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On the wave theory of matter

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With 1 figure.

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Some of the consequences of five-dimensional geometry that concern the problem of matter are given.

The tendency to regard matter as a complicated wave process was first shaped into a clear program in the work of **de Broglie** ([1]). As is known, the mathematical methods that are required for that were worked out by **Schrödinger** ([2]). He introduced a special wave equation in place of the usual **Hamilton** partial differential equation. As **O. Klein** ([3]) and **V. Fock** ([4]) showed, the **Schrödinger** wave equation can be put into the simple form:

$$\Box\psi = 0 \tag{1}$$

by introducing the fifth coordinate.

One can attempt to construct the solutions to that equation that are similar to point-like material structures.

§ 1. – Here, we would like to treat the simple case of only one spatial coordinate and look for the solution of equation (1) in the three-dimensional space of (x, t, p). Here, t means time and p means the new coordinate.

We will assume that our space is Euclidian (i.e., gravitational and electromagnetic fields are absent), and its metric is determined to have the form:

$$ds^2 = dx_0^2 + dx_1^2 - dx_2^2, \tag{2}$$

([1]) **De Broglie**, Ann. de phys. (10) **3** (1925), pp. 22.
([2]) **E. Schrödinger**, Ann. Phys. (Leipzig) **79** (1926), 361, 489.
([3]) **O. Klein**, Zeit. Phys. **37** (1926), pp. 895.
([4]) **V. Fock**, Zeit. Phys. **39** (1926), pp. 226. We shall preserve the notations of **V. Fock** in this notice. We are very grateful to **V. Fock** for placing himself at our disposal in the proofreading of it.

in which:

$$x_0 = \frac{1}{mc}\,p\,, \qquad x_1 = x\,, \qquad x_2 = c\,t\,. \tag{3}$$

Here, *m* is the elementary mass, and *c* is the basic velocity.

We demand ([1]) that the solutions to equation (1) are purely-periodic in the coordinate *p* with the **Planck** constant *h* as its period.

We can write the particular solution to that solution in the form of plane waves:

$$\psi = e^{2\pi i \nu_2 [t + (z_0 \cos\varphi - z_1 \sin\varphi)/c]} = e^{2\pi i \nu_2 \left(t - \frac{x_1}{c/\beta}\right)} \cdot e^{2\pi i \nu_2 x_0 \frac{\sqrt{1-\beta^2}}{c}}\,, \tag{4}$$

in which $\beta = \sin\varphi$ and φ is the angle between the wave surface and the *x*-axis.

The periodicity condition for *p* implies that:

$$\frac{p}{h} = \nu\, x_0 \frac{\sqrt{1-\beta^2}}{c} = \nu\, p\, \frac{\sqrt{1-\beta^2}}{m c^2}\,,$$

from which, it follows that:

$$\nu_2 = \frac{m c^2}{h} \frac{1}{\sqrt{1-\beta^2}}\,. \tag{5}$$

In that way, we get a section in the form of a sine wave that propagates along the *x*-axis with speed $c\,/\,\beta$ whose frequency is given by the formula (2). Those waves are identical to the **de Broglie** phase waves [([1]), pp. 1].

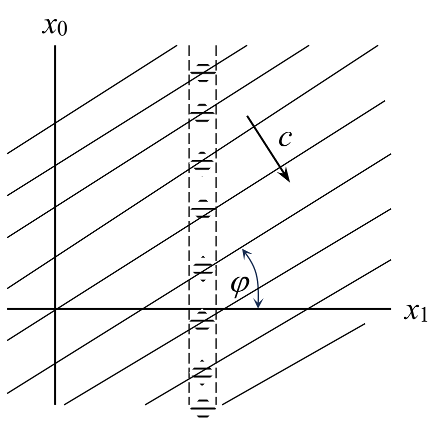

Figure 1.

§ 2. – We shall consider a wave bundle whose direction of propagation lies within a small angle. As is known, we can then determine the amplitudes and phases of those waves such that the **de Broglie** waves that are included as sections with the *x-axis* (except for a small region on the *x*-axis) are reduced to zero everywhere by interference. [Conversely, that wave-bundle can be obtained from the Fourier decomposition of a function of x_1 (e.g., e^{-x^2}).]

Due to the periodicity in the *p*-coordinate, ψ will assume non-zero values only within a narrow strip in the *px*-plane and vary periodically in that strip. Fig. 1 gives a crude intuitive picture of that situation.

([1]) **V. Fock**, *loc. cit.*

It is easy to see that when this strip of waves advances with the speed of light, its section with the x-axis is a propagating impulse with speed $c\,\beta$ whose form will vary with the frequency:

$$\nu_1 = \frac{mc^2}{h}\sqrt{1-\beta^2}\,. \tag{6}$$

However, that is known to be the frequency of **de Broglie**'s ([1]) "internal processes," and up to now, that is probably the single property of moving mass-points that cannot be understood from the standpoint of the wave theory of matter ([2]). We will then arrive at the consideration of wave processes in matter upon starting from suitably-normalized coordinates.

§ 3. – One can attempt to extend this representation to the general case of a give-dimensional space whose metric is given in the form:

$$d\sigma^2 = \gamma_{ik}\, dx^i\, dx^k \qquad (i, k = 0, 1, \ldots, 4). \tag{7}$$

One can write $d\sigma^2$ in the form ([3]):

$$d\sigma^2 = -\,c^2\, d\tau^2 + (d'\Omega)^2\,, \tag{8}$$

in which:

$$d'\Omega = \frac{e}{mc^2}\varphi_j\, dx^j + \frac{1}{mc}dp \qquad (j = 1, 2, 3, 4). \tag{9}$$

Here, τ is the proper time, and φ_j are the components of the electromagnetic potential.

If the mass-particle is constructed from waves in our space then due to the periodicity requirement for the waves, we must assume that the parameter of the internal structure to the mass-particle is assigned the factor:

$$\vartheta = e^{2\pi i\, p/h}\,. \tag{10}$$

According to **Klein** and **Fock**, the paths of mass-particles are given by geodetic lines.

We get from formulas (8) and (9) that ([4]):

$$dp = mc^2\, d\tau - \frac{e}{c}\,\varphi_j\, dx^j = mc^2\sqrt{1-\beta^2}\; dt - \frac{e}{c}\,\varphi_j\, dx^j\,. \tag{11}$$

([1]) **De Broglie**, *loc. cit.*, pp. 34.

([2]) The same result can be obtained without introducing the fifth coordinates. Here, we can employ the theory of waves that propagate in dispersive media. The speed of our impulse behaves just like the group velocity, which is equal to $c\,\beta$ for the **de Broglie** waves (**de Broglie**, *loc. cit.*). **Schuster** showed (**A. Schuster**, *Boltzmann Festschrift*, 1904, pp. 569) that the impulse will change form with a well-defined frequency during its motion. In the case of the **de Broglie** waves, calculation will show that this frequency coincides with that of the "internal processes."

([3]) **V. Fock**, *loc. cit.*

([4]) Let it be remarked that $dp = |\,L\,dt\,|$, where L is the relativistic Lagrange function for the electron, such that p coincides with **Hamilton**'s action.

Hence:

$$\vartheta = e^{2\pi i \frac{i}{h}\int dp} = e^{2\pi i \frac{mc^2}{h}\sqrt{1-\beta^2}\, t} \times e^{-2\pi i \frac{e}{ch}\int \varphi_j\, dx^j} . \tag{12}$$

The first factor once more gives **de Broglie**'s "internal processes." According to **Weyl**, the second one gives the change in a linear quantity that is coupled with the mass-particle, which is closely connected with the quantum conditions, as **Schrödinger** remarked in 1923 ([1]).

Added in proof: As one can see from this notice, one must ascribe a well-defined value of $1/(mc)^2$ to the coefficient γ_{00} (= α, according to **O. Klein**) in order to get the **de Broglie** waves. On other hand, **O. Klein** arrived at the gravitational equation by setting $\alpha\,\beta^2 = 2x$, which will establish the value of β. With that metric, the wave equations (equations of motion, resp.) will be valid for only mass-points with a charge $\approx 1.4 \times 10^{-30}$ esu. One can attempt to obtain a normal electron along the detour of the anisotropic properties of space for the wave processes.

Leningrad, Phys. Inst. of the University, 10 September 1926.

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([1]) **E. Schrödinger**, Zeit. Phys. **12** (1923), pp. 13.