"Modéle de la théorie causale des micro-objects relativistes de spin quelconque au moyen d'un fluid relativiste doté de moment cinétique interne," C. R. Acad. Sci. **241** (1955), 744-747.

Model for the causal theory of relativistic micro-objects of arbitrary spin by means of a relativistic fluid endowed with an internal kinetic moment

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The application of some results of a preceding note ([1]) to the linear equations of spinning particles, presented in von Neumann form, allows one to give a hydrodynamical representation for the causal theory of spinning particles as a function of the usual relativistic variables. In the case of particles of spin 1/2, one will recover the Takabayasi model ([2]).

The results that were obtained in our preceding note permit us to extend the hydrodynamical model of the causal interpretation ([3]) to the general theory of spinning particles. That model supposes that each individual micro-object (electron, meson, etc.) presents both an extended real aspect that can be assimilated to a particular fluid that is endowed with stresses that correspond to a quantum potential [which constitutes a hydrodynamical representation of Louis de Broglie's non-singular wave *v*, which is a particular solution of the usual linear wave equations ([4])] and a particule aspect that corresponds to a stable singularity that represents the wave u_0 that satisfies a nonlinear equation in the singular region and constrained to follow a well-defined streamline of the continuous fluid.

The introduction of spin amounts to assuming that the continuous fluid is endowed with a continuous density of internal kinetic moment, and that the singular region is endowed with the same rotational motion as the region of the fluid that follows it in the course of its motion. In summary, that amounts to adding a third "hidden parameter" (viz., the proper rotation) to the two parameters (viz., position and velocity) that were previously introduced into the causal interpretation. The kinetic moment of the particle aspect of the micro-object will then vary in a continuous fashion in the course of time, and one can show that the operations of macroscopic measurement will provide the

([1]) F. HALBWACHS, G. LOCHAK, and J.-P. VIGIER, Comptes rendus **241** (1955), pp. 692.
([2]) Private communication of the author. To appear in Prog. Theor. Phys. (Japan).
([3]) BOHM and VIGIER, Phys. Rev. **96** (1954), pp. 208, 216.
([4]) The problem that is posed by the description of a medium that is capable of transmitting the waves of wave mechanics was addressed by David Bohm and two of us (viz., Lochak and Vigier) in an article that will be published later. [Translator: Upon perusing a list of papers by Bohm, it would appear that the paper in question never got published.]

quantized values of that kinetic moment that are predicted by the non-deterministic interpretation of quantum theory ([5]).

For example, in the case of particles with spin 0, the spin (i.e., the hidden parameter) is non-zero, while the *measured* spin is always zero.

In order to describe a relativistic micro-object that is endowed with spin, we shall utilize the variational method that was indicated in our preceding note. We take the Lagrangian to be ([6]):

$$\mathcal{L} = q^{+}\,\gamma^{\mu}\,\partial_{\mu}\,q + \frac{mc}{\hbar}\,q^{+}\,q,$$

in which q represents a spinor, q^{+} is its conjugate, and γ^{μ} is an irreducible representation of the rotation group.

The wave equation will then become:

$$\gamma^{\mu}\,\partial_{\mu}\,q + \frac{mc}{\hbar}\,q = 0.$$

It describes the evolution of the continuous wave $v = q$ and the dynamical variables ρ, u_{μ}, g_{μ}, $\sigma_{\lambda\mu\nu}$, $s_{\mu\nu}$, and $K_{\mu\nu}$, which characterize the associated continuous fluid ([7]).

By way of example, we shall then justify the hydrodynamical representation of the Dirac equation that was proposed by Takabayasi, which is a problem that was addressed by several authors already ([8]). We show that the Takabayasi fluid is a Møller-Weyssenhoff fluid that is endowed with internal stresses, and we shall especially justify the impulse density that was proposed by Yvon and Takabayasi.

Takabayasi decomposed the canonical (or orbital) energy-impulse tensor with the aid of the wave equation, and obtained:

$$T_{\mu\nu} = mc^{2}\,P\,k_{\mu}\,u_{\nu} + cP\,\frac{\hbar}{2}\,(s_{\nu}\,\partial_{\mu}\theta + i\,\varepsilon_{\nu\alpha\beta\gamma}\,u^{\alpha}\,s^{\nu}\,\partial_{\mu}\,u^{\gamma}),$$

in which one has:

$$s_{\mu} = \text{unit vector collinear to spin} \qquad (s_{\mu}\,s^{\mu} = 1 \text{ and } s_{\mu}\,u^{\mu} = 0),$$

and $\varepsilon_{\nu\alpha\beta\gamma}$ is the Levi-Civita symbol.

P and θ are such that:

$$P\cos\theta = q^{+}q, \qquad P\sin\theta = q^{+}\,\gamma^{5}\,q.$$

He postulated that $P\,k_{\mu}$ is the impulse density of the fluid and got:

([5]) BOHM, TIOMNO, and SCHILLER, Nuovo Cimento, supp. 1 (1955).
([6]) BHABHA, Rev. Mod. Phys. **21** (1949), pp. 451.
([7]) HALBWACHS, LOCHAK, and VIGIER, *loc. cit.*
([8]) J. YVON, J. Phys. Rad. **1** (1940), pp. 18.
O. COSTA DE BEAUREGARD, *Thése*, Paris, 1943.
L. DE BROGLIE, *Théorie des particules de spin* 1/2, Paris, 1952.

$$k_\mu = u_\mu \cos\,\theta + mc\,\frac{\hbar}{2}\left\{(u_\mu s_\nu - u_\nu s_\mu)\partial^\nu\theta - \frac{i}{P}\varepsilon_{\mu\alpha\beta\gamma}\partial^\alpha(P u^\beta s^\gamma)\right\}.$$

Yvon and Takabayasi have likewise established the relation:

$$\cos\,\theta + \frac{\hbar}{2mc}s^\nu\,\partial_\nu\,\theta + k^\nu\,u_\nu + \frac{i\hbar}{2mc}\,\varepsilon_{\alpha\beta\gamma\delta}\,u^\alpha\,s^\beta\,u^\delta = 0,$$

which permits one to write:

$$k_\mu = -\,u_\mu\,u_\nu\,k^\nu - \frac{i\hbar}{2mc}\,u_\mu\,\varepsilon_{\alpha\beta\gamma\delta}\,u^\alpha\,s^\beta\,\partial^\gamma u^\delta - \frac{\hbar}{2mc}\,s_\mu\,u^\nu\,\partial_\nu\theta$$
$$-\frac{i\hbar}{2mc}\,\varepsilon_{\alpha\beta\gamma\delta}\,s^\gamma\,\partial^\alpha u^\beta - \frac{\hbar}{2mc}\,P\;\varepsilon_{\alpha\beta\gamma\delta}\,u^\beta\,\partial^\alpha\,(P\,s^\gamma).$$

Now introduce the expression:

$$T_{\mu\nu} = g_\mu\,u_\nu + \Theta_{\mu\nu}, \qquad \text{with } \Theta_{\mu\nu}\,u^\nu = 0,$$

$$T_{[\mu\nu]} = \tfrac{1}{2}(\,g_\mu\,u_\nu - g_\nu\,u_\mu) + \Theta_{[\mu\nu]},$$

and

$$g_\mu = -\,u_\mu\,g_\nu\,u^\nu - 2\,T_{[\mu\nu]}\,u^\nu + 2\,\Theta_{[\mu\nu]}\,u^\nu.$$

One infers $T_{[\mu\nu]}$ and $\Theta_{[\mu\nu]}$ from the Takabaysi expression:

$$2T_{\mu\nu} = \frac{ic\hbar}{2}\,\varepsilon_{\mu\alpha\nu\gamma}\partial^\alpha\,(P\,s^\gamma),$$
$$\Theta_{\mu\nu} = \frac{c\hbar}{2}\,P\,(s_\nu\,\partial_\mu\,\theta + i\;\varepsilon_{\nu\alpha\beta\gamma}\,u^\alpha\,s^\beta\,\partial_\mu\,u^\gamma),$$

so

$$g_\mu = -\,u_\mu\,g_\nu\,u^\nu - \frac{i\hbar c}{2}\,\varepsilon_{\mu\alpha\nu\gamma}\,u^\lambda\,\partial^\alpha\,(P\,s^\gamma) - \frac{\hbar c}{2}\,P\{s_\mu\,u^\nu\,\partial_\nu\,\theta + i\;\varepsilon_{\mu\alpha\beta\gamma}\,u^\lambda\,\partial^\alpha\,(P\,s^\gamma)\}.$$

Now the decomposition $T_{\mu\nu} = g_\mu\,u_\nu + \Theta_{\mu\nu}(\Theta_{\mu\nu}u^\nu = 0)$ is unique. One will then have:

$$g_\mu = mc^2\,P\,k_\mu$$

and

$$u_\mu\;\varepsilon_{\nu\alpha\beta\gamma}\,u^\alpha\,s^\beta\,\partial^\gamma\,u^\delta + \varepsilon_{\mu\alpha\beta\gamma}\,(s^\gamma\,\partial^\alpha\,u^\beta - u^\alpha\,s^\beta\,u^\nu\,\partial_\nu\,u^\gamma) = 0.$$

Formula (III) of our preceding note gives us Weyssenhoff's "density of kinetic moment" as:

$$s_{\mu\nu} = \tfrac{1}{2}\,\varepsilon_{\nu\alpha\beta\gamma}\,u^\alpha\,\sigma^\beta \qquad \left(\text{with } \sigma^\beta = \frac{\hbar c}{2}\,P\,s^\beta\right).$$

The Møller relations $s_{\mu\nu} u^{\nu} = 0$ is obviously verified.