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## On the theory of the anomalous Zeeman and magneto-mechanical effects.

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With two figures

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The theory of the anomalous Zeeman effect and its conversion into the **Paschen-Back** effect was reduced to a few basic assumptions about the quantum structure of the atom and its behavior in an external magnetic field in a significant investigation by **W. Heisenberg** (<sup>1</sup>). Obviously, those basic assumptions:

a) Half-integer quantum jumps between atomic shells and the atomic nucleus  $(^2)$ .

b) Ineffectiveness of the nuclear momentum under spatial quantization of the magnetic field (<sup>3</sup>).

c) The axis of the nuclear momentum does not affect the the resultant of external and internal fields (<sup>4</sup>),

<sup>(&</sup>lt;sup>1</sup>) **W. Heisenberg**, "Zur Quantentheorie der Linienstruktur und der anomalen Zeemaneffekte," Zeit. Phys. 8 (1922), pp. 273. Cf., also **A. Sommerfeld**, *Atombau und Spektralinien*, 3<sup>rd</sup> ed., pp. 494, *et seq*.

<sup>(&</sup>lt;sup>2</sup>) Although the rotational momentum of the nucleus and the luminous electron should be intrinsically half-integer only in their *time average* for continuously whole-number total momenta of the atom, the nucleus can continually maintain half-integer momentum for a gradual separation of the luminous electron and an increasingly-weak interaction.

<sup>(&</sup>lt;sup>3</sup>) At the same time, that will imply a contradiction to the momentum coupling of the atomic and ether momenta (**Rubinowicz**), against which even **Heisenberg**'s *arithmetic mean* of the radiated momentum would not help, due to the variable intensity ratios of the multiple lines. However, according to **Wood**, e.g., pure  $D_1$  line resonances can be created in the absence of  $D_2$ .

<sup>(&</sup>lt;sup>4</sup>) From **Larmor**'s theorem, which is based in mechanics, no such effect can take place, but only a precession around the external field direction while conserving the orientation of the internal field.

do not seem to be compatible with the general principles that have been known up to now from the coupling of mechanics and electrodynamics that were employed by **Bohr** (<sup>1</sup>) in his new systematic theory of atomic structure. However, with a closer examination of the problem of magnetic line splitting, despite all misgivings, one will always once more arrive at the conviction that the Zeeman types, with their deep-rooted symmetry of the term splittings, can already be interpreted as being hardly essentially different from those of **Heisenberg** from the formal, as well as the model-theoretic viewpoint. For instance, according to **Heisenberg**, the formal symmetry between two associated doublet terms  $\mathfrak{x}_1$  and  $\mathfrak{x}_2$  whose splitting formulas [cf., *infra*, eq. (6)] emerge

from each other by only a change of sign  $\pm$  can be reduced to two *symmetric* positions of the nuclear axis, which obviously behaves anomalously in some way, with respect to the atomic axis, which remains invariable. By contrast, according to **Bohr** (lectures at Göttingen, June 1922), for the  $\mathfrak{x}_1$  term, the nuclear axis will be parallel to the invariable atomic axis, while for the  $\mathfrak{x}_2$  term, it will be perpendicular, which is an arrangement that cannot be suited to a model-based understanding of the aforementioned symmetry from the outset, in my opinion.

With that state of affairs, in what follows, we shall attempt to give the content of **Heisenberg**'s theory a firm foundation in such a way that when the postulates a), b), c) are approached from the standpoint of normal intuition, they will present themselves as necessary consequences of a basic assumption about the kinematics of the nucleus that the empirical facts will compel one to acknowledge, especially on the grounds of *principle of analogy* between classical and quantum radiation. Namely, from that principle of analogy, one can directly read off the type of anomaly in the motion of the luminous electron from the empirical Zeeman types and Paschen-Back transitions (§ 1). The conclusion that there are "anomalous" forces that create them is no longer complicated then (§ 2) and will lead, on the one hand, to the *origin* of those forces and furthermore to information about atomic structure, on the other, and indeed once more to Heisenberg's conception of things, which then find its kinematical and quantum-theoretical basis.

**§ 1.** – We initially restrict ourselves to weak external fields and consider only Zeeman types from the following series of *doublet line atoms:* 

 $(\mathfrak{s} \mathfrak{p}_1) \qquad (\mathfrak{p}_1 \mathfrak{d}_1) \qquad (\mathfrak{d}_1 \mathfrak{b}_1) \qquad \dots \\ (\mathfrak{p}_2 \mathfrak{d}_2) \qquad (\mathfrak{d}_2 \mathfrak{b}_2) \qquad \dots$ 

whose asymptotic approximation to classical radiation (<sup>2</sup>) is to be expected, because a quantum number *k* will jump by  $\Delta k = 1$  for it, which will make  $\lim \Delta k / k = 0$ .

The splitting of those line series – say, of  $(p_1 \ \vartheta_1)$  and  $(p_2 \ \vartheta_2)$  [see eq. (1)] – can be regarded as being constructed from several intertwined *sub-triplets* that each consist of a  $\pi$ -component that is

<sup>(&</sup>lt;sup>1</sup>) **N. Bohr**, *Drei Aufsätze über Spektren und Atombau*, Braunschweig, Friedrich Vieweg & A.-G., 1922. In particular, see the remarks in it about **Heisenberg**'s theory, pp. 98.

<sup>(&</sup>lt;sup>2</sup>) No analogy should be expected with the combination of an  $\mathfrak{x}_1$  term with an  $\mathfrak{x}_2$  term, which is linked with a jump in the nuclear position, because the quantum number for the nuclear orientation does not go to infinity asymptotically.

surrounded by two  $\sigma$ -components, which is suggested by  $\sigma - \pi - \sigma$  in eq. (1). It is characteristic of those sub-triplets that:

- 1. Their line-width is not that of a normal **Lorentzian** triplet.
- 2. Their center line ( $\pi$ -component) is displaced with respect to the field-free center line.

$$(\mathfrak{p}_{1},\mathfrak{d}_{1}) \begin{cases} -21/15\cdots(-3/15)\cdots+11/15 \\ -19/15\cdots(-1/15)\cdots+17/15 \\ -17/15\cdots(+1/15)\cdots+19/15 \\ -15/15\cdots(+3/15)\cdots+21/15 \\ \end{cases}$$
(1)  
$$(\mathfrak{p}_{2},\mathfrak{d}_{2}) \begin{cases} -13/15\cdots(-1/15)\cdots+11/15 \\ -11/15\cdots(+1/15)\cdots+13/15 \\ \end{cases}$$

In summary, the observations (by extrapolation of the Bergmann series) yield the following Table I for the displacements of the center lines of the sub-triplets and their line-widths, which are constant within a Zeeman type. [The line-width ( $\pi$ ) –  $\sigma$  of the normal triplet is set equal to 1.]

Series	$\Delta \omega / \mathfrak{o} = displacement$	$1 + \Delta b / o$ = line-width
$(\mathfrak{s} \ \mathfrak{p}_1) \dots (\mathfrak{p}_1 \ \mathfrak{d}_1) \dots (\mathfrak{d}_1 \ \mathfrak{b}_1) \dots$	$\begin{array}{l} \pm 1/3 \\ \pm 1/5,  \pm 1/15 \\ \pm 1/2,  \pm 3/25,  \pm 1/35 \end{array}$	1 + 1/3 1 + 1/5 1 + 1/7
$(\mathfrak{p}_2 \mathfrak{d}_2) \dots$ $(\mathfrak{d}_2 \mathfrak{b}_2) \dots$	$[\mp 1/5], \mp 1/15$ $[\mp 1/7], \mp 3/35, \mp 1/35$	1 - 1/5 1 - 1/7

Т	a	bl	e	I.

[The sub-triplets that belong to the bracketed displacements for the  $(\mathfrak{x}_2, \mathfrak{y}_2)$  types are unobservable since they have vanishing intensity.]

This structure of a Zeeman type as something that consists of displaced and expanded subtriplets was initially indicated under the assumption of *classical* radiation from following *motions* of the luminous electron: Each particular sub-triplet corresponds to a rotation of the luminous electron in its orbit and a superimposed precession of the orbital plane around the field direction for a particular inclination  $\mathcal{G}$  between the normal to the orbit and the magnetic field. However, the anomalous line-width of the sub-triplet shows that it is not the normal **Larmor** precession  $\mathfrak{o}$  that one expects from the ordinary theory (<sup>1</sup>) that takes place, but a modified precession of the frequency  $\mathfrak{o} + \Delta \mathfrak{o}$  around the field direction. The displacement of the  $\pi$ -components of the subtriplet further shows that the field-free orbital frequency  $\omega$  of the luminous electron has been converted into a field-free orbital frequency of  $\omega + \Delta \omega$ . The values in Table I give the corresponding ratios  $\Delta \omega / \mathfrak{o}$  and  $(\mathfrak{o} + \Delta \mathfrak{o}) / \mathfrak{o}$  for the sub-triplets of the Zeeman type. In that way, from Table I, the maximum amount of additional rotation  $(\Delta \omega / \mathfrak{o})_{max}$  will be equal to  $\Delta \mathfrak{o} / \mathfrak{o}$ , moreover. Obviously, that comes from the *parallel placement* of the atomic axis and the field direction (cos  $\mathcal{G} = \pm 1$ )., while the remaining small additional rotation  $\Delta \omega$  is attributed to a factor cos  $\mathcal{G}$  that belongs to a *skew* placement of the axes (<sup>2</sup>).

*Quantum-theoretically*, this picture that was obtained from the line splitting can be adapted to the terms in the following way: If an external magnetic field acts on a doublet line atom in the term state:

then the luminous electron will not take on the **Larmor** precession  $\sigma$  that one would expect from electrodynamics for an unchanged rotation  $\omega$ , but an additional precession  $\Delta \sigma$  will be added to  $\sigma$  and an additional rotation  $\Delta \omega$  will be added to  $\omega$ (<sup>3</sup>). As will be shown shortly, the following Ansatz for the additional quantities in the doublet terms will lead to the empirical term splitting:

$$\frac{\Delta \mathfrak{o}}{\mathfrak{o}} = \pm \frac{1}{2n-1} = \frac{\pm \frac{1}{2}}{n-\frac{1}{2}} = \frac{(\Delta \omega)_{\max}}{\mathfrak{o}}, \qquad \qquad \frac{\Delta \omega}{\mathfrak{o}} = \frac{(\Delta \omega)_{\max}}{\mathfrak{o}} \cdot \cos \vartheta, \tag{3}$$

which does, in fact, agree with Tab. I asymptotically.

[The upper sign shall always be true for the  $\mathfrak{x}_1$  terms, while the lower one shall be true for the  $\mathfrak{x}_2$  term. According to eq. (2), *n* is assigned to the individual terms.] Here as well, the additional precession  $\Delta \mathfrak{o}$  and the additional rotation  $(\Delta \omega)_{\text{max}}$  cancel each other in the positions  $\cos \vartheta = +1$ 

<sup>(&</sup>lt;sup>1</sup>) **P. Debye**, Göttinger Nachr., 3 June 1916. **A. Sommerfeld**, Phys. Zeit. **17** (1916), pp. 491.

<sup>(&</sup>lt;sup>2</sup>) In the event that one leaves behind the normal precession  $\mathfrak{o}$  and the field-free rotation  $\omega$ , the relationships are the same as they are for the rotation  $\Delta \omega$  of the **Foucault** pendulum relative to the Earth, which rotates by  $\Delta \mathfrak{o}$ . At the poles ( $\cos \vartheta = \pm 1$ ),  $\Delta \mathfrak{o}$  cancels out ( $\Delta \omega$ )<sub>max</sub> precisely, but in general, one has |  $\Delta \omega$  | = |  $\Delta \mathfrak{o} \cdot \cos \vartheta$  |. I have Herrn **A. D. Fokker** in Delft to thank for that comparison.

<sup>(&</sup>lt;sup>3</sup>) The author has suggested that the anomalous character of the Zeeman effect is due to an additional precession or rotation in Zeit. Phys. **7** (1921), pp. 398, (Part II), and in fact, he got around the dilemma that was mentioned above on pp. 1, rem. 3 in a different way. In connection with the paper by **Heisenberg** that appeared then (*loc. cit.*), we shall now assign  $(\Delta \omega)_{\text{max}}$  to the *parallel placement* cos  $\mathcal{G} = \pm 1$ .

and  $\cos \theta = -1$ . The motion remains a normal **Larmor** precession here, and the associated magnetic energy is calculated in the entirely normal way [eqs. (5) and (5')].

From the basic principles of quantum theory (<sup>1</sup>), along with the parallel and opposite positions of the field and atomic axis, one should also expect the skew positions  $\mathcal{G}$  for which the magnetic energy terms E / h differ from the maximal energy term by integer multiples of the perturbation, i.e., the orbital number of precession  $o + \Delta o$ , in this case:

$$\frac{E_{\max}}{h} - \frac{E}{h} = z \cdot (\mathfrak{o} + \Delta \mathfrak{o}) \qquad (z = \text{whole number}). \tag{4}$$

The total momentum  $\Im$  of the atom around its invariable axis is set to:

$$\Im = \frac{h}{2\pi} \left( n - \frac{1}{2} \pm \frac{1}{2} \right) \tag{5}$$

for the  $\mathfrak{x}_1$  term ( $\mathfrak{x}_2$  term, resp.) (<sup>2</sup>). That  $\mathfrak{I}$  belongs to the value:

$$\frac{E_{\max}}{h} = \mathfrak{o}(n - \frac{1}{2} \pm \frac{1}{2}) \tag{5'}$$

for the maximum magnetic energy term in the normal way in which the parallel position where the normal **Larmor** precession takes place.

Therefore, eqs. (3) and (4) yield the following (empirically confirmed) series of term splittings:

$$E / h = \mathfrak{o}(n - \frac{1}{2} \pm \frac{1}{2}) - z (\mathfrak{o} + \Delta \mathfrak{o}) \qquad (z = 0, 1, 2, ..., 2n - 1).$$

This can also be written as:

$$\frac{E}{h} = \mathfrak{o}(n - \frac{1}{2} \pm \frac{1}{2}) \cdot \frac{n - \frac{1}{2} - z}{n - \frac{1}{2}}$$
(6)

with the use of eq. (3).

(6) is associated with the following angle between the atomic axis and the field:

$$\cos \vartheta = \frac{E}{E_{\max}} = \frac{n - \frac{1}{2} - z}{n - \frac{1}{2}},$$
 (6')

<sup>(&</sup>lt;sup>1</sup>) **N. Bohr**, On the quantum theory of line spectra, Copenhagen 1916, Part II, 32.

<sup>(&</sup>lt;sup>2</sup>) Due to the field-less choice of combination in **Sommerfeld**'s "combined doublet" [Ann. Phys. (Leipzig) **63** (1920), pp. 221], k = n (n - 1, resp.) was introduced as an "internal quantum number." On the basis of the intensity ratios within the individual Zeeman types, the author [Zeit. Phys. **5** (1921), pp. 231, Part I, § **5**] interpreted k as a total momentum quantum number of the atom.

such that the equatorial momentum quantum number  $J \cos \vartheta$  will become a *fractional quantum number*. (6') gives precisely the position of the axes that **Heisenberg** introduced in order to explain the empirical (<sup>1</sup>) splitting term eq. (6), and which he derived theoretically from his basic assumption *b*) that it is not the equatorial component of the *total* atomic momentum that advances by whole quantum steps, but only that of the luminous electron, although the nucleus should contribute to the total magnetic energy in the field  $\mathfrak{H}$  in the usual way. The goal of the considerations in the present § 1 was to represent the consequences of **Heisenberg**'s hypothesis *b*), which is initially debatable from the standpoint of quantum theory, as *consequences* of **Bohr**'s quantum demand (4) with the help of the anomalous precession frequency  $\mathfrak{o} + \Delta \mathfrak{o}$ . The question of what sort of *forces* would generate the additional precession  $\Delta \mathfrak{o}$  and the additional rotation  $\Delta \omega$  shall be treated in § 2, while the origin of those forces will be treated in § 3.

**2.** – We recall some familiar theorems from the theory of tops. For a rigid symmetric top with its axis fixed at one point, the magnitude and direction parallel to that axis of its rotational



momentum will be represented by the momentum vector  $\mathfrak{I}_1$ , which subtends an angle of  $\mathcal{G}_1$  with the fixed  $\mathfrak{z}$ -direction (Fig. 1). A rotational moment  $\Delta \mathfrak{M}$ around a  $\Delta \mathfrak{M}$  axis that is kept perpendicular to the  $\mathfrak{z}$   $\mathfrak{I}_1$ -plane is now exerted on a top that rotates with an orbital number of  $\omega$ . It is then known that axis of the top  $\mathfrak{I}_1$  will then exhibit a precession  $\Delta \mathfrak{o}$  around the  $\mathfrak{z}$ -axis with an angle  $\theta_1$  that stays constant and whose orbital frequency is given by the vector equation:

Figure 1.

$$\frac{\Delta \mathfrak{o}}{\mathfrak{o}} = \frac{\Delta \mathfrak{M}}{[\mathfrak{I}_1 \mathfrak{o}]}, \quad \Delta \mathfrak{o} = \mathfrak{z} \frac{\Delta \mathfrak{M}}{[\mathfrak{I}_1 \mathfrak{o}]} \quad \mathfrak{o} \text{ and } \Delta \mathfrak{o} = \operatorname{vector} \| z \\ \Delta \mathfrak{M} = \| \bot \mathfrak{I}_1 \text{ and } \bot z. \end{cases}$$
(7)

Since the rotational moment  $\Delta \mathfrak{M}$  can produce no change of momentum around the  $\mathfrak{z}$ -axis,  $\Delta \mathfrak{o}$  will be coupled with a simultaneous change  $\Delta \omega$  in  $\omega$ , in such a way that  $\Delta \omega$  will vanish when  $\cos \vartheta_1 = 0$ , while  $\Delta \omega$  will cancel with  $\Delta \mathfrak{o}$  when  $\cos \vartheta_1 = 1$  and -1, and in general there will be the corresponding vector equation:

$$|\Delta \omega| = |\Delta \mathfrak{o} \cdot \cos \vartheta|, \qquad (8')$$

$$\Delta \omega = -\frac{(I_1 \cdot \Delta \sigma) \,\mathfrak{I}_1}{\mathfrak{I}_1^2} \qquad (\Delta \omega = \text{vector} \parallel \mathfrak{I}_1) \tag{8}$$

<sup>(&</sup>lt;sup>1</sup>) A. Landé, "Über den anomalen Zeemaneffekt," Part I, Zeit. Phys. 5 (1921), pp. 231.

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(cf., **Foucault** pendulum, see above). Moreover, the coupling of precession  $\Delta \sigma$  and the additional rotation  $\Delta \omega$  acts in such a way that when the rotational moment  $\Delta \mathfrak{M}$  increases from zero, it can do no work since the increase is kinetic energy  $\Delta \sigma$  will cancel with the decrease in kinetic energy  $\Delta \omega$ . The additional motion is therefore "free of work and impulse."

It is known that things are different when a rotational moment  $[i_1 \mathfrak{H}]$  that is due to a magnetic field  $\mathfrak{H}$  that is parallel to  $\mathfrak{z}$  acts upon a system of masses that is endowed with a charge whose ratio is  $\varepsilon : \mu$ , which will then appear to be a magnet whose moment is:

$$\mathfrak{i}_1 = \mathfrak{I}_1 \cdot \frac{\varepsilon}{2c\mu} \ . \tag{9}$$

Corresponding to eq. (7), a precession of:

$$\mathfrak{o} = \frac{\mathfrak{H}[\mathfrak{i}_1 \mathfrak{H}]}{[\mathfrak{I}_1 \mathfrak{H}]} = \frac{\mathfrak{i}_1}{\mathfrak{I}_1} \mathfrak{H} = \frac{\varepsilon}{2e\,\mu} \mathfrak{H}$$
(10)

will appear, but the additional rotation  $\Delta \omega$  will be missing, and the magnetic field will do work:

$$A = (\mathfrak{i}_1 \,\mathfrak{H}) \tag{10'}$$

when it increases from zero to full strength  $(^{1})$ .

If an external magnetic field  $\mathfrak{H}$  that is parallel to  $\mathfrak{z}$  acts simultaneously with a mechanical rotational moment  $\Delta \mathfrak{M}$  around an axis that is perpendicular to  $\mathfrak{I}_1$  and  $\mathfrak{z}$  then both [effects eqs. (7), (8), and (10)] will combine and lead to the following result:

Precession: 
$$\mathfrak{o} + \Delta \mathfrak{o}$$
,  $\mathfrak{o} = \frac{\varepsilon}{2c\mu} \mathfrak{H}$ ,  $\frac{\Delta \mathfrak{o}}{\mathfrak{o}} = \frac{\Delta \mathfrak{M}}{[\mathfrak{I}_1 \mathfrak{o}]}$ ,  
Additional rotation:  $\Delta \omega$ ,  $|\Delta \omega| = |\Delta \mathfrak{o} \cdot \cos \vartheta|$   
due to the magnetic field  $\mathfrak{H}$  and rotational moment  $\Delta \mathfrak{M}$ .  
(11)

The result eq. (3) of § 1 can then be interpreted in the following way: Along with the external field  $\mathfrak{H}$ , which leads to a normal Larmor precession of the luminous electron  $\mathfrak{I}_1$  around  $\mathfrak{H}$  with a

<sup>(&</sup>lt;sup>1</sup>) One can, in fact, regard  $\mathfrak{H}$  as a vector sum  $\mathfrak{H} = \mathfrak{H}_p + \mathfrak{H}_s$  (Fig. 1). The component  $\mathfrak{H}_s = \mathfrak{H} \cdot \cos \mathfrak{G}_1$  that is perpendicular to  $\mathfrak{I}_1$  acts like a mechanical rotational moment  $[\mathfrak{i}_1 \mathfrak{H}] = [\mathfrak{i}_1 \mathfrak{H}_s]$ , so from eq. (7), the precession frequency  $\mathfrak{o}$  will be eq. (10), and likewise the additional rotation around the  $\mathfrak{I}_1$  axis will be  $-\mathfrak{o} \cdot \cos \mathfrak{G}_1$ . The component  $\mathfrak{H}_p$  that is parallel to  $\mathfrak{I}_1$  will give an additional rotation around the  $\mathfrak{I}_1$ -axis of  $+\mathfrak{o} \cos \mathfrak{G}_1$ , such that the pure **Larmor** precession  $\mathfrak{o}$  will remain the same with an associated normal increase ( $\mathfrak{i}_1 \mathfrak{H}$ ) in the kinetic energy.

normal magnetic energy (10'), there exists a mechanical rotational moment  $\Delta \mathfrak{M}$  whose axis always stays perpendicular to  $\mathfrak{H}$  and  $\mathfrak{I}_1$ , so it will not increase the momentum around  $\mathfrak{Z}$  or do any work. When one sets eq. (3) equal to eq. (7):

$$\frac{\Delta \mathfrak{o}}{\mathfrak{o}} = \frac{\Delta \mathfrak{M}}{[\mathfrak{I}_1 \mathfrak{o}]} = \frac{\pm \frac{1}{2}}{n - \frac{1}{2}} , \qquad |\Delta \omega| = |\Delta \mathfrak{o} \cos \vartheta| , \qquad (12)$$

its magnitude is determined to be:

$$\Delta \mathfrak{M} = \frac{\pm \frac{1}{2}}{n - \frac{1}{2}} \cdot [\mathfrak{I}_1 \mathfrak{o}] , \qquad (13)$$

or, due to eqs. (9) and (10):

$$\Delta \mathfrak{M} = \frac{\pm \frac{1}{2}}{n - \frac{1}{2}} \cdot [\mathfrak{i}_1 \,\mathfrak{H}] \ . \tag{14}$$

That mechanical additional rotational moment, whose origin was explained in  $\S$  **3** and might be referred to as the "internal rotational moment," can then be identified as the cause of the additional precession  $\Delta o$  and the additional rotation in § 1.

§ 3. – The atomic nucleus might possess mechanical angular momentum  $\Im_2$  and therefore a magnetic moment of  $i_2 = \Im_2 \varepsilon / 2m c$ . With **Heisenberg**, we assume that the vector  $\Im_2$  is positioned *parallel* (for the  $x_1$  term) or *opposite* (for the  $x_2$  term) to the resultant  $\mathfrak{H} + \mathfrak{H}_i$  of the internal and



external field, where the internal field  $\mathfrak{H}_i$  should point parallel to  $\mathfrak{H}^+ \mathfrak{H}^{\mathfrak{H}}$  the momentum  $\mathfrak{I}_1$  of the luminous electron (Fig. 2). The rotational moment [ $\mathfrak{i}_2 \mathfrak{H}$ ] that  $\mathfrak{H}$  exerts upon  $\mathfrak{i}_2$  will then be cancelled by the the momentum  $\mathfrak{I}_1$  of the luminous electron (Fig. 2). The rotational opposite rotational moment –  $[i_2 \mathfrak{H}]$  that the luminous electron exerts upon the nucleus in the event that i<sub>2</sub> is parallel (anti-parallel,

resp.) to the resultant  $\mathfrak{H} + \mathfrak{H}_i$  in equilibrium, corresponding to **Heisenberg**'s assumption. However, the latter rotational moment, as a reaction, corresponds to an opposite rotational moment  $[i_2 \ \mathfrak{H}]$ that the nucleus  $\mathfrak{I}_2$  exerts upon the luminous electron  $\mathfrak{I}_1$ :

$$\Delta \mathfrak{M} = [\mathfrak{i}_2 \,\mathfrak{H}] \,. \tag{15}$$

In particular, if  $\mathfrak{H}$  is small compared to  $\mathfrak{H}_i$  (weak external field) then the resultant  $\mathfrak{H} + \mathfrak{H}_i$  will become parallel to  $\mathfrak{H}_i$ , so  $\mathfrak{i}_2$  will also become parallel to  $\mathfrak{i}_1$ , and:

$$\Delta \mathfrak{M} = \pm [\mathfrak{i}_1 \mathfrak{H}] \frac{\mathfrak{i}_2}{\mathfrak{i}_1} \qquad (\mathfrak{i}_2 \parallel \pm \mathfrak{i}_1). \tag{16}$$

We identify this rotational moment, which does no work or impulse, that the nuclear orientation exerts upon the luminous electron with the internal rotational moment that was found in § 2 eq. (14) for a weak field (<sup>1</sup>). In particular, equating the eqs. (14) and (16) will then yield the condition:

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$$\frac{\pm \frac{1}{2}}{n-\frac{1}{2}} = \frac{\pm \mathfrak{i}_2}{\mathfrak{i}_1} = \frac{\pm \mathfrak{I}_2}{\mathfrak{I}_1} \qquad (\mathfrak{I}_2 \parallel \pm \mathfrak{I}_1) . \tag{17}$$

Since the total momentum of the atom in eq. (5) was:

$$\Im = \Im_1 + \Im_2 = \frac{h}{2\pi} (n - \frac{1}{2} \pm \frac{1}{2}),$$
 (17')

what will ultimately follow is the special result that:

$$\mathfrak{I}_{1} = \frac{h}{2\pi} (n - \frac{1}{2}), \qquad \mathfrak{I}_{2} = \frac{h}{2\pi} \cdot \frac{1}{2}$$
(18)

are the individual momenta of the nucleus and luminous electron, resp., which corresponds to what **Heisenberg** found a). The basic assumption c) that the nucleus does not affect the resultant of  $\mathfrak{H}$  and  $\mathfrak{H}_i$  will then suffice to make **Heisenberg**'s further assumption b) irrelevant, since the momentum  $\mathfrak{I}_2$  of the nucleus does not affect the spatial quantization in the magnetic field. Rather, b) follows from c) by means of (4). Imagining the nucleus to be a *bar magnet* (i.e., a magnetic dipole, and in particular with a moment of 1/2 magneton here) contradicts **Larmor**'s theorem, which requires only a precession  $\mathfrak{o}$  in a fixed field-free position for the nucleus, as a *top* (i.e., an **Ampèrian** molecular current), but only leads to the *normal* Zeeman effect.

In the resultant position, which already contradicts the kinematics of tops then, the nucleus cannot be treated as a top then either. That is because no resultant rotational moment at all acts upon the nucleus in the resultant position. As a top, the nucleus shall then remain at rest in the instantaneous direction of the resultant in space, since its normal Larmor precession  $\sigma$  that is produced by the external field  $\mathfrak{H}$  shall be superimposed with an opposite additional precession  $\Delta \sigma = -\sigma$  and an additional rotation  $|\Delta \omega| = |-\sigma \cos \vartheta_2|$  as a result of the internal field  $\mathfrak{H}_i$ . Those additional quantities of the nucleus that do no work or impulse will then differ from the additional quantities eq. (3) that do no work or impulse for the luminous electron. The fact that, in reality, the

<sup>(&</sup>lt;sup>1</sup>) The ratio of the anomalous precession number to the normal one  $\mathfrak{o} + \Delta \mathfrak{o} / \mathfrak{o}$  is identical to the ratio  $(\mathfrak{I}_1 + \mathfrak{I}_2) / \mathfrak{I}_1$  and identical to the splitting factor *g* that appeared in the term analysis of the Zeeman types (cf., Landé, *loc. cit.*, Part I).

nucleus (in addition to the normal energy increase  $h \cdot \frac{1}{2} \circ \cos \circ$  of the Larmor precession that enters into the potential energy against the internal field as an energy term) brings with it the additional quantities eq. (3) that do no work or impulse for the luminous electron if the atom is to still maintain its integrity, shows just that the normal treatment of the nucleus as a *top* does not indeed contradict the laws of energy and momentum, but probably kinematics, and must be replaced with a nonmechanical picture of the nucleus.

§ 4. – If one applies eq. (15), which is true for all fields, instead of eq. (16), which is true for weak fields, then one will get, entirely in connection with **Sommerfeld**'s (<sup>1</sup>) formal adaptation of **Voigt**'s coupling theory and **Heisenberg**'s model-based theory, the transition to the **Paschen-Back** effect, since the resultant  $\mathfrak{H} + \mathfrak{H}_i$  goes from the direction  $\mathfrak{H}_i$  ( $\mathfrak{H} \ll \mathfrak{H}_i$ ) to the direction  $\mathfrak{H}$  ( $\mathfrak{H} \gg \mathfrak{H}_i$ ), during which the additional rotational moment and the additional precession and rotation and magnetic energy will change correspondingly (<sup>2</sup>).

§ 5. – The explanation for the anomalous nature of the magneto-mechanical effect of **Barnett** (<sup>3</sup>) and **Einstein-de Haas** (<sup>4</sup>) is very simple now. From eq. (3), a well-defined magnetization is coupled with a precession of the atomic axis around the field direction that is  $g = \left(1 + \frac{1}{2n-1}\right)$ 

times larger than what one would normally expect for the magnetization in question, so for the  $\mathfrak{s}_1$  term (normal state n = 1), it would be twice as large. [The same thing is also true for the  $s_1$  term state of a triple line atom, which likewise possesses the splitting factor g = 2 (cf., footnote on pp. 9]. In **Barnett**'s experiment, it is then necessary to have a twice-normal angular velocity of the atomic axis and the nuclear mass arrangement that is coupled with it in order to create (cancel, resp.) a well-defined magnetization. In the **Einstein-de Haas** experiment, a twice-normal rotational velocity of the atomic axis and the nuclear mass arrangement that is coupled with it as a reaction to a well-defined produced magnetization must enter into the magnetization (reverse magnetization, resp.) of a body.

<sup>(&</sup>lt;sup>1</sup>) A. Sommerfeld, Zeit. Phys. 8 (1922), pp. 257. Atombau und Spektrallinien, 3<sup>rd</sup> ed., Chap. 6.

<sup>(&</sup>lt;sup>2</sup>) The adaptation of the previous arguments to triplet line atoms will be touched upon in a later article where, among other things, the thesis that is derived from the Zeeman effect for chrome and neon will be maintained that one must generally replace **Bohr**'s  $n_k$ -orbits with  $n_{k-1/2}$ -orbits, as **Heisenberg** had proposed for doublet and triplet line atoms.

<sup>(&</sup>lt;sup>3</sup>) S. J. Barnett, Phys. Rev. 6 (1915), pp. 239. I. G. Stewart, Phys. Rev. 11 (1918), pp. 100.

<sup>(&</sup>lt;sup>4</sup>) Einstein and de Haas, Verh. d. D. Phys. Ges. 17 (1918), pp. 152. Emil Beck, Ann. Phys. (Leipzig) 60 (1919), pp. 109. G. Arvidsson, Phys. Zeit. 21 (1920), pp. 88.

## **Summary**

The non-mechanical contribution of the atomic nucleus to the resultant of the external ( $\mathfrak{H}$ ) and internal field ( $\mathfrak{H}_i$ ), which is due to the luminous electron that **Heisenberg** assumed, in contradiction to **Larmor**'s mechanically-based theorem, leads to the appearance of a *rotational moment*  $\Delta \mathfrak{M}$ that is exerted upon the luminous electron.  $\Delta \mathfrak{M}$  creates an additional precession  $\Delta \mathfrak{o}$  around  $\mathfrak{H}$  and an additional rotation  $\Delta \omega$  around  $\mathfrak{H}_i$  that "does no work or impulse." That additional motion can be read off from the empirical Zeeman type using the principle of analogy, and with the help of **Bohr**'s quantization of the perturbation (4), it justifies the apparently anomalous spatial quantization in the magnetic field that **Heisenberg** introduced as a special hypothesis b), along with the cancellation hypothesis c). The special form of the Zeeman term splitting can then be used in order to establish the individual momenta of the electrons and will lead uniquely to **Heisenberg**'s half-integer quantum division a) between the nucleus and luminous electron in a doublet line atom. At the same time, the anomalous additional precession explains the anomalous nature of the **Barnett** and **Einstein-de Haas** effect. For the non-mechanical nuclear properties, **Bohr**'s quantum perturbation requirement (4) implies the demand of an integer-quantized equatorial momentum (which is usually identical to it).