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# The inertia of energy and its consequences ([1])

By P. Langevin

The notion of mass, which is fundamental in mechanics, can be introduced in three different ways that correspond to the three aspects of the phenomenon of inertia. One can define mass to be:

1. The coefficient of proportionality that relates force to acceleration.
2. The capacity for impulse or quantity of motion.
3. The capacity for *vis viva* or kinetic energy.

Rational mechanics demands that there should be agreement between those different definitions and assumes, moreover, the absolute invariability of the mass of the same portion of matter under all of the changes to which it can be subjected: e.g., physical, chemical, or mechanical (motion that is more or less rapid).

**1. Mass as coefficient of inertia.** – One ordinarily intends inertia to mean the property that matter possesses that it tends to preserve the motion that it has acquired: It resists changes in its velocity in such a way that an external action or force is necessary if one is to modify the magnitude or direction of the velocity.

Newton assumed that there was a proportionality between the force that acts upon a body and the change in velocity per unit time that the force communicated on that body, or the acceleration. The constant quotient of those two quantities serves to define the mass of the body. It necessarily results from the fundamental law that Newton assumed (viz., the independence of the effects of the forces and the motion that was acquired before) that acceleration would always point along the force that that produced it, no matter what its direction with respect to the velocity that was acquired would be, whether it be longitudinal (tangential acceleration), transverse (normal acceleration), or oblique to the trajectory.

**2. Mass as the capacity for impulse.** – One can make each portion of matter, each material point in motion correspond to a directed quantity, namely, its *impulse G*, which is zero at rest and for which, by definition, *the magnitude and direction of its variation per unit time is given by the resultant force that acts upon that portion of matter*. In other words, the magnitude and direction of the impulse that is communicated by a force $f$ during the time $dt$ is defined by the product $f\,dt$,

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which is the *impulse of a body*, and is by definition the geometric sum of the elementary impulses that were communicated to it upon starting at rest by the various forces that were exerted upon it.

For a system of bodies that are generally capable of acting upon each other, the total impulse is defined to be the geometric sum of the individual impulses. It would then result from the principle of the equality of action and reaction that if the system is closed, so removed from all external actions, then that total impulse would remain invariable, so it is conserved in the course of time. It does not change, although the individual impulses of the parts of the system change as a result of the forces that they exert upon each other.

It results from the law of inertia that the impulse $G$ that is imparted to a body upon starting at rest is equal to its quantity of motion, i.e., the product of its mass with its velocity with the same direction as the latter, so the vectorial relation:

$$G = m\,v\,.$$

One can also define the mass by that relation, namely, as the *capacity for impulse*, so it is the quotient of the impulse over the velocity (the Maupertuisian mass, to Poincaré). One can further call it the capacity for the quantity of motion if one considers the latter expression to be synonymous with impulse.

I shall insist upon the point that the principle of the conservation of impulse or total quantity of motion *that is carried by matter* in a closed system is a consequence of the principle of the equality of action and reaction and will cease to be exact at the same time as the latter principle.

**3. Mass as the capacity for kinetic energy.** – Just as the motion of the impulse communicated by a force leads one to state the principle of the conservation of the quantity of motion, the notion of work will lead one to state the principle of the conservation of energy in a more or less general form.

By definition, *the kinetic energy of a material point in motion is equal to the algebraic sum of the works done by the forces that have acted upon it when starting at rest.* If one is dealing with a body with finite dimensions then that definition will persist with no modifications only if the body remains the same for all observers that are connected with it. One should note that it is not necessary to introduce any restriction of that type in the definition of impulse that was given above: We will see that the same simplicity is also found in the relationship between mass and energy when one introduces the *total* energy of the body in motion in place of its kinetic energy.

In the meantime, we define the kinetic energy $w$ of a body, as usual, by the total work that must be expended in order to take it from a state of rest to its present state of motion of the body *while in its present configuration*.

Under those conditions, in rational mechanics, if $m$ is the mass of the body and $v$ is its speed then one will have:

$$w = \tfrac{1}{2}\,mv^2\,.$$

One can also utilize that relation in order to define the mass as the *capacity for kinetic energy*, namely, as the quotient of twice the kinetic energy or *vis viva* by the square of the speed (kinetic mass, to H. Poincaré)

**4. Inertia as a fundamental property and a mechanism.** – Since the time of Newton, one has assumed (and this is how every treatise on physics begins) that inertia is a fundamental property of matter whose existence cannot be reduced to simpler phenomena and which must be, on the contrary, accepted as a principle of explanation. For more than two centuries (and this is what is essential in the mechanistic doctrine), one has likewise believed that a physical phenomenon can be explained completely only after one has reduced it to motions that are governed by the laws of rational mechanics, and in particular, the law of inertia.

Despite the perfection of its form and the secular services that it has rendered, nowadays we can no longer preserve the edifice of Newtonian dynamics, in which inertia, as measured by an invariable mass, is included as its principal foundation.

Inertia is no longer a fundamental property since it is possible to account for it (or at least some of it) by starting from some laws of electromagnetism that are probably simpler and more primitive. Mass is no longer invariable since its various definitions cease to coincide when the speed of matter ceases to be small with respect to that of light, and for the same portion of matter, all three of those definitions will lead to values that vary as a function of speed according to three different laws.

There is more to it than that: When one is close to a state of rest (so for small speeds), the three definitions will coincide and lead to a certain *initial mass* $m_0$ for a given portion of matter: However, *that initial mass depends upon the physical or chemical state of the system and varies under any modification that accompanies an exchange of energy with the external environment*; radiation, for example.

We will be led to conclude that any increase $\Delta E$ in the total energy of a body at rest or in motion will translate into a proportional increase $\Delta m$ in its mass according to the remarkably simple relation:

$$\Delta m = \frac{\Delta E}{V^2},$$

in which $V$ represents the speed of light *in vacuo*. The mass of a body will remain constant only to the extent that the internal energy does not change, and even for low speeds.

In an isolated system whose various parts exchange energy with each other, the individual masses will not be conserved. Only the total mass will remain invariable under the condition that the system neither gains nor loses total energy.

The conservation of mass ceases to be a distinct principle. It becomes confused with the conservation of energy.

Does it not seem that our edifice has gained some simplicity and harmony by that unification of the two principles in question, which were originally considered to be independent? The most recent of the concepts, and the one that was laboriously attained by its successive generalizations, namely, that of energy, now seems to us to be much more fundamental and richer than the other one, which reduces to nothing more than one aspect of the former.

Moreover, it is understood that the departures from the results that are given by rational mechanics will become perceptible only under exceptional circumstances.

Mechanics preserves its immense practical value in all situations where one is not dealing with either speed above 30,000 kilometers per second or changes in state that bring energy into play, such as the ones that take place in radioactive bodies or the ones that must accompany the formation of atoms. With those reservations, one can consider mass to be invariable and the equations of dynamics to be exact. Rational mechanics will only have to lose the explanatory power that defines its supremacy and will always be satisfied in a first approximation.

**5. Electromagnetic inertia.** – The insufficiency of the mechanism will be neatly manifested when one is unsuccessfully forced to provide an explanation for electromagnetic and optical phenomena.

Today, we see that the deep reason for those difficulties lies in the fact that the equations of dynamics, on the one hand, and those of electromagnetism, on the other, do not involve the same conceptions of space and time. It even seems that electromagnetism is the more correct of the two and that the other one, viz., rational mechanics, is valid only in the first approximation. For the last decade, it has then seemed to be much more fruitful to search for an electromagnetic interpretation of inertia rather than a mechanical explanation for the laws of electromagnetism. The latter have the character of great simplicity that qualifies them to serve as the basis for physics and the explanatory principle. Although we are still far from being able to state that such an electromagnetic synthesis is possible, the effort that has gone into formulating it and the change in viewpoint that it implies have already exhibited some very rich consequences and new insights, some of which we would like to point out.

The first indication of any possibility of explaining inertia was given by J.-J. Thomson in 1881 around his twentieth year. Full of Maxwell's ideas that he had absorbed when he was at Cambridge, it included the fact that in order for a body to be electrified, a certain inertia of electromagnetic origin must be communicated to it. That would result from the law of convection of current, i.e., from the property that an electrified body possesses that it will create a magnetic field around it when it is in motion. We can presently consider that property, which Maxwell deduced from the equations that he established, and which served as the basis for Thomson's calculations, to be an experimental fact.

The experiments of Rowland and his students have indeed verified the following consequence of the theory qualitatively and quantitatively: An electrified body – a sphere, for example – of radius $a$ and charge $e$ that moves uniformly with a velocity $v$ will create a magnetic field around it that is distributed along circular lines of force that have their plane normal to the direction of the velocity and are centered on the trajectory of the center of the sphere.

In other words, the magnetic field at a point $A$ is normal to the plane $AOv$ that passes through the point $A$ and the trajectory of the center and has a magnitude of:

$$H = \frac{ev\sin\alpha}{r^2}, \tag{1}$$

at least *as long as the speed v is very small with respect to the speed of light V*.

If the sphere is charged only on its surface then it will produce no magnetic field in its interior, any more than an electric field, and the formula (1) will be valid only when $r > a$.

On the other hand, as one knows, the extension of the principle of conservation of energy to electromagnetic phenomena would demand that the production of an electric field $h$ in a medium of specific inductive power $K$ would represent a localized expenditure of energy in the medium of order $\frac{Kh^2}{8\pi}$ per unit volume and a magnetic field $H$ in a medium of constant permeability $\mu$ would likewise represent a localization of energy with a density of $\frac{\mu H^2}{8\pi}$.

In order to find the electric $W_e$ and magnetic $W_m$ energies that are localized in a portion of a medium of finite extent, if $d\tau$ represents a volume element then one must calculate the following integrals over that extent:

$$W_e = \int \frac{Kh^2}{8\pi} d\tau \qquad \text{and} \qquad W_m = \int \frac{\mu H^2}{8\pi} d\tau .$$

One easily deduces from this that, conforming to a well-known result in electrostatics, the electric field that surrounds our sphere of surface charge $e$ and radius $a$ (which is supposed to be *in vacuo*, whose inductive power is $K_0$ and whose permeability is $\mu_0$) represents an electrostatic potential energy of:

$$W_0 = \frac{e^2}{2K_0 a}$$

when it is at rest.

When the sphere is in motion with a speed that is small compared to that of light, that electric field will remain distributed as it was at rest and will accompany the sphere in its motion: It carries with it the radial lines of force that are arranged symmetrically about it like hair. The electrostatic energy, which will be constant for as long as the speed remains small, will then displace along with the charge.

However, from Rowland's experiment, that displacement of the electric field will imply the production of a magnetic field that surrounds the electrified body and will also accompany its motion. That field, which is proportional to the speed according to the law of current convection (1), represents an energy that is proportional to the square of the speed, and an easy calculation that is based upon the expression for the volume density $\frac{\mu_0 H^2}{8\pi}$ will show that it is equal to:

$$W_m = \frac{\mu_0 e^2}{3a} v^2 . \tag{3}$$

At the moment when the charged sphere is put into motion, that energy must be provided by the external actions that communicated the speed $v$ to it. It will remain coupled with them along its evolution and must be restored at the moment when it stops in the form of the work that is done against the retarding actions. Furthermore, despite putting the electrified body in motion or stopping it, those exchanges of work will leave it identical to itself for the observers that are coupled with it since for them it will remain surrounded by only its electrostatic field ([1]).

The energy $W_m$ will then present all of the character of a kinetic energy since it has the character of being proportional to the square of velocity and corresponds to the existence of kinetic mass, namely, a supplementary inertia of electromagnetic origin:

$$m_0 = \frac{2\mu_0 e^2}{3a}, \tag{4}$$

which results solely from the fact that the sphere is electrified and is added to the inertia that it might represent on the other hand.

*We must note immediately that a comparison of* (4) *with* (2) *will shows that this electromagnetic inertia, which is due to the presence of an electric field around the sphere, is proportional to the energy $W_0$ that the electric field represents and that the sphere carries with it.* Any variation of the charge or radius, and as a result, of the energy that is stored around the sphere at rest, implies a variation that is proportional to its inertia. Later on, we shall examine the relationship that this points to between the inertia of a system and the potential energy that it might include more precisely.

We will also see that for speeds of the same order as that of light, the electric field will no longer be distributed around the sphere in motion in the same way that it was at rest. It will represent an energy $W_e$ that differs from $W_0$. If we continue to let $W_m$ denote the energy of the magnetic field then the work that would be necessary to put the body into motion as a result of its charge would be:

$$w = W_e + W_m - W_0 ,$$

and it would indeed represent a kinetic energy since the electrified body would always have the same appearance for the observers that are coupled with it.

The preceding very simple argument leads us to predict that when we are close to a state of rest, there will be an initial supplementary mass $m_0$ when we take the viewpoint of our third definition, namely, that of *kinetic mass*: We have shown that a body will possess a supplementary capacity for energy of motion due to the fact that it is electrified. In order to do that, we utilized the well-known localization of energy in a magnetic field.

That result will remain exact even when one adopts the viewpoint of Maupertuisian mass: Due to its charge, the electrified body will take on a supplementary capacity for a quantity of motion. In order to show that, it is necessary to recall how Henri Poincaré was led to localize the quantity of motion in an electromagnetic field in the same way that one does in order to represent the

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([1]) In order to be fully rigorous, that last statement must involve the principle of relativity, at which point, one will then begin to glimpse the role of the statement in that theory.

conservation of energy, namely, by the well-known localization of energy that we just appealed to in order to preserve the principle of the conservation of the quantity of motion. If we choose that path then we will encounter some general results that will be useful to us in what follows.

**6. The ether and electromagnetic waves.** – First of all, we must rise to a level that is a little higher than what we have had need for up to now and briefly recall the fundamental electromagnetic properties of the vacuum that were asserted by Maxwell and verified experimentally by Hertz.

The two electric and magnetic fields $h$ and $H$, resp., that might be based in the ether and that correspond to the localizations of energy that were recalled above are not independent of each other and are coupled in such a fashion that each of them can exist only under the condition that it does not vary: Any variation in time of one of the two fields would imply the production of the other. The variation of the magnetic field in a given region of the ether would produce an electric field whose lines of force would rotate around the direction in which the magnetic field varies: That is the phenomenon of static induction. If the space in which that electric field is thus created is found to be occupied by a conductor then induced currents would result in it that would rotate around the direction in which the magnetic field varies. The quantitative relation that translates that relationship is given by the well-known law of induction: *The electromotive force or work done by the electric field along an arbitrary closed contour C is equal to the variation per unit time of the magnetic induction flux across an arbitrary surface S that is bounded by the contour C*. The magnetic induction $B$ is equal to $\mu_0$ $H$ *in vacuo*, and if $\Phi$ is its flux across the surface $S$ then one will have:

(5) $$\int_C h \cos \alpha \, dl = -\frac{d\Phi}{dt}.$$

Analogously, the variation of the electric field in a given region of the ether will produce a magnetic field whose lines of force rotate around the direction in which the electric field varies. That is the primordial phenomenon of the *displacement current* that Maxwell predicted and which we shall see can be deduced immediately from the law of the *convection current*, and it was verified experimentally by Rowland, as well as the propagation of electromagnetic waves with the speed of light, which was verified experimentally by Hertz.

The quantitative relation that translates the law of the displacement current is symmetric to the preceding one: *The magnetomotive force or work done by the magnetic field along an arbitrary closed contour C is equal to the variation per unit time of the electric induction flux across an arbitrary surface S that is drawn in the vacuum and is bounded by the contour C*. The electric induction $b$ is equal to $K_0$ $h$ *in vacuo*, and if $\varphi$ is its flux across the surface $S$ then one will have:

(5) $$\int_C H \cos \beta \, dl = \frac{d\varphi}{dt}.$$

The two relations (5) and (6) imply the immediate consequence that an electromagnetic perturbation that is produced by, for example, a Hertz exciter, will propagate spherically in the vacuum with a speed $\frac{1}{\sqrt{K_0\,\mu_0}}$. That quantity can be deduced from a comparison of the two systems of electrostatic units and the electromagnetic CGS units, and experiments have shown it to be numerically equal to the speed of light.

The other characteristics of the perturbation will be shown to be particularly simple when one takes the distance to the source to be large enough that one has the right to equate it to a plane wave that propagates normally to its plane. The same equations will show that the perturbation that the wave represents is composed of an electric field $h$ that is located in the plane of the wave, which is consequently transverse to the direction of propagation, and a magnetic field $H$ that is likewise transverse and perpendicular to the direction of the electric field in the plane of the wave. The three directions, when arranged in alphabetical order: electric field, magnetic field, propagation, then form a tri-rectangular trihedron whose sense is obtained by making them correspond to the first three fingers of the right hand in the order that was indicated. Moreover, the magnitudes of the two fields are such that they represent equal energies per unit volume in all of the plane wave when it propagates freely, so one has the relation:

$$(7) \qquad K_0\,h^2 = \mu_0\,H^2 \qquad \text{or} \qquad H = \sqrt{\frac{K_0}{\mu_0}}\,h\,.$$

All of the characteristics of the electromagnetic waves that were thus predicted, viz., the magnitude of their speed of propagation and the transversality of the vectors that represent the state of the medium, will give an excellent representation of light waves: The analogy will become even more striking when one considers the fact that Hertz had succeeded in producing such waves along a purely-electric path and studying their properties. Nowadays, we must consider the electromagnetic nature of light to be an established fact and assume that its waves have the structure that I just recalled. Hertz's experiment was the first and most spectacular verification of Maxwell's law of the displacement current.

**7. The convection current.** – That same law leads one to predict the production of a magnetic field by the motion of an electrified body. Indeed, it would carry with it the electric field that is produced by its charge, so the intensity of the field would vary at a fixed point in the medium according to whether the electrified body was approaching it, passing it, or moving away. The electric field would then vary with time in the medium, and as a result a magnetic field would be produced by virtue of the law of the displacement current. Upon applying the statement of that law that was given by formula (6) to the case of a sphere and supposing that the speed is low enough that the electric field will remain distributed around it during the motion as it was at rest, one will easily recover the relation (1) in a way that would be accessible to experiment.

One also imagines that this expression for the magnetic field that is produced by convection will cease to be exact when the speed $v$ becomes sufficiently large: The electrified body will carry its magnetic field along with it just as it carried its electric field, so the intensity of the magnetic field will vary at a fixed point in the medium, and as a result an induced electric field will be produced by virtue of the law of induction that is expressed by (5), which must be added to the electrostatic field in order to modify the distribution. Calculation will show that this modification must become appreciable only when the speed $v$ ceases to be small with respect to the speed of light:

(8) $$V = \frac{1}{\sqrt{K_0 \, \mu_0}} .$$

To the extent that the speed $v$ increases, the lines of force of the electric field, which remain radial at distances from the sphere that are sufficiently large with respect to its radius, would cease to be distributed uniformly around it and would tend to accumulate, as J.-J. Thomson was the first to show, in the vicinity of an equatorial plane that is perpendicular to the direction of the velocity: At large speeds, those lines of force that are attached to the electrified body will tend to be placed transversally with respect to the direction of motion and will achieve that state completely when $v$ attains the speed of light, as long as the energy of the electromagnetic field that corresponds to that bounded configuration does not become infinite. The speed of light will then represent an upper limit on the speed that an electrified body can take, which is a limit at which the kinetic energy that is defined by:

$$w = W_e + W_m - W_0$$

would become infinite. That kinetic energy would then increase more rapidly than formula (3) would suggest and would cease to be proportional to the square of the speed when the latter became of the same order as the speed of light. We shall return to the exact form for its expression later on.

**8. The Lorentz force.** – Just as an electrified body in motion will produce a magnetic field and can act upon a magnet when it passes through the neighborhood of one (Rowland's experiment), it is likewise subject to a reaction to the latter that is determined the intensity of the magnetic field that is produced by the magnet at the point where the electrified body is found, the charge of the latter, and its velocity. In other words, an electrified body of charge $e$ that moves with a speed $v$ in an external magnetic field $H$ that makes an angle of $\beta$ with the velocity will be subject to a force:

(9) $$f = \mu_0 \, H \, e \, v \sin \beta$$

that points perpendicular to the plane that contains $H$ and $v$ in the sense that is indicated by the middle finger of the left hand when the thumb points in the direction of $H$ and the index finger points in that of $v$ while the charge $e$ is supposed to be positive. It is the action of that force that produces the deflection of cathode particles and the $\alpha$ and $\beta$ rays of radioactive bodies in an external magnetic field. If there exists an external electric field $h$ at the point where the moving electrified body is found at the same time as the magnetic field then the preceding force will be combined with the electric force $h\,e$, which points along the field $h$. Naturally, the latter will exist only when the magnetic field is zero or when the electrified body is immobile.

**9. Action and reaction.** – The forces that are thus exerted upon the moving electrified body and whose existence has been verified by experiment both qualitatively and quantitatively do not satisfy the principle of the equality of action and reaction.

One can already verify that this is indeed the case for the mutual action of two electrified bodies in motion with speeds that are low enough that formula (1) will remain applicable to both of them. The mutual electric force that is given by Coulomb's law satisfies the equality of action and reaction, but the same thing is not true for the electromagnetic Lorentz forces that result from the motion of each of the two bodies in the magnetic field that is produced by the other. The same thing is true *a fortiori* for large speeds at which neither the electric force, nor the electromagnetic force, nor their resultant will obey the principle.

That fact will become particularly clear when radiation is involved: One easily convinces oneself of that by examining the phenomenon of radiation pressure, whose existence can be predicted by an application of the preceding results. Indeed, imagine a source of electromagnetic radiation that is emitted around it symmetrically with respect to an axis: e.g., a Hertz exciter or an incandescent body that radiates light. From symmetry, the radiation will exert no force on its source at the moment of emission. Suppose that the emitted perturbation, which can be replaced with a plane wave at some distance in a certain direction, proceeds to encounter an obstacle, such as a metallic plate or a receiving antenna. The electric field that is present in the wave that acts upon the obstacle will produce current in the latter with the same direction as the field, and the magnetic field that is present in the same wave will exert a force on the current, and it is easy to see that it will point in the sense of propagation. In that way, one can predict the radiation pressure whose existence has been established quite well experimentally. It will push the obstacle in the direction of propagation of the radiation, and the action thus-exerted will not be compensated by any reaction that is exerted upon an arbitrary portion of the matter. If one considers only the matter then a quantity of motion will then appear on the obstacle at the moment when the radiation reaches it: There will be action without reaction, so there will be no conservation of the quantity of motion that the matter possesses.

One can also look at things from a different angle by supposing that the obstacle receives and absorbs all of the radiation that is emitted by the source, which is no longer supposed to be symmetric and radiates only in the direction of the obstacle that is placed at a distance. The obstacle will be pushed by the radiation that it receives at the moment when it receives the radiation: Contrary to what happened in the previous case, a reaction will exist, but it will be ahead of the action by a time interval that is equal to the duration of the propagation from the source to the obstacle. Indeed, theory shows that the source submits to a recoil at the moment of emission that is equivalent to the impulse that is exerted on the obstacle, like the quantity of motion, but there is no equality of action and reaction at each instant: A quantity of motion will disappear from the matter (source) at the moment of emission and reappear much later on the obstacles at the moment of absorption.

**10. The electromagnetic quantity of motion.** – One can obviously accept that way of describing the facts and abandon the conservation of the quantity of motion, at least as far as electromagnetism is concerned. However, as Henri Poincaré showed quite well, one can also safeguard the principles whose simplicity makes their use convenient by a suitable generalization of the notions that they imply. Indeed, it suffices to assume that the radiation, namely, the electromagnetic perturbation *in vacuo*, represents a quantity of motion that propagates with it and disappears when it is absorbed.

In our first case, the impulse that the obstacle is subjected to was compensated by the perturbation that it brought to the radiation and the quantity of motion that the radiation represented, which was originally zero by symmetry. In the second case, at the moment of emission, the recoil of the source was compensated by the electromagnetic quantity of motion of the radiation that appeared, and that, in turn, transformed into an impulse that the obstacle was subjected to at the moment of absorption.

We can then recover conservation by considering the radiation to be a vehicle for the quantity of motion. One can proceed in exactly the same way by considering it to be a vehicle for energy. Indeed, it is even obvious that we assumed localizations of the energy in the electric and magnetic fields outside of the matter in order to maintain the validity of the principle of the conservation of energy exactly as we seek to do here with the quantity of motion. The matter will lose energy at the moment when the source emits radiation and it will receive energy when the obstacle absorbs it, and there will be conservation at each instant only if we assume that the electromagnetic field is a form of energy that equals $\frac{1}{8\pi} K_0 \, h^2 + \frac{1}{8\pi} \mu_0 \, H^2$ per unit volume that it occupies. With Poincaré, we will also be led quite naturally to assume that it is a form of quantity of motion.

Furthermore, the statement about the localization of the quantity of motion is as simple as the one about the localization of energy. In a region of the ether that is occupied by an electric field $h$ and a magnetic field $H$ whose direction form an angle of $\alpha$ between them, one must assume the presence of a quantity of motion per unit volume of:

$$g = \frac{K_0 \, \mu_0 H \, h \sin \alpha}{4\pi} = \frac{S}{4\pi V^2}, \tag{10}$$

in which $S$ represents the area of a parallelogram that is constructed from $h$ and $H$. That quantity of motion points normally to the plane of that parallelogram in the sense that is indicated by the middle finger of the right hand when the thumb and index finger point along $h$ and $H$, respectively.

Before we address the calculation of the electromagnetic Maupertuisian mass of our electrified sphere, it is particularly interesting to apply it to the case of a plane wave. Given the description of such a wave that we made above, we will easily see from the preceding statement that it represents a quantity of motion that *points in the sense of propagation*, which is perpendicular to the plane of the wave that contains the two electric and magnetic fields. On the other hand, since those fields are mutually perpendicular, that angle $\alpha$ will be a right angle, and if one takes the relation (7) that exists between their magnitudes into account then one will have that the density $g$ of the quantity of motion is:

$$g = \frac{K_0 \, h^2}{4\pi V}. \tag{11}$$

By virtue of (7), the value of the energy density will be:

(12) $$E = \frac{1}{8\pi} K_0 h^2 + \frac{1}{8\pi} \mu_0 H^2 = \frac{1}{4\pi} K_0 h^2 .$$

One then has the very simple relation that relates to the case of a plane wave:

(13) $$g = \frac{E}{V} .$$

The radiation pressure is calculated immediately upon starting from that result: It is due to the transmission to the obstacle of the quantity of motion that the absorbed radiation represents. If the absorption is complete and the incidence is normal then one will have that the radiation pressure or quantity of motion that is transmitted per unit area and per unit time is the amount that is contained in the radiation along a length that is equal to its speed of propagation $V$, i.e., $g\,V$ or $E$, from the relation (13). That implies the result that *the radiation pressure under normal incidence is equal to the energy of radiation per unit volume.*

**11. Electromagnetic mass.** – Since we know the distribution of the electric and magnetic fields around an electrified body in motion with velocity $v$, it is easy to find the total electromagnetic quantity of motion that is localized in its wake by an application of formula (10): By virtue of the principle of conservation, it is provided by the forces that put the body into motion, and will be restored at the moment when it stops. The state of electrification of the body that communicates it is then a supplementary capacity for the quantity of motion, so a Maupertuisian mass of electromagnetic origin.

In the case of a sphere whose speed is low, the electric field, which is zero in the interior when the charge is on its surface, will have the value at a distance of $r$ : $\frac{e}{K_0 r^2}$ when $r > a$. Since that field is radial and the magnetic field that is given by (1) is normal to the meridian plane that passes through the trajectory of the center and through the point where one considers the field, the density $g$ of the quantity of motion at that point will be situated in the meridian plane and normal to the radius in a sense such that its projection onto the velocity will be in that direction at any point of the field. That density will have the value:

$$g = \frac{\mu_0 e^2 v \sin\alpha}{r^4} .$$

One will easily see that the total quantity of motion that is localized in the wake points along the velocity due to symmetry. One will obtain it by integrating the product of each volume element with the projection $g \sin \alpha$ of the corresponding density onto the direction of velocity over the entire external volume, and one will get:

$$G = \frac{2\mu_0 e^2}{3a} v ,$$

in total.

One will indeed find that for low speeds one has a Maupertuisian mass $G / v$ that agrees with the kinetic mass that was obtained before:

$$m_0 = \frac{2\mu_0 e^2}{3a}.$$

**12. Case of large speeds.** – The various definitions of mass will no longer coincide for the electromagnetic part of inertia when the speed ceases to small with respect to $V$, and everything will lead to values for it that start from $m_0$ for small speeds and increase with $v$ until they become infinite in the limit of $V$.

Similarly, the kinetic energy $w = W_e + W_m - W_0$ will cease to be proportional to the square of velocity, the total quantity of motion $G$ will cease to be proportional to the velocity and will increase faster than the latter. We shall continue to define a Maupertuisian mass that is a function of the velocity by the relation:

$$m = \frac{G}{v}. \tag{14}$$

We shall see in an instant the relation that exists between the kinetic mass $m_e$ and the Maupertuisian mass or the mass that is properly called $m$.

On the other hand, the definition of the mass as a coefficient of inertia by the equation $f = m\gamma$ will lead, first of all, to an electromagnetic mass that is a function of just the velocity only when the set of the two fields that surrounds the electrified body is determined at each instant and any distance by the *present* value of the velocity.

That will be true only if the variations of the velocity are very slow, so the acceleration is very small. One then says that the motion is *quasi-stationary*. Since the condition for the motion to be like that is that the electrified body should not emit any appreciable radiation by virtue of its acceleration ([1]), that radiation will carry neither energy nor quantity of motion to infinity, and one can apply the conservation principles while taking into account only the electromagnetic wake that is determined by its velocity. One can define the coefficients of inertia, which are functions of only the velocity, by dividing the force by the acceleration that it produces.

It is easy to show that the result obtained will differ according to whether forces acts in the direction of velocity and will produce a change in its magnitude (tangential force $f_l$, longitudinal mass $m_l$) or in a direction that is normal to the velocity in order to produce a change in its direction (normal force $f_t$, transverse mass $m_t$). In the first case, the quantity of motion changes in magnitude and not in direction, and one will have:

$$dG = f_l \, dt$$

by the definition of impulse, and on the other hand, the definition of longitudinal mass as the coefficient of inertia in this case:

([1]) See P. Langevin, J. de Phys. (4) **4** (1905).

$$f_l = m_l \frac{dv}{dt},$$

so:

(15) $$m_l = \frac{dG}{dv}.$$

In the second case, the quantity of motion will change only in direction without changing in magnitude. If $d\alpha$ represents the angle through which the direction of the velocity turns, and as a result the quantity of motion, during the time interval $dt$ then one will have:

$$f_t\, dt = G\, d\alpha$$

by the definition of impulse, and:

$$f_t = m_t\, \gamma = m_t\, v \frac{d\alpha}{dt}$$

from the definition of transversal mass $m_t$, so:

$$m_t = \frac{G}{v}.$$

The transversal mass will then coincide with the Maupertuisian mass.

Indeed, upon applying the conservation of energy, one will get:

$$dw = f_l\, dt = m_l\, v\, dv = v\, dG,$$

so the kinetic energy will be:

$$w = \int_0^v v\, dG,$$

and the kinetic mass will be:

(16) $$m_c = \frac{2w}{v^2} = \frac{2}{v^2}\int_0^v v\, dG.$$

In the case where $G$ ceases to be proportional to $v$, while still remaining a function of velocity, equations (14), (15), and (16) will determine the relations between the various definitions of mass. However, it is much simpler to keep just one of those definitions since they will cease to coincide.

For various reasons, the most important of which is simplicity, we shall ordinarily intend that the word *mass* should mean the Maupertuisian, or transverse, mass $m = G / v$. Moreover, that is the one that is involved with the relative measurements of electrified particles in motion (cathode rays and $\beta$ rays) when one observes the deflections that are produced by electric or electromagnetic forces that are perpendicular to the direction of the ray.

We give the name of *initial mass* to the value $m_0$, which will be common to the various definitions of mass for low speeds.

The *electromagnetic mass* is obtained as a function of velocity upon dividing the quantity of motion that is localized in the electromagnetic wake of the electrified body by the speed.

Moreover, the function of velocity that is obtained for that quantity of motion depends upon the manner by which one supposes that the electrified body behaves when its velocity varies according to whether one supposes that its form is fixed or not:

For Max Abraham, the simplest hypothesis seemed to be undeformability: For example, if our sphere preserves its form for all velocities then upon letting $\beta$ denote the ratio $v / V$, we will get:

$$m = \frac{G}{v} = \frac{3m_0}{4\beta^2}\left(\frac{1+\beta^2}{2\beta}\log\frac{1+\beta}{1-\beta} - 1\right) .$$

However, on the other hand, one knows that in order to account for the absence of any action of the motion of the Earth on electromagnetic and optical phenomena, Lorentz was led to assume that all bodies have different forms when they are in uniform translatory motion or at rest, so a body in motion respect to some observers will appear to be contracted in the direction of its velocity by the ratio $\sqrt{1-\beta^2}$, in such a way that the sphere in motion will appear to be a flattened ellipsoid. Upon taking that change in form into account in the calculation of the electromagnetic quantity of motion:

(17) $$m = \frac{m_0}{\sqrt{1-\beta^2}} .$$

That formula is much simpler than that of Max Abraham, and *experiments that were performed on $\beta$ rays that were produced by radioactive bodies have shown that the variation of the mass of negative electrons with velocity is indeed represented by the Lorentz formula, to the degree of precision in the measurements.* Moreover, from the standpoint of the principle of relativity, it even seems certain that any mass must vary with velocity according to that law, regardless of it origin.

If one starts from (17) then it would be easy to get the laws of variation that correspond to the other definitions of mass: One will immediately obtain:

$$G = \frac{m_0\, v}{\sqrt{1-\beta^2}} , \qquad m_l = \frac{dG}{dv} = \frac{m_0}{(1-\beta^2)^{3/2}} ,$$

and then:

(17′) $$w = \int_0^v v\, dG = m_0\, V_2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) ,$$

so:

$$m_c = \frac{2\, m_0}{\beta^2}\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) .$$

**13. Poincaré pressure.** – We just saw that experiments confirm the hypothesis of Lorentz contraction for cathode particles or free electrons: Their mass varies with velocity according to the law that predicted that hypothesis for electromagnetic inertia. The idea that such a contraction is produced by the single fact that a body is in motion might seem singular at first. Einstein then showed that it corresponds to only one aspect of the new notions of space and time that are imposed by the principle of relativity.

An important remark by Henri Poincaré illuminated the mechanism itself of that contraction from an entirely different viewpoint. We know that there exist free negative electrons or cathode corpuscles, which are all identical and carry an individual charge that is equal to around $4 \times 10^{-10}$ electrostatic units (CGS). We also know that their initial mass is equal to $10^{-27}$ grams, which is very probably of solely electromagnetic origin, but we ignore all of their structure, as well as the nature of the actions that maintain their unity. Meanwhile, it is certain that apart from the electromagnetic actions, some other ones would be necessary inside of the electron in order to prevent the dispersion of its charge by the mutual repulsion of the elements that comprise it, and it is natural to assume (and this is the first thing that one attempts) that we can extend the laws of electrostatics that were established by experiments performed at our own scale to the interior of the electron.

Poincaré pressure provides at least a simple image for some similar actions. Imagine that the charge $e$ of the electron at rest is distributed over the surface of a sphere of radius $a$.

In the absence of motion, the mutual electromagnetic actions between the various elements of that charge reduce to the well-known electrostatic repulsion, which tends to push the electrified shell outwards with a force per unit area that is equal to $2\pi\sigma^2 / K_0$, in which $\sigma$ represents the surface density $e / 4\pi a^2$. That repulsion can be equilibrated if the electron is subject to a pressure that comes from the external ether and has the value:

$$p = \frac{2\pi\sigma^2}{K_0} = \frac{e^2}{8\pi K_0 a^4}. \tag{18}$$

If one imagines that such a pressure prevails in all of the space that is occupied by the ether external to the electrons then each of them will be in equilibrium at rest when one takes the spherical form to have a radius $a$ that is determined by the preceding relation as a function of its charge and the universal pressure $p$. I shall not go into the details of the difficulties that the question of the stability of that equilibrium might raise.

The remarkable fact that Henri Poincaré pointed out is that if one supposes that the electron is in motion and the various elements of its electrified surface layer are subject to *that same uniform external pressure*, as well as the electrostatic and electromagnetic Lorentz forces under the action of the other elements then *the equilibrium form will change and become Lorentz's flattened ellipsoid precisely*. Moreover, it is easy for one to imagine that this can be true: Indeed, we have seen that the electric field is not distributed when it is in motion as it was when at rest, and that it will weaken at the poles in order to strengthen at the equator. The electrostatic pressure will then diminish at the poles, and the Poincaré pressure will become the one that prevails externally: The electron will flatten at the poles until one recovers a new equilibrium. It will not dilate at the equator because the Lorentz electromagnetic force acts in the opposite sense to the accrued electrostatic pressure, and it will bring about equilibrium for the same equatorial radius $a$ at rest when the polar semi-axis is equal to:

$$a\sqrt{1-\beta^2}\ .$$

We thus get a provisional, but simple, image of the electron that implies the Lorentz contraction and agrees with the experimental variation of the inertia of the electron as a function of its velocity. We would then be justified in thinking that the pressure $p$ is one of the essential factors in the equilibrium of the electron.

We can further imagine the situation in the following way: The equilibrium configuration of the electron at rest is the one that minimizes the total potential energy of the actions that are superimposed in it, which are electrostatic repulsions and the Poincaré pressure. When the form is spherical and of radius $a$, the potential energy of the former is $\frac{e^2}{2K_0\,a}$, while that of the external pressure is equal to the product of that pressure with the volume of the electron $\frac{4}{3}\pi a^3$, so the total potential energy at rest will be:

$$E_0 = \frac{e^2}{2K_0\,a} + \tfrac{4}{3}\pi a^3 p\ .$$

The value of $a$ that makes that expression a minimum is given by the relation (18) precisely. If we now replace the pressure $p$ in $E_0$ with a function of the equilibrium radius then we will get the total potential energy of the electron in equilibrium at rest as:

$$E_0 = \frac{e^2}{2K_0\,a} + \frac{e^2}{6K_0\,a} = \frac{2e^2}{3K_0\,a}\ . \tag{19}$$

**14. Mass and energy.** – A comparison of that expression with that of the initial electromagnetic mass $m_0$ that is given by (4) will immediately lead us to the remarkable relation:

$$m_0 = K_0\,\mu_0\,E_0 = \frac{E_0}{V^2}\ . \tag{20}$$

*The initial electromagnetic mass of an electron is equal to the quotient of its total potential energy by the square of the speed of light.*

We will recover that same relation if, instead of a spherical electron, we consider a system that is in equilibrium under the superimposed action of electromagnetic forces and actions such that the equilibrium configuration is modified according to Lorentz contraction when the system is put into motion.

Einstein could generalize the preceding relation and extend it to some other cases besides those of electrostatic equilibrium that were envisioned up to now by one of the most beautiful and important applications of the principle of relativity. I myself developed the following considerations independently of him in 1906, and I presented them in my course at the Collège de France in a form that was less elementary and more general than the one here.

We shall show that the presence of electromagnetic radiation inside of a cavity that is devoid of matter corresponds, in the envelope, to a supplementary inertia that is determined by the energy of radiation, and on the other hand, that the emission or absorption of radiation by a material system translates into a variation of its energy, namely its capacity for quantity of motion, that is proportional to the energy of radiation.

**15. The principle of relativity.** – All of the experiments that have tried to exhibit the motion of terrestrial observers with respect to the medium that transmits the electromagnetic actions have given only negative results, despite the extreme precision that was possible to attain. Meanwhile, we know that for two positions of the Earth in its orbit that are diametrically opposite to each other (in January and July, for example), the speed will change by around 60 kilometers per second. That negative result will then permit one to state that when electromagnetic phenomena are referred to axes that are fixed in the Earth, they will present the same character in any season, so the fundamental laws that were recalled above will be exact in both cases with respect to two systems of axes that move at a speed of 60 kilometers per second with respect to each other. In other words, everything will happen in the same fashion for various groups of observers that are in uniform translational motion with respect to each other, so everything will happen *for each of them* as if it were immobile with respect to the ether.

In particular, it will be true for all of them together that it is a consequence of fundamental laws that a luminous or electromagnetic perturbation will propagate in all directions with the same velocity:

$$V = \frac{1}{\sqrt{K_0 \, \mu_0}} .$$

That single fact implies some consequences in terms of one's conception of space and time that might seem singular on first glance since they are consequences that are incompatible with the space and time that rational mechanics demands and implies. First of all, one has the Lorentz contraction, which takes the following form: When the same object is measured by two groups of observers that move uniformly with respect to each other with a speed of $v = \beta V$, it will be shorter in the direction motion by the ratio $\sqrt{1-\beta^2}$ for the observers that pass by it than it is for the ones that move with it. Some analogous consequences exist for the time interval between two events: For example, events that are simultaneous for certain observers will cease to be so for other observers that are in motion with respect to the former ones.

As singular as those consequences might seem (and we shall not need to appeal to them here, moreover), they are imposed by the facts, so they are implied by the structure itself of the fundamental equations of electromagnetism: Those equations can have the same form for all groups of observers that are in motion with respect to each other only if the measurements of space time that they make have the mutual relationships that I have just recalled. In other words, those equations will possess a group of transformations that was discovered by Lorentz, and the part of it that relates to space and time is profoundly different from the one that corresponds to the group of transformation of the equations of ordinary mechanics: The two groups are then incompatible, and everything leads us to think that the ones for electromagnetism are the only exact ones, i.e., that experiments will give us only the space and time of the Lorentz group.

We shall utilize the principle of relativity in a very simple form, and we will need to preserve only those terms in the equations of transformation that relate the measurements of the same quantity that are made by two groups of observers that are in uniform motion with respect to each other that have first order as functions of their relative velocity.

Imagine an electromagnetic plane wave that is studied simultaneously by two observers $O_0$ and $O_1$ whose relative velocity $v = \beta V$ is supposed to be normal to the plane of the wave and suppose that the velocity has a sense such that $O_0$ sees $O_1$ as moving in the opposite sense to the propagation, so it moves behind the wave. Conversely, $O_1$ will see $O_0$ as moving with the same velocity $v$ in the sense of propagation, so it moves ahead of the wave and will reach it at a given moment since $v$ is always supposed to be less than $V$ because $\beta$ is always less than 1. By virtue of the principle of relativity, all of the results that we have obtained will be exact for both $O_0$ and $O_1$. In particular, the wave will propagate with the same speed $V$ for both of them, but that same wave will not have the same intensity for both of them: It will be more intense for the one that moves ahead of the wave than it is for the one that moves behind it because it contains electric and magnetic fields that are greater for $O_1$ than they are for $O_0$.

We can deduce that result from the group of Lorentz transformations, which gives the correspondence between the measurements of the same quantity (the electric field of the wave, for example) that are made simultaneously by the two observers. However, we can find that relation in the following manner while being content with the first-order terms, which will suffice for our purposes:

By definition, the electric field of the wave is measured by the observer $O_1$ as the force that the wave exerts upon a unit electric charge that is *at rest with respect to him*.

Let that force be $h_1$. For the observer $O_0$, the electric charge in question is in motion in the opposite sense to the propagation and normally to the wave with a speed of $v$. To him, it is therefore subject to not only the electric force, but also the electromagnetic Lorentz force that results from its motion in the presence of the magnetic field of the wave. Since that magnetic field is perpendicular to the electric field, an application of the law that translated into formula (9) will show that the electromagnetic force has the same direction and the opposite sense to the electric force. If $h_0$ and $H_0$ are the two fields that are present in the wave for the observer $O_0$ then the forces that it exerts on the unit charge in question will have the value:

$$h_0 + \mu_0 H_0 v .$$

However, since one is dealing with a plane wave, one will have the relation (7) between the two fields, so when one takes (8) into account, the result will be the value $h_0 (1 + \beta)$ that is measured by the observer $O_0$. Moreover, since it results from the principle of relativity that the measurements of *the same* force that are made by two observers in motion with respect to each other will differ only to second order as functions of the relative velocity, one can write:

$$h_1 = h_0\,(1 + \beta)\,, \tag{21}$$

to the same degree of approximation.

On the contrary, for a wave that propagates in the opposite sense with respect to the same observers, one will have:

$$h_1 = h_0\,(1 - \beta)\,. \tag{21}$$

Therefore, the wave will always be less intense for that one of the two observers who seems to move in front of it than it is for the other one.

The densities of energy and quantity of motion in the wave that are given by the relations (11) and (12), which are exact for both of the two observers, will not be the same for both of them and will have a ratio of $(1 + \beta)^2$ or $(1 - \beta)^2$ according to the sense in which the wave propagates.

**16. The inertia of radiation.** – Now consider a container of the particularly simple form that consists of two parallel plane plates whose faces are perfectly reflecting and located at a distance $d$ from each other. Suppose that electromagnetic radiation of arbitrary spectral composition is contained between them and that it propagates in the form of planes parallel to the plates. Those waves must alternately reflect from the two plates, and they will constitute radiation that is imprisoned within the container. Let $W_0$ be the electromagnetic energy per unit area of the plates that it represents for the observer $O_0$, who we suppose to be at rest with respect to the container. Each reflection from the immobile plates will simply change the sense of propagation of a wave without changing its energy, so it is obvious that, in the mean, for the observer $O_0$, half of the energy exists in the container in the form of waves that propagate in one sense, while half of it is in the form of waves that propagate in the opposite sense.

By virtue of the relation (13), the first half represents a quantity of motion:

$$G_0 = \frac{W_0}{2V}\,,$$

that points in the sense of propagation, and the second half represents a quantity of motion that is equal and has the opposite sense. For the observer $O_0$, the radiation will then represent a quantity of motion that is equal to zero, in the mean.

Furthermore, the reflection from the plates is accompanied by a radiation pressure that results from the changing of the sense of the electromagnetic quantity of motion of the waves at the moment when they are reflected. That pressure is equal, in the mean, to the energy density of radiation $W_0 / d$. In order to keep the plates in equilibrium, it would then be necessary to exert a pressure on them from the outside that is equal to the radiation pressure that is exerted from the inside. That external pressure that is necessary for equilibrium represents a potential energy per unit area of the plates that equals $p\,d$, i.e., $W_0$, in such a way that the total energy $E_0$ that is available in the system at rest will have the value $2\,W_0$. In particular, that would be the energy of the radiation that would leave the container if the plates were slightly transparent so they would allow the radiation that is contained between them to slowly escape and a constant external pressure acting on them would bring them closer together as the radiation left them. One will easily see that under those conditions, the work $p\,d$ that is done by the external pressure will be transformed into energy of radiation during the reflections that are kept on the slowly-moving plates, and that at the moment when they meet each other, the radiant energy outside of them will be $2W_0$, which is twice the original electromagnetic energy.

The external pressure plays a role here that is analogous to that of the Poincaré pressure in the case of the electron. It must intervene in the same way in the calculation of the total energy that is available in the system, which is supposed to be kept at constant external pressure.

*Just as it is only when that condition of constant external pressure is fulfilled during the process of putting things in motion*, one can consider the system to remain *the same* when it is in motion as it was at rest, and to have preserved *the same* appearance for the observers that were put in motion at the same time as it was and coupled with it ([1]).

We shall now see that the observer $O_1$ who passes the container with a speed $v$ will attribute a quantity of motion to it of electromagnetic origin that is localized in the radiation that is found between the plates. Indeed, for that observer, the waves propagate in the sense of the motion of the container with respect to him, so for him it will represent a quantity of motion with that same sense that is greater than $G_0$ by the ratio $(1 + \beta)^2$:

$$G_1 = G_0\,(1 + \beta)^2,$$

and the waves that propagate in the opposite sense will represent the quantity:

$$G_1' = G_0\,(1 - \beta)^2$$

for him.

Therefore, in total, there is a quantity of motion for the radiation *that points in the sense of the motion of the container* and has the value:

$$G = G_1 - G_1' = 4\,G_0\,\beta = \frac{2W_0}{V^2}\,v = \frac{E_0}{V^2}\,v\,.$$

The presence of radiation of energy $E_0$ will then correspond to a capacity for quantity of motion with an electromagnetic mass that is once more equal to the quotient $E_0 / V^2$.

**17. Variation of mass under absorption or emission of radiation.** – We shall now examine one last case: That of a body that emits radiation into its environment, which represents a reduction $\Delta E_0$ in the internal energy of the radiation that is measured by an observer that is coupled with it. We shall see that a reduction in its initial mass $m_0$ that is equal to $\Delta E_0 / V^2$ will result from that.

([1]) This results from the fact that the pressure is invariant under the Lorentz transformation.

Conversely, the increase in internal energy by the absorption of radiation will be accompanied by an analogous increase in initial mass.

Consider a source that consists of a planar plate that radiates plane waves from its two faces that propagate on one side and the other normally to its plane and suppose that the radiation is symmetric for an observer $O_0$ that is coupled to the source. It will send out electromagnetic quantities of motion from both sides that are equal and opposite, and from an application of the conservation law, it will remain immobile for $O_0$ during the entire time that is emitting – a second, for example. If the total radiation energy per unit area of the source during that time is $\Delta E_0$ then the wave trains that propagate to the right and left will each represent an energy of $\Delta E_0 / 2$ for $O_0$.

How will the phenomenon look to the observer $O_1$? He will see the source as moving to the right, for example, with a speed of $v$. The waves that are emitted to the right, which have an energy density of $\frac{\Delta E_0}{2V}$ for $O_0$ since they occupy a length $V$ for him, will have a density of $\frac{\Delta E_0}{2V}(1+\beta)^2$ for $O_1$. Moreover, for him, they will occupy a length of only $V - v$ since the head of the wave that is emitted to the right at the beginning of the unit time will be found to be at a distance from his starting point that is equal to the speed $V$ of propagation at the end of it, and the tail of that same wave train will be found on the plate, which has traversed the distance $v$ in the same sense. The wave train on the right will then represent an energy:

$$\frac{\Delta E_0}{2V}(1+\beta)^2 (V - v) = \frac{\Delta E_0}{2}(1+\beta)$$

for $O_1$, if one neglects the terms in $\beta^2$.

For $O_1$, that train will then represent a quantity of motion that is emitted by the source to the right:

$$\Delta G_1 = \frac{\Delta E_0}{2V}(1+\beta) .$$

One likewise sees that the wave train on the right, which has length $V + v$ for $O_1$, will represent a quantity of motion that is emitted to the left of:

$$\Delta G_1' = \frac{\Delta E_0}{2V}(1-\beta)$$

for him.

In total, to $O_1$, the source has emitted a quantity of motion towards the right (i.e., in the sense of his motion) of:

$$\Delta G = \Delta G_1 - \Delta G_1' = \frac{\Delta E_0}{V}\beta = \frac{\Delta E_0}{V^2}v .$$

By virtue of the conservation principle, the source must lose that quantity of motion: Now, its velocity with respect to $O_1$ has not changed since it remains immobile with respect to $O_0$.

Therefore, the quotient of his quantity of motion by its velocity, so consequently its mass, has diminished by:

$$\Delta m_0 = \frac{\Delta G}{v} = \frac{\Delta E_0}{V^2}. \tag{23}$$

We have thus arrived at this supremely important result:

*Any variation of internal energy of a material system by the emission or absorption of radiation will be accompanied by a proportional variation of its inertia.*

**18. Variation of mass with temperature.** – Let us first examine some consequences of the last statement, namely, some situations in which a change of state in an isolated material system accompanies a variation in internal energy by the emission or absorption of radiation.

The same portion of matter, when taken at two different temperatures, can pass from one to the other of them by emitting or absorbing radiant heat. We can evaluate the variation of mass that results from that by dividing the quantity of heat that is exchanged with the external environment by $V^2$. Take water, for example, whose calorific capacity is particularly high. We see that a mass of water at $0^{\circ}$ that has an inertia equal to 1 gram will have a greater inertia at $100^{\circ}$, and the difference will be obtained by dividing the absorbed heat (viz., 100 calorie-gram-degrees, which is equivalent to $4.18 \times 10^9$ ergs) by $V^2$, or $9 \times 10^{20}$ cm/s, which will give around $5 \times 10^{-12}$ g, i.e., a variation that is entirely imperceptible.

That example shows quite well that despite the smallness of the predicted effect, the notion of mass ceases to agree with that of a quantity of matter from the theoretical viewpoint. Two equal masses of water, one of which is taken at $100^{\circ}$, while the other is taken at $0^{\circ}$, will not contain the same quantity of matter, since will cease to be equal when one reduces both of them to the same temperature. Two masses of water that contain the same number of molecules will have the same inertia only if they are taken at the same temperature, so only when their internal energies are equal.

We shall now examine the more profound changes of state that correspond to more important variation of internal energy, and consequently, of inertia.

**19. Case of chemical reactions.** – Imagine that a chemical reaction is produced inside of a closed container – a sealed glass tube, for example. The heat that is released by the reaction will dissipate by radiation from its envelope, and from the preceding, a reduction in inertia must result that is easy to calculate by formula (23) and is proportional to the radiant energy. The order of magnitude of the heat of reaction is such that the variations of mass, thus-predicted, will again be extremely small and inaccessible to measurement.

Indeed, take one of the more exothermic reactions per unit mass of reactant: That of the formation of liquid water upon starting from its elements when taken in the gaseous state. The formation of 18 grams of water will release 69.000 calorie-gram-degrees, which is equivalent to around $3 \times 10^{12}$ ergs. Since $V^2$ is equal to $9 \times 10^{20}$ in the same system of units, the theory would predict a reduction in mass equal to $\frac{1}{3} \times 10^{-8}$ grams, which is a relative difference of one five billionth between the mass of the explosion gas and that of the water that it can form when taken at the same temperature. We can no longer hope to exhibit a variation in inertia of that order.

**20. Case of radioactive transformations.** – Radioactive bodies are the seat of spontaneous phenomena that bring into play energies that are enormous with respect to the ones that are released in ordinary chemical reactions. For example, one knows that a gram of metallic radium will release 130 calories per hour at the same time that it transforms into radium $D$ through the successive emanation forms of radium $A$, $B$, and $C$, and that it will emit helium in the form of $\alpha$ particles. The quantity of radium that transforms in one hour in 1 gram of that metal is extraordinarily small, moreover, given the slowness of the spontaneous destruction of radium: The mean life of an atom of radium is, in fact, around 2600 years, in such a way that in one hour, the reduction in 1 gram of mass would be represented by $\frac{1}{2600 \times 365 \times 24}$, and the complete transformation of a gram of radium into helium and radium $D$ would give an energy in ergs of:

$$130 \times 2600 \times 24 \times 4.18 \times 10^7 = 1.1 \times 10^{17}.$$

Furthermore, that represents only one step in the transformations that start from uranium and conclude with lead. We must then conclude that the ultimate products (viz., helium and lead) of the evolution of a given quantity of uranium have a global inertia that is less by more than one ten-millionth of that of the original uranium since one has a reduction in mass equal to:

$$\Delta m_0 = \frac{1.1 \times 10^7}{9 \times 10^{20}} = 1.2 \times 19^{-4}$$

per gram for just the radium-radium $D$ step.

How does one exhibit such a variation in inertia? Suppose that the loss of energy by radiation (so the resulting variation in mass) is not accompanied by any variation in weight (so the internal energy), so we know that it contributes to the inertia without bringing with it any contribution to the weight. It will then result that a certain quantity of uranium and the products of its transformation, namely, helium and lead, will have equal weight, but different inertias: Consequently, it will not take on the same acceleration under the action of gravity. At the same location, there must exist differences in the corresponding values for the acceleration of gravity $g$ that are equal to most one ten-millionth for different substances – uranium, helium, lead, etc. – which are differences that are entirely accessible to measurement.

Now, experiments have shown that such differences do not exist since the law of the constancy of $g$ for all bodies at the same place has been verified in an extremely precise manner: At a given location, there is an exact proportionality between weight and inertia.

The best verification of that law was made by Eotvös by means of a torsion balance: In short, his process amounts to verifying that the direction of the vertical is the same for all bodies. That direction will be the direction of the of the resultant of the real weight and the centrifugal force that is due to the rotation of the Earth: Since the latter force is proportional to the inertia, the resultant, i.e., the observed vertical, would not have exactly the same direction for all bodies if one did not have a constant ratio between weight and inertia for all of them. Eotvös confirmed that this constancy is exact to at least one twenty-millionth. That is far from deviations that are greater than one ten-millionth.

We can conclude from this that if energy is inertia then it will have, at the same time, a weight that is proportional to it, so a variation in internal energy will be accompanied by a simultaneous variation in mass and a variation of weight, so for example the same body would weigh a little more when it is hot than when it is cold, water would weigh a little less than the explosive gas that it came from, and uranium would weight appreciable more than the material products of its spontaneous destruction.

That weight of internal energy, which prevents us from exhibiting the variations in inertia by means of the variations of $g$ can, from a different viewpoint, make it easier to experimentally verify some consequences of the theory by permitting one to replace the observation of a variation of the inertia with the much easier observation of a variation in the weight that is connected with any variation of the internal energy $\Delta E_0$ and is measured in grams by $\Delta E_0 / V^2$, like the variation of mass.

That clearly agrees with the negative result of the experiments that were performed by Landolt and others in order to see whether a chemical reaction that is performed in a closed container would be accompanied by a variation in weight. We can essentially predict that it would be one ten-billionth of the original weight, but that is much less than what might register on even the most sensitive balances, and it has not been achieved for even the more energetic reactions.

It still remains for us to address the radioactive transformations. We can obviously enclose radium or uranium in a sealed tube in order to confirm that there is a variation in weight by confirming that the spontaneous transformation will terminate differently, at least noticeably sooner: That would require several centuries for radium and millions of centuries for uranium. Fortunately, such a lengthy experiment is hardly necessary, and in order to verify the exactness of our predictions, it would suffice to measure the atomic weight with a precision that is greater than one ten-millionth, which is not at all impossible to attain.

Indeed, if the mass, and consequently the weight, of the same portion of matter is preserved exactly in the course of radioactive transformations that might take place in it then that fact would result from some simple relationships between the atomic weights of the elements that are generated in succession. For a transformation that is accompanied by only the emission of $\beta$ and $\gamma$ rays, the atomic weight must not change since if the new atom is to become electrically neutral then it must get back one or more of the negative electrons (which always have the same mass) that were lost by the transformed atom. In the case of the emission of an $\alpha$ particle or a helium atom, the atomic weight will decrease by exactly the atomic weight of helium. The differences between the atomic weights of uranium or radium and that of the lead that results from their transformation must be simple integer multiples of the atomic weight of helium. On the contrary, if our conclusions are exact then those differences must be greater by quantities that are proportional to the energies that are lost during the intermediate transformations.

Given the order of magnitude of those atomic weights, which is greater than 200, and that of the heat that is released during the transformations, the deviations will affect the tenth decimal place. One sees the great theoretical importance that measurements that are made with that degree of precision can have.

**21. Prout's law.** – However, is it necessary to expect that one would arrive at that as a conclusion? It even seems that *nowadays the experimental proof of the inertia and weight of internal energy is carried by the existence of certain deviations from Prout's law*, namely, by the fact that atomic weights, although they are reasonably close to simple integer multiples of the same quantity, nonetheless present certain small irregularities when one starts from that law. It is very remarkable that the atomic weights of most elements, when calculated by taking that of hydrogen to be unity, are grouped around integers without actually coinciding with them:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| C | = 11.94 | Az | = 13.90 | O | = 15.87 | F | = 18.90 |
| He | = 4 | Li | = 6.94 | Gl | = 9 | Bo | = 10.90 |
| Na | = 22.80 | Mg | = 24.12 | Al | = 26.90 | Si | = 28.10 |

That fact is certainly not random in character, and it poses an important question in natural philosophy, namely, that of the unity of matter, and we would now like to address the possibility of answering that question.

For as long as the principle of the conservation of material mass was assumed without discussion, it seemed impossible to reconcile the existence of certain departures from Prout's law with the seductive hypothesis that the various atoms are constructed by starting from one or more primordial elements. The discovery of radioactive transformations brings with it a decisive argument in favor of that hypothesis, but the deviations will persist, and one must understand the reason for them.

The explanation that I shall propose results immediately from all of the foregoing: *The deviations that are due to the formation of atoms by starting from primordial elements* (by disintegration, as we see in radioactivity, or by an inverse process that has not been observed that would give rise to heavy atoms) *are accompanied by variations of internal energy by the emission or absorption of radiation*. The sum of the weights of the atoms differs from that of the transformed quantities by a quantity that is equal to the quotient of the variation of energy by the square of the speed of light. Moreover, the deviations are such that the energies that are thus brought into play will have exactly the same order as the ones that are effectively observed in the course of radioactive transformations.

For example, if the oxygen atom results from the condensation of 16 atoms of hydrogen or 4 atoms of helium then if one is to explain the atomic weight of 15.87, which is *less than* 16, it would suffice to assume that this condensation is accompanied by a *loss* of energy that is only five times greater than the energy that is released during the transformation of an atom of radium into radium *D*.

Far from posing an enigma, those deviations in which we see the experimental proof of the inertia and weight of energy, will, on the contrary, give us valuable information about the possible parentage of the elements and the magnitude of the energies that are brought into play during their transformations.

**22. Matter as a reservoir of energy.** – We saw that any variation of the internal energy of a body by radiation is accompanied by a proportional variation of its mass and its weight, and on the other hand, that the presence of accumulated energy in the form of radiation in a closed container or in the form of electrostatic energy corresponds to the existence of a mass, and consequently of a weight, that are always coupled to the energy that is present in the same way.

Must one believe that none of the inertia of matter has any other origin? The principle of relativity has led one to think that the variation of mass with speed $m = \frac{m_0}{\sqrt{1-\beta^2}}$, which was first established for electromagnetic inertia, applies to more general things. The same thing is probably true for the relation $m_0 = E_0 / V^2$. Any inertia will correspond to the presence in the system that possesses it of an energy that is equal to the product of the mass with the square of the speed of light, which is an energy that would have to correspond to the complete destruction of the material structure if were released.

Without prejudice, if we can one day acquire that destructive power and exhaust the reserves of energy that are present in matter then from the preceding hypothesis, we can evaluate the importance and immensity of those reserves. Every gram of matter, no matter what its nature might be, corresponds to the presence of an internal energy that is equal to $9 \times 10^{20}$ ergs, i.e., it is equivalent to the heat that is produced by the combustion of $3 \times 10^9$ grams or three million kilograms of coal.

That result, viz., that matter has inertia and weight in proportion to the energy that it contains, makes the principle of the conservation of mass coincide with that of energy. In a closed system that exchanges no energy with its external environment, the total mass will be conserved, but the individual masses of the various portions of the system will vary in proportion to the exchanges of energy that are produced between them.

The new dynamics rests upon the two fundamental laws of the conservation of energy and the conservation of impulse or quantity of motion. Moreover, those two laws are not independent: From the standpoint of the principle of relativity, they appear to be two different aspects of a single law, namely, the conservation of the world-impulse.

The individuality of one portion of matter can no longer be characterized by its mass as before: One must seek to preserve the number and structure of the elements – i.e., atoms or molecules – that define it. The transmutations to which we witness in radioactive bodies might permit us to extend that structure beyond that of changing atoms, which is where chemistry stops, and to find the individuality of a portion of matter in the number and nature of the primordial elements from which the atoms, cathode corpuscles, and maybe the positive nuclei of helium or hydrogen atoms are constructed when one starts with them. Only the number and nature of those elements will remain invariant under all of the changes that matter is subjected to, and only they can serve to define it.

**23. Case of free radiation.** – We have been led to attribute to radiation that propagates freely with the speed of light $V$, not only an energy of $E_0$ that is distributed with density that is given by (12), but also a quantity of motion whose density is given by (13) and represented by:

$$G = \frac{E_0}{V^2}$$

in the direction if propagation. We can also attribute a mass to it that is always defined by the quotient of the quantity of motion by its speed, which is equal to $V$. That mass will then be $E_0 / V^2$, which is always related to the energy by the same relation as in the case of matter, except that although matter, whether or not it includes radiation, can take a speed whose absolute value varies from 0 to $V$, the energy that is represented by free radiation can displace only with a speed of $V$.

**24. The weight of light.** – The energy that is contained in matter takes the form of weight, as well as inertia. Might one wonder whether the same thing is true for radiant energy that propagates freely? Might one wonder whether light waves or electromagnetic ones are susceptible to the action of a gravitational field, as is they had an equivalent mass to that of matter? Indeed, it is reasonable to think that we can respond in the affirmative, namely, that light rays will deflect when they are in the neighborhood of matter by virtue of gravitation and that Newton's universal law of attraction basically asserts the attraction of energy to energy. That is the conclusion that Einstein reached: He could calculate the refraction that would result in the light from a star that is seen in a direction that is close to the Sun as a result its propagation in the gravitational field that is produced by the Sun. The verification of that refraction might not be impossible to attempt.

There are indeed some problems whose solution is undoubtedly forthcoming. One can only admire the singular detour by which the theory of light waves, which is otherwise neatly opposed to the Newtonian theory of emission, is found to lead one to conclude from its connection with electromagnetism that radiation has inertia and weight and possesses all of the attributes by which one otherwise distinguishes. Meanwhile, we are quite far from our starting point since those are some consequences that are based upon properties of the medium itself that transmits the wave, which are properties that the work of Maxwell and Hertz has revealed to us. The distinction between matter and radiation remains fundamental, and it must be sought in the notion of structure, in the presence or absence of electrified centers in matter that are capable of moving with a speed that varies with the medium, although radiation propagates in it with a speed that is defined completely.

**Generalization.** – To conclude, we can generalize the relation that was established between the initial mass $m_0$ of a body and its internal energy $E_0$, as evaluated by observers $O_0$ that are at rest with respect to it. The same proportionality relationship persists between its mass $m$ and its total energy $E$, as measured by arbitrary observers $O_1$ with respect to which it is in motion. One will always have:

$$m = \frac{E}{V^2}.$$

Indeed, we saw that there exists the relation (17) between the mass of a body in motion and its initial mass, so between the masses of the same body as measured simultaneously by $O_0$ and $O_1$:

$$m = \frac{m_0}{\sqrt{1-\beta^2}}.$$

On the other hand, it results from the principle of relativity that the same relation exists between the energies $E_0$ and $E$ that are measured by $O_0$ and $O_1$ :

$$E = \frac{E_0}{\sqrt{1-\beta^2}}. \tag{24}$$

The relation (20) will lead to the indicated generalization by virtue of (17) and (24). One can then say that the total energy of a body, whether at rest or in motion, is equal to the product of its mass with $V^2$. If one agrees (as has been proposed many times with just cause) to take the speed of light *in vacuo* to be a fundamental unit then one can say: *The mass of a body is equal to its total energy*, which will the translate completely into a numerical equality with the same character as the one that we obtained between mass and energy: *The mass of a body measures its internal energy*.

Since $\sqrt{1-\beta^2}$ is always less than unity, it results from (24) that the energy of *the same body* (which is put into motion without changing its appearance to the observers that are coupled with it) is greater when it is motion than when it is at rest. By definition, the difference represents the kinetic energy, and that result will explain the form of (17′) that we have obtained for the kinetic energy $w$, which appears to be slightly complex. The two terms, $\frac{m_0 V^2}{\sqrt{1-\beta^2}}$ and $m_0 V^2$, whose difference is that kinetic energy, are nothing but the two measurements of the total energy of the same body that are made successively when it is in motion and at rest.