"Über neuere englische Arbeiten zur Mechanik," Jahresbericht der Deutschen Mathematiker-Vereinigung, Bd. 1 (1891/92).

On recent English papers on mechanics [1])

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The distinguishing characteristic of the English work on mechanics, as compared to that on the continent, is, to my understanding, the fact that it leads towards an immediate grasp of its importance and the thoroughly intuitive nature of its development. As a result, this work must be particularly stimulating to mathematicians that are accustomed to abstract trains of thought, and nothing is lost in that respect, since it is often perfectly useful, if the stated investigations are not carried out in a manner that is as methodical or rigorous as we would have wished. When given the level of detail that I devoted to it, one might make a remark of general interest on the history of the roots of Hamilton's theory of integration in mechanics. It seems to be completely unknown whether Hamilton himself extended his research with sufficient clarity at various points, in particular, his first treatment of ray systems (1828) [2]. Hamilton discovered the conception of the theory of emission, in which the determination of light rays that pass through an arbitrary inhomogeneous (but isotropic) medium is a special case of an ordinary mechanical problem relating to the motion of a mass point; we can even assume that the specialization that was made is hardly essential, since when one goes on to higher spaces one can often convert any mechanical problem into the determination of light rays passing through a certain medium. Thus, Hamilton's discovery that *the integration of the dynamical differential equations is linked with the integration of a certain partial differential equation* rests simply upon the fact that Hamilton, in accord with the great currents of physics in his era, showed that well-known results of geometrical optics could be derived, in emissive form, from the standpoint of undulation theory. Hamilton's theory of the integration of dynamical differential equations is therefore nothing but an analytically general formulation of the well-known relation between *light rays* and *light waves* in physical form. – By means of the starting point that is thus given, the unnecessarily specific form in which Hamilton published his theory, and with which Jacobi began, becomes understandable. In his investigations into ray systems, Hamilton

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[1]) Report on a lecture that was given before the Naturforscher-Versammlung on 22 Sept. 1891. (cf., Official Reports, part II, pp. 4) [In a lecture on mechanics in the summer semester of 1891, which has since been developed by various mathematicians, I derived the development of the Jacobi theory from quasi-optical considerations in higher spaces that was mentioned in the talk. K.]

[2]) [The most important of Hamilton's work that was mentioned above are the following: Essay on the theory of systems of rays, Transactions of the Royal Irish Academy, v. 15 (1828), pp. 69-174. Also, three supplements in ibid., v. 16 (1830), pp. 3-62, v. 16, pp. 93-126, as well as v. 17 (1832/37), pp. 1-144.

On a general method in dynamics, Philosophical Transactions of the Royal Society of London, 1834, pp. 247-308. Second essay on a general method in dynamics, ibid. 1835, pp. 95-144.]

had entirely practical questions about the theory of instruments in mind. Therefore, he operated exclusively with light waves that emanated from single *points*. Jacobi's generalization then proceeded from the notion that *one may use any other light waves in the definition of light rays.* In optics, as is well-known, one constructs the general waves from the special waves by means of the so-called principle of Huygens; this construction is precisely equivalent for the analytical process, by means of which one goes from any "complete" solution to a "general" solution in the theory of first-order partial differential equations.

[Optics, as we understand it above, is *geometrical optics*, which operates with the concept of light rays (hence, diffraction phenomena are principally excluded), and by the use of ordinary rectangular coordinate systems, they are governed by the first-order partial differential equation of degree two:

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2 - \frac{1}{c^2}\left(\frac{\partial f}{\partial t}\right)^2 = 0. \tag{1}$$

This is therefore completely distinct from *physical optics*, whose central focus is on the second-order partial differential equation of degree one:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} = 0; \tag{2}$$

however, one can regard the former as a limiting case of the latter for infinitely small wavelengths. In fact: if one substitutes the expression $e^{2\pi ik\, f(x,\, y,\, z,\, t)}$ for Φ in (2) and allows k to become infinite then one will obtain the differential equation (1) in the limit. Cf., Debye, in an article by A. Sommerfeld and I. Runge, Annalen der Physik, series 4, v. 35 (1911), pp. 290. K]