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ON THE NEUTRINO THEORY OF LIGHT

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Up to now, there has been no direct connection between the nuclear neutrinos, which were introduced in order to recover the conservation laws in β-decay, and the neutrinos that act in pairs and should replace light quanta, as **de Broglie** has proposed. The light-neutrinos must also have a rest mass of zero and a spin of 1/2 and obey Fermi statistics. If we assume anti-neutrinos (–), along with neutrinos (+), then both kinds of neutrinos can arise during nuclear reactions. However, in the atomic domain, one must naturally assume that the difference between the numbers of both kinds of particles will remain constant, since two neutrinos always interact with the atom. It does not seem to be uninteresting then to investigate the statistical equilibrium of the neutrinos with the constraint:

$$N^+ - N^- = B. \tag{1}$$

In what follows, we will give a simple statistical-thermodynamical derivation of the formulas for the energy of the neutrino field, and in fact, in three dimensions. First, however, we shall treat the instructive case of the one-dimensional problem, which was examined already by **Jordan** ([1]) using a different method, that is related closely to **Einstein**’s consideration of the equilibrium of atoms and light quanta. We then demand that the maximum of entropy S with the constraints ε = const., B = const:

$$S = \sum \left\{ a_i^+ \log a_i^+ - (a_i^+ - n_i^+) \log (a_i^+ - n_i^+) - n_i^+ \log n_i^+ \cdots \right\}$$

(and analogous terms for (–) particles)

$$\sum \left\{ \varepsilon_i^+ n_i^+ + \varepsilon_i^- n_i^- \right\} = E,$$
$$\sum \left\{ n_i^+ - n_i^- \right\} = B. \tag{2}$$

a_i means the number of cells, n_i, the density, and ε_i means the energy of the neutrinos (anti-neutrinos, resp.). We obtain from that, in the usual way:

([1]) **P. Jordan**, Zeit. Phys. **98** (1935), 759. One will also find all citations in this.

$$n_i^+ = \frac{a_i^+}{\exp(-\beta + \varepsilon_i^+ / KT) + 1}, \quad n_i^- = \frac{a_i^-}{\exp(\beta + \varepsilon_i^- / KT) + 1}, \tag{3}$$

in which β is the chemical potential.

For what follows, we will need some integrals that we can assume will take the following two forms:

$$I_{2n} = \int_0^\infty \frac{\zeta^{2n} d\zeta}{e^{-\beta+\zeta}+1} - \int_0^\infty \frac{\zeta^{2n+1} d\zeta}{e^{\beta+\zeta}+1} = 2\sum_{k=0}^{n-1} \binom{2n}{2k+1} \beta^{2k+1} \int_0^\infty \frac{\zeta^{2n-2k-1} d\zeta}{e^{\zeta}+1} + \frac{\beta^{2n+1}}{2n+1}, \tag{4}$$

$$I_{2n+1} = \int_0^\infty \frac{\zeta^{2n+1} d\zeta}{e^{-\beta+\zeta}+1} + \int_0^\infty \frac{\zeta^{2n+1} d\zeta}{e^{\beta+\zeta}+1} = 2\sum_{k=0}^{n} \binom{2n+1}{2k} \beta^{2k} \int_0^\infty \frac{\zeta^{2n+1-2k} d\zeta}{e^{\zeta}+1} + \frac{\beta^{2n+2}}{2n+2}. \tag{4.1}$$

It should be remarked that in contrast to the known integrals of **Fermi** statistics, the given sums and differences can be evaluated precisely, as one can verify by elementary means. With the help of (2), (3), and (4), (4.1), and when we set $n = 1$ in the last expressions, we will get:

$$B = \frac{2KTV}{ch}\left\{\int_0^\infty \frac{d\zeta}{e^{-\beta+1}+1} - \int_0^\infty \frac{d\zeta}{e^{\beta+1}+1}\right\} = \frac{KTV}{ch}\beta, \quad \beta = \frac{ch}{2KTV}B \tag{5}$$

$$W = \frac{2K^2T^2V}{ch}\left\{\int_0^\infty \frac{\zeta\, d\zeta}{e^{-\beta+1}+1} - \int_0^\infty \frac{\zeta\, d\zeta}{e^{\beta+1}+1}\right\} = \frac{\pi^2 K^2 T^2 V}{3ch} + \frac{hc\,B^2}{2V} \tag{6}$$

for the chemical potential and energy, resp., in the one-dimensional case. If we now separate the light energy of the photon gas $\varepsilon = \frac{\pi^2 K^2 T^2 V}{3ch}$ from the neutrino field then we will directly get the important relationship:

$$W = \varepsilon + \frac{hc\,B^2}{2V} \tag{7}$$

that was first presented by **Kronig** ([1]). In the three-dimensional case, we assume that the expressions for B and ε have the following form:

$$B = \frac{8\pi K^3 T^3 V}{c^3 h^3}\left\{\int_0^\infty \frac{\zeta^2\, d\zeta}{e^{-\beta+1}+1} - \int_0^\infty \frac{\zeta^2\, d\zeta}{e^{\beta+1}+1}\right\},$$

$$\varepsilon = \frac{8\pi V}{(hc)^3}(KT)^4 \left\{\int_0^\infty \frac{\zeta^3\, d\zeta}{e^{-\beta+1}+1} + \int_0^\infty \frac{\zeta^3\, d\zeta}{e^{\beta+1}+1}\right\}. \tag{8}$$

([1]) **R. de Le Kronig**, Physica **2** (1935), 491, 854, 965; unfortunately, this article was known to us only by its citation.

If we now set *n* equal to 1 in the auxiliary formulas (4) and (4.1) then we will get:

$$B = \frac{8\pi^4 K^3T^3V}{3c^3h^3}\left\{\frac{\beta^2}{\pi^2}+\frac{\beta}{\pi}\right\},$$

$$\varepsilon = \frac{14\pi^2}{15}\cdot\frac{K^4T^4V}{h^3c^2}+\frac{4\pi^5K^4T^4}{h^3c^3}\left\{\frac{\beta^2}{\pi^2}+\frac{1}{2}\frac{\beta^4}{\pi^4}\right\}. \tag{8}$$

If we also separate the energy of the light field $\varepsilon = \frac{8\pi^5}{15} K^4\ TV\, ab$ here then that will yield the following connection between that energy and the neutrino energy:

$$W = \varepsilon + \frac{2\pi^5K^4T^4V}{h^3c^3}\left\{\frac{\beta^4}{\pi^4}+2\frac{\beta^2}{\pi^2}+\frac{1}{5}\right\}. \tag{9}$$

In the domain of low temperatures, we will get:

$$W = \varepsilon + \frac{3BchV}{8}\left(\frac{3B}{\pi V}\right)^{1/2}+\frac{\pi^2K^2}{3ch}\left(\frac{3B}{\pi V}\right)^{2/3}T^2V, \tag{10}$$

and, in particular, for $T = 0$:

$$W_0 = \frac{3BchV}{8}\cdot\left(\frac{3B}{\pi V}\right)^{1/2}, \tag{10.1}$$

which was to be expected naturally from the Fermi statistics of relativistic gases. By contrast, for high temperatures, we will get:

$$W = \varepsilon + \frac{2}{5}\frac{\pi^5K^4T^4V}{h^3c^3}+\frac{9}{16}\cdot\frac{c^3h^3B^3}{\pi^3K^2T^2V}. \tag{10.2}$$

It is easy to see that for very high temperatures, the energies of the neutrinos and anti-neutrinos will be equal to each other, and will have the ratios 7 : 7 : 8 with the photon energy. The cases of equilibrium of electrons-positrons and neutrinos-anti-neutrinos can be treated in an entirely analogous manner, and in fact, at high temperatures, the energies of all four kinds of particles will be equal to each other.

If we inquire about the equilibrium of the atomic nucleus and neutrinos then we must introduce, in addition to the processes of absorption and emission of two neutrinos (which is analogous to **Einstein**'s *B* and *A* processes) and the new specific neutrino-Raman effect processes with the absorption of one neutrino and emission of the other, also the processes that are characteristic of the nucleus; e.g., mixed Raman processes with the absorption of a neutrino and the emission of an electron. The requirement of equilibrium will then determine the behavior of all probability coefficients, and we will then arrive at the comparison of the light neutrino and nuclear neutrino effects.

In conclusion, we stress that when one heeds a verbal remark by **Bohr** and regards the nuclear neutrinos as "a convenient method for describing non-conservation" and identifies those particles with light neutrinos, one must mainly work with non-conservation in the case of the generalized theory of light, as well. We have seen that only part of the total energy of the neutrino field will coincide with the corresponding immediately-interpretable light energy, and that is true only in the mean in the case of equilibrium.

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