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Nonuniform motions of electricity without magnetic or radiation fields

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If one attempts to think through certain questions in the dynamics of electrons in a purely-conceptual way, and with as little computational effort as possible, then one might be led to address the corresponding one-dimensional cases, instead of the three-dimensional problem. However, in so doing, one would encounter a peculiar situation: In the event that an infinite planar sheet of electrons always moves only perpendicularly to its two bounding planes, the magnetic field strength $\mathfrak{H}$ would be equal to zero everywhere. Therefore, for an oscillatory motion, for example, it would also lack a radiation field (and the electromagnetic quantity of motion!).

The fact that the electric charges reach to infinity here can make that result seem absurd. The following example is free of that reservation: Let spatial (everywhere-positive) charges be distributed over a spherical shell in such a way that the charge density is a function of only the distance from the center. That spherical sheet might perform purely-radial pulsations in some way, and indeed in such a way that the charge densities and radial velocities possess spherical symmetry at any moment. One sees with no further analysis that the magnetic field must be absent: If it were present then the magnetic force lines would have to run purely radially, due to the symmetry of the configuration, so there would have to be some sort of source, which is excluded by div $\mathfrak{H} = 0$.

The most general class of magnetic field-free motions of electricity (except for some inessential restrictions) is provided by the following construction: One imagines that one has chosen any sort of finite region (*A*) in space and calls rest space (*B*). One takes a function Φ (x, y, z, t) on it whose choice is not restricted by any constraints other than the following ones (which are very easy to fulfill):

a) Let Φ be temporally constant in the region (*B*) and fulfill the potential equation $\Delta\,\Phi = 0$.

b) Let Φ be temporally variable in the region (*A*) and let $\Delta\,\Phi$ have an arbitrary non-zero value there.

c) When one goes from (A) to (B), Φ and $\Delta\,\Phi$ are connected continuously with the values that prevail in (B) ([1]).

Finally, one derives the electric field strength $\mathfrak{E}$, the charge density ρ, and the velocity $\mathfrak{v}$ of the convective motion of electricity from Φ in the following way:

$$\mathfrak{E} = -\operatorname{grad}\Phi\,, \qquad 4\pi\rho = -\Delta\,\Phi\,,$$

$$\mathfrak{v} = \frac{\frac{\partial}{\partial t}(\operatorname{grad}\Phi)}{\Delta\Phi} \quad (\text{in } A) \qquad \mathfrak{v} = 0 \quad (\text{in } B).$$

The expressions that are defined in that way, together with $\mathfrak{H} = 0$, $\partial\,\mathfrak{H} / \partial t = 0$, will fulfill all of **Maxwell**'s equations (as one convinces oneself by substitution). The ponderomotive force that must be attached to the individual volume elements is $\rho\,\mathfrak{E}$.

All of those motions of electricity are characterized essentially by the fact that *the convection and displacement currents cancel each other at every point of space*. The configuration is not required to possess any symmetry for that, as the construction above shows.

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([1]) For example, if the region (A) were taken to be an ellipsoid and the function Φ were taken to be the **Newtonian** potential of the ellipsoid for external space in (B) then the requirement (c) could be fulfilled in an infinitude of ways by representing Φ graphically for the internal space, and indeed differently at every moment.