"Über die Streuung von Licht an Licht nach der Diracschen Theorie," Naturwiss. **23** (1935), 246-247.

The scattering of light by light in Dirac's theory

By H. Euler and B. Köckel

Translated by D. H. Delphenich

HALPERN[1] and DEBYE[2] have remarked that in Dirac's theory there must be the scattering of visible light by light. Thus, there will be processes in which two light quanta create a virtual pair (positron and electron), which then immediately radiates again. This process then converts two light quanta (ν_1, ν_2) into two other light quanta (ν_1', ν_2'), and this can also happen when their energy is not sufficient for the creation of an actual pair.

We have determined the interaction cross-section for the collision of two light quanta[3] for this case, which can characterized for all light quanta by the condition:

$$\nu_1\nu_2(1 - \cos \angle\nu_1, \nu_2) \ll \frac{2(mc^2)^2}{h^2},$$

hence, in a particular coordinate system by $h\nu \ll mc^2$.

$$\begin{pmatrix} m = \text{mass of the electron}, & e = \text{charge of the electron}, \\ c = \text{velocity of light}, & h = 2\pi\hbar = \text{PLANCK's quantum of action.} \end{pmatrix}$$

Next, from this, the ordinary perturbation term in DIRAC's theory for the fourth-order matrix element for this process was calculated and developed in light quantum energies $h\nu / mc^2$.

The zeroth-order term in $h\nu / mc^2$ proves to be equal and opposite to the term:

$$H_4 = -\frac{1}{12\pi^2}\left(\frac{e^2}{hc}\right)^4 \cdot \frac{1}{hc} \cdot \lim_{\xi\to 0}\int d\xi \left(\mathfrak{A}(\xi), \frac{\mathfrak{r}}{r}\right)^4$$

($\mathfrak{A}$ = potential of the radiation field), which, according to HEISENBERG[4], must be added to the ordinary fourth-order matrix element in order to give a real result.

The terms of order 1, 2, and 3 vanish, and the fourth-order term in $h\nu / mc^2$ may be formally represented as the matrix element of a function of the radiation field, such that

[1] O. Halpern, Phys. Rev. **44** (1933), 855 – also cf. G. Breit and J. Wheeler, Phys. Rev. **46** (1934), 1087.
[2] In a discussion with Herrn Prof. HEISENBERG.
[3] The detailed analysis will appear later.
[4] W. Heisenberg, Z. Phys. **90** (1934), 209, formula 61; **92** (1934), 692.

for the process that is under consideration here, the ordinary Hamiltonian function, which includes the energy of light *and* matter[1], can be replaced by the following one, which depends only upon the radiation field:

$$\int U\,dV = \int \frac{\mathfrak{B}^2 + \mathfrak{D}^2}{8\pi}\,dV - \frac{1}{360\pi^2}\frac{\hbar c}{e^2}\frac{1}{E_0^2}\int [(\mathfrak{B}^2 - \mathfrak{D}^2)^2 + 7(\mathfrak{B}\mathfrak{D})^2]\,dV\,. \tag{1}$$

In this, $\mathfrak{D}$ is the electric displacement, $\mathfrak{B}$ is the magnetic induction, $E_0 = e/(e^2/mc^2)^2$ is the magnitude of the "field strength at the edge of the electron." One has $\mathfrak{B} = \text{rot}\,\mathfrak{A}$, and $\mathfrak{D}$ is canonically conjugate to $\mathfrak{A}$, i.e.[2]:

$$\mathfrak{D}_i(P)\mathfrak{A}_k(P') - \mathfrak{D}_k(P)\mathfrak{A}_i(P') = \delta_{ik}\cdot\delta(P - P')\cdot\frac{2\hbar c}{i}\,.$$

If one introduces the quantities $\mathfrak{E}$ and $\mathfrak{H}$ in the usual way by the equations:

$$-\frac{1}{c}\dot{\mathfrak{A}} = \mathrm{E}, \qquad \frac{1}{c}\dot{\mathfrak{B}} + \text{rot}\,\mathfrak{E} = 0, \qquad -\frac{1}{c}\dot{\mathfrak{D}} + \text{rot} = 0 \tag{2}$$

then it follows that:

$$\begin{cases} \dfrac{\mathfrak{E}}{4\pi} = \dfrac{\partial\mathfrak{U}}{\partial\mathfrak{D}} = \dfrac{\mathfrak{D}}{4\pi} - \dfrac{1}{360\pi^2}\cdot\dfrac{\hbar c}{e^2}\cdot\dfrac{1}{E_0^2}[-4(\mathfrak{B}^2 - \mathfrak{D}^2)\mathfrak{D} + 14(\mathfrak{B}\mathfrak{D})\mathfrak{B}], \\[2ex] \dfrac{\mathfrak{H}}{4\pi} = \dfrac{\partial\mathfrak{U}}{\partial\mathfrak{B}} = \dfrac{\mathfrak{B}}{4\pi} - \dfrac{1}{360\pi^2}\cdot\dfrac{\hbar c}{e^2}\cdot\dfrac{1}{E_0^2}[+4(\mathfrak{B}^2 - \mathfrak{D}^2)\mathfrak{B} + 14(\mathfrak{B}\mathfrak{D})\mathfrak{D}]. \end{cases} \tag{3}$$

The connection between the quantities $\mathfrak{B}$ and $\mathfrak{D}$, on the one hand, and $\mathfrak{E}$ and $\mathfrak{H}$, on the other, is therefore nonlinear in this theory, since the scattering of light implies a deviation from the superposition principle.

The term in addition to the MAXWELLian energy in (1):

$$-\frac{1}{360\pi^2}\cdot\frac{\hbar c}{e^2}\frac{1}{E_0^2}\cdot\int [(\mathfrak{B}^2 - \mathfrak{D}^2)^2 + 7(\mathfrak{B}\mathfrak{D})^2]\,dV \tag{4}$$

can be intuitively understood as the interaction energy of the light quantum. It is the analogue of the COULOMB interaction of the electrons:

$$\iint \frac{\text{density}_1\cdot\text{density}_2}{\text{distance}_{12}}\,dV_1 dV_2\,. \tag{5}$$

[1] Cf., HEISENBERG and PAULI, Z. Phys. **56** (1930), 1; **59** (1930), 168.
[2] See previous footnote.

The fact that there is only one simple integral in (4), as opposed to (5), means that two light quanta can only interact at the same point.

The nonlinear correction to the MAXWELL equations of the vacuum becomes essential when the field strengths approach the one at "the edge of the electron;" the formulas that were derived here are therefore valid only as long as they do not become too large. ($|\mathfrak{E}|, |\mathfrak{B}|, |\mathfrak{D}|, |\mathfrak{H}| \ll E_0$).

It is interesting to compare this supplementary term to the MAXWELL energy, which arises from the quantum-mechanical possibility of pair creation, with the one that BORN[1] obtained in the framework of classical theory, and the first term in its development is:

$$\frac{(1.2361)^4}{32\pi}\cdot\frac{1}{E_0^2}\int[(\mathfrak{B}^2-\mathfrak{D}^2)^2+4(\mathfrak{B}\mathfrak{D})^2]\,dV\,.$$

Disregarding the fact that the ratio of the coefficients of the two supplementary terms is 1:4 for BORN and 1:7 for us, the two expressions differ only by the factor:

$$\frac{1}{45\pi\cdot(1.261)^4}\cdot\frac{\hbar c}{e^2}\,.$$

Considering the actual value of the SOMMERFELD fine structure constant, the numerical value of this factor is ~ 1.7, and it is remarkable that the quantum-theoretic change in the MAXWELL equations is, in any event, of the same order of magnitude as one would expect in the classical presentation on the basis of self-energy.

The equations (1), (2), (3) that follow from the DIRAC theory are valid under the assumption that the wavelength of light is large compared to the COMPTON wavelength. Otherwise, in contrast to the BORN theory, higher-order terms in development in light quantum energies, hence, supplementary terms will appear in the interaction, in addition to the ones that are of fourth order in the derivatives of the field strengths (multiplied by $\hbar/mc$).

The experimental test of the deviation from the MAXWELL theory is difficult since the noteworthy effects are extraordinarily small. The interaction cross-section for the scattering of light by light with the mean wavelength λ is, from (1), of the order of magnitude:

$$Q\sim\left(\frac{e^2}{mc^2}\right)^4\left(\frac{\hbar}{mc}\right)^4\cdot\frac{1}{\lambda^2}\,,$$

in DIRAC's theory, hence, about 10^{-28} cm^2 for γ-rays and 10^{-76} cm^2 for visible light.

We would like to cordially thank Herrn Prof. HEISENBERG for the problem definition, his ongoing interest, and his numerous suggestions on the work.

Leipzig. Institute for Theoretical Physics, 11 February 1935.

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[1] M. BORN, Proc. Roy. Soc. Lond., M. BORN and L. INFELD, ibid., A143 (1933), 410, A144, 423; A147 (1934), 522.