubes. **Maxwell** showed further that the reductions in the vortex intensity on the one hand and its increases on the other could be reduced to the driving motions that a current conductor experiences that its (concentric) system of force lines will produce in a field of magnetic lines of force. One can conclude immediately from the theorem that was proved **§ 1** that a system of concentric (oppositely-directed) lines of force (vortices, resp.) around the conductor will be created (induced) when it is led through field of magnetic lines of force by an external force in the sense of the previously-sought motion.

However, from the more-recent theory of vortex motion, one will also get a migration of energy in the sense of **Poynting**'s theory. I appealed to **Helmholtz**'s formulas on pp. 297, but not with complete justification because they were derived essentially under the assumptions that dissipation of energy would not take place anywhere in the entire field of vortices.

By contrast, **R. Reiff** has investigated vortex motion in viscous fluids and shows that one would arrive at a formula for the migration of energy in that way that would be completely analogous to **Poynting**'s fundamental formula ([2]).

**4. The stationary electric current and its relationship with temperature.** – If one uses the intuitions that were presented here as a foundation then the process of "electrical current" can be viewed in an essentially different light from the previous theories. Namely, the concepts of "conductivity" and that of "resistance" will take on a different meaning, and even the opposite one under some circumstances. It was already shown before by **Poynting** that this entirely-different way of looking at things is a necessary consequence of **Maxwell**'s theory.

In the older theories, electrical current was a phenomenon that essentially played out inside of the conductor. One imagined that a fluid actually flowed inside of it and that the terms resistance and conductivity would have meanings that would be analogous to their meanings in hydrodynamics.

The newer intuitions place the process more and more in the space external to the conductor. What migrates and flows is the energy that starts from electromotively-active surfaces and approaches the conductor between the level surfaces. Those level surfaces are perpendicular to the conductor, so when it is in its stationary state, the electric force tubes will run along the conductors and will move transversely to the conductor along its entire length. In that way, the energy that is

([1]) **J. C. Maxwell**, Phil. Mag. (4) **21** (1861), pp. 168.
([2]) **R. Reiff**, Mittheilungen des math.naturwissenschaftl. Vereins in Württemberg, Sep.-Abz. (1893), pp. 13.

contained in it will be transformed into heat. The number of force tubes that migrate per second will determine the current strength. What gives a body the property of being a conductor is, in essence, its ability to transform the energy of the electromagnetic field in such a way that the pressure from new energy quanta will be created such that a continuing "introduction of energy" will come about. In this way of looking at things, the "insulating" gutta percha sheath of a cable will be the "conductor." Its copper core facilitates only the flow of energy by continually transforming it. It holds the latter together and indeed it first introduces the migration of energy by the aforementioned transformation, i.e., the formation of current (in the older sense) ([1]).

However, the better a wire conducts (in the older sense), the less energy will flow per unit length in one second. We imagine that a current of the same intensity flows in, e.g., an iron wire and a copper wire of exactly the same dimensions, but eight times better conductivity. One can also express the fact that the copper wire is eight times better as a conductor as follows: Along the same length (e.g., a unit length), the potential drop in it will be eight times less than in the iron wire. However, the energy that is converted into heat per unit length per second will be determined by the product of that drop with the current strength. If eight times less energy were converted in the copper wire then we would also need to introduce only one-eighth as much of it as a replacement. If we then relate the concept of conductivity to the energy, which is the only thing that actually flows according to our conception of things, then we must say that copper is eight times worse than iron as a conductor ([2]). The migration of energy in the wire will then be more allowed as the "conductivity" becomes greater. Since one does not intend to consider that migration in the transfer of electrical energy, but one also cannot get around it if an advance of field energy is to be possible at all, one would do best to choose "good conductors" in the older sense, and even preserve that terminology with regard to the practical purposes, although one must actually call then "bad conductors."

In my previous paper, I sought (pp. 296, *et seq.*) to connect the capacity of a metal to conduct energy with the mutual motions of its molecules. It was the likely that this capacity would go away to the extent that we restrict the molecular motions (by cooling) and in that way impede the fact the energy that is absorbed by the parts that lie on the upper surface will be passed along to the lower-lying ones and converted into heat. I referred to the fact that the experiment of **Cailletet** and **Bouty**, **Wroblewski** and **Dewar** seems to me to have proved that this is the case in *loc. cit.*, pp. 298. They showed that the temperature coefficient $\alpha$ in the formula for the resistance $r_t = r_0 (1 + \alpha t)$ will increase appreciably for sharp drops in temperature, such that the resistance will drop faster than the temperature and approach zero. Therefore, with decreasing temperature, the conductivity (in the sense of the older theories) will continually increase without one being able to say that one can actually produce a conductor of infinite conductivity by cooling it strongly enough ([3]). Therefore, with decreasing temperature, from the foregoing, as far as the streaming of energy

([1]) **J. H. Poynting**, Phil. Trans. **175**, Pt. II (1884), pp. 354.

([2]) In regard to similar apparent paradoxes that nonetheless only come about because "one declines to say what is being conducted," cf., **H. Hertz**, Wied. Ann. **37** (1889), pp. 408; *Untersuchungen*, 1893, pp. 183.

([3]) Cf., **G. Wiedemann**, *Electricität*, 4th ed., **1**, 1893, pp. 472.

is concerned, the "conductivity" (in the new sense) will be less ([1]), so that would, in fact, seem to be most closely connected with the molecular motions in the conductor.

The fact that the thermal motion in the conductor is closely related to its electrical conductivity seems to me to further point to the fact that after removing the secondary perturbing influences, the resistance $r_t$ of the metal will be almost proportional to the absolute temperature (**Clausius**) ([2]). However, the latter is proportional to the kinetic energy of molecular motion for not only gases, but also for all bodies ([3]), so $r_t$ is also proportional to it, as well.

Furthermore, the parallelism between electric and thermal conductivity might be mentioned. The study of electric valence charges in atoms and molecules might be best suited to justifying the relationships that were presented here.

**Erlangen**, February 1894.

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([1]) On pp. 298 of my article, that experimental fact was expressed in a way that could easily lead to misunderstandings, as **Ostwald** was kind enough to bring to my attention. In the cited place, it was not clearly specified what the word "conducting" was referring to. For the sake of clarity, the words "electric field energy" should be repeated once more after "it no longer conducts."

([2]) Cf., **G. Wiedemann**, *Electricität*, 4th ed., **1**, 1893, pp. 474. Cf., also **F. Auerbach**, Wied. Ann. **8** (1879), pp. 479.

([3]) **H. Poincaré**, *Thermodynamique*, 1892, pp. 408.