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Fundamental propositions in the statics of elastic bodies

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It is an honor for me to communicate some propositions to the Circolo, while reserving the publication of their proofs for a paper that I hope to send to press soon.

1. In a homogeneous, elastic body that is indefinite in all directions, and in which the elastic force admits a potential, consider the three following deformations, which I call *typical deformations*.

1. A deformation that is subjected to the following conditions:

a) The components u, v, w of the displacement of an arbitrary point are finite and continuous, monodromic functions of its coordinates; they are annulled at infinity and the nine products of them with the coordinates do not diverge at infinity.

b) The nine components of the surface tensions X_x, Y_y, Z_z, X_y, Y_z, Z_x (*Kirchhoff's* notations) are all finite and continuous, are annulled at infinity, and the nine products:

$$X_x\,x^2,\quad Y_y\,y^2,\quad Z_z\,z^2,\quad X_y\,x^2,\quad X_y\,y^2,\quad Y_z\,y^2,\quad Y_z\,z^2,\quad Z_x\,x^2,\quad Z_x\,z^2$$

do not diverge at infinity.

c) The second derivatives of the u, v, w satisfy the equations:

$$\frac{\partial X_x}{\partial x}+\frac{\partial X_y}{\partial y}+\frac{\partial Z_x}{\partial z} = X,$$
$$\frac{\partial X_y}{\partial x}+\frac{\partial Y_y}{\partial y}+\frac{\partial Y_z}{\partial z} = Y,$$
$$\frac{\partial Z_x}{\partial x}+\frac{\partial Y_z}{\partial y}+\frac{\partial Z_z}{\partial z} = Z,$$

where X, Y, Z are three finite functions that are defined on a three-dimensional portion S of the body and are zero outside of that portion.

This deformation can be regarded as being provoked in the indefinite body by the force whose components X, Y, Z act upon the portion S of it. I call it a *deformation due to cubic effects* or *deformation of the first type.*

2) A deformation that satisfies the following conditions:

a) The components u, v, w of the displacement of a point answer to condition *a*) of the deformation of the first type.

b) The nine components of surface tension are all finite and continuous, except that upon crossing a surface that is denoted by σ, they present discontinuities that satisfy the equations:

$$\mathcal{D}\,(X_x\,\alpha + X_y\,\beta + Z_x\,\gamma) = L,$$
$$\mathcal{D}\,(X_y\,\alpha + Y_y\,\beta + Y_z\,\gamma) = M,$$
$$\mathcal{D}\,(Z_x\,\alpha + Y_z\,\beta + Z_z\,\gamma) = N,$$

where the symbol $\mathcal{D}(f)$ means the difference between the values that a function f takes at points that are infinitely close to a point of the surface σ, first, on the positive normal and secondly on the negative one. α, β, γ are the direction cosines of the positive normal. L, M, N are three finite functions that are defined for all points of σ. In addition, the nine components of the tensions are annulled at infinity and the nine products of them with the squares of the coordinates, as well as the products of distinct ones, do not diverge at infinity.

c) The second derivatives of the u, v, w satisfy the equations of the preceding condition *c*), except that X, Y, Z are zero in all of the indefinite body.

This deformation can be regarded as being provoked in the indefinite body by a force with components L, M, N that acts upon the surface σ. I call it a *deformation due to a surface effect* or *deformation of the second type.*

3. A deformation that is endowed with the following properties:

a) The components u, v, w of the displacement of a point are finite and continuous monodromic functions of its coordinates, except that upon crossing a surface that is denoted by σ they will present the discontinuities:

$$\mathcal{D}\,u = \bar{u}\,, \quad \mathcal{D}\,v = \bar{v}\,, \quad \mathcal{D}\,w = \bar{w}\,,$$

where $\bar{u}$, $\bar{v}$, $\bar{w}$ are three finite functions that are defined for all points of σ. In addition, these components are annulled at infinity, and the nine products of them with the coordinates do not diverge at infinity.

b) The nine components of the tensions satisfy condition *b*) of the deformation of the first type.

c) The second derivatives of the u, v, w satisfy condition *c*) of the deformation of the second type.

I call this a *deformation of the third type.*

Assuming the existence of the three indicated deformations, the conditions that define each of them are determined uniquely.

These three deformations play the same role in the statics of elastic bodies that the three-dimensional *volume, single-layer*, and *double-layer* potential functions do in the theory of Newtonian forces.

II. Consider a homogeneous, elastic body such that the elastic force admits a potential, and suppose that it occupies a space S that is bounded by a surface σ. A force with components X, Y, Z acts upon all of S and a force with components L, M, N acts upon the surface σ, which both determine a deformation uniquely (*Clebsch's* Theorem) for which u, v, w will denote the components of the displacement of a point and $\bar{u}$, $\bar{v}$, $\bar{w}$ will distinguish the values of u, v, w relative to the points of σ.

Imagine that the body S has been extended indefinitely in all directions and that the extension also affects its elastic behavior; let S_∞ denote the indefinite body thus obtained.

The deformation (u, v, w) *of* S *can be decomposed into three typical deformations of* S_∞ : *The deformations of the first type that are provoked by the force* (X, Y, Z) *act upon the portion* S, *the deformations of the second type that are provoked by the force* (L, M, N) *that acts upon* σ, *and the deformations of the third type, for which the components of the displacement present the discontinuities* $\bar{u}$, $\bar{v}$, $\bar{w}$ *upon crossing* σ.

This theorem is analogous to that of *Green* on the decomposition of a function that is finite and continuous, although it is defined within a field, into a sum of three potential functions.

III. In a homogeneous, elastic body that is indefinite in all directions, for which the elastic force admits a potential, consider two deformations of the first or second type that are provoked by the forces that act upon two bounded extensions S, S' that are three or two-dimensional.

The work that is the product of the force that acts in S *with the deformation that is provoked by the force that acts in* S' *is equal to the work that is the product of the force that acts in* S' *with the deformation that is provoked by the force that acts in* S.

This theorem is analogous to the *Gauss's* reciprocity theorem for Newtonian force.