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On the origin of radiation and electromagnetic inertia

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Translated by D. H. Delphenich

I. – Here, I propose to show how the study of the electromagnetic perturbation that is produced in the ether – which is assumed to be immobile, following Lorentz – by a given arbitrary motion of electric charges or electrons will permit one to penetrate the phenomena of radiation and inertia in detail, and to analyze the mechanism of the connection that electrons establish between the matter that contains them and the electromagnetic ether, in the sense that was described by Larmor ([1]).

The results that follow have been presented at the Collége de France over two years. I have since then found that some of it exists in the publications of Liénard and Schwarzschild ([2]), but I believe that it is necessary to present them completely in order to direct attention to their importance and to make the new viewpoint that I have assumed seem clearer.

Indeed, it seems to be well-established today that – without speaking of purely electromagnetic phenomena – the fundamental properties of matter, inertia, and the power to emit and absorb radiation are linked with the presence of electrified particles in motion – i.e., electrons – whose displacement upon traversing the ether will modify the electric and magnetic fields that define the state of that medium.

From the experiments of Kaufmann ([3]), the inertia of negative electrons – i.e., cathode corpuscles – is of entirely electromagnetic origin, due to the necessity of creating or destroying the magnetic field that is known to accompany its motion in order to modify the motion of the corpuscle. In order to not seek two different explanations for the same phenomenon, it is tempting to extend this result to all matter by considering its inertia to be the total electromagnetic inertia of the positive and negative electrons that comprise it.

On the other hand, radiation is present in the ether only at a great distance from the source, and it can decompose into plane waves that are perpendicular to the direction along which it propagates with the velocity of light *V*, and it is composed of two transversal electric and magnetic fields whose mutually-perpendicular directions are contained in the plane of the wave. These two fields always represent equal energies per unit volume of the medium, in such a way that the propagation of such a wave will not correspond to any exchange of energy between the two fields or any oscillation of radiated energy between the electric and magnetic forms.

([1]) J. Larmor, *Aether and Matter*, pp. 229.

([2]) A. LIÉNARD, *l'Ecl. elect.*, t. XVI, 1898; pp. 5, 53, 106. – K. Schwarzschild, *Götting. Nachr. math. phys. Klasse* (1903).

([3]) W. KAUFMANN, *Götting. Nachr. math. phys. Klasse* (1903), pp. 90.

A similar exchange of energy is possible only in the presence of matter – by the intermediary of the electrified centers that comprise it and which act in some way as catalysts; i.e., as agents that are necessary, but not modified by the transformation. In other words, *only matter can be the source of radiation.*

II. – From this double viewpoint of the electromagnetic origin and limits of validity of mechanics, on the one hand, and all of the phenomena of radiation, on the other, it is then important to know the electromagnetic perturbation that is produced by an element of electric charge in a given, but arbitrary, state of motion in the ether that is governed by the Hertz-Maxwell equations, and in a form that is simple and general as possible.

The electric potential (Ψ) and vector (F, G, H) that the two fields that constitute that perturbation depend upon were given by Lorentz ([1]) in a very simple form by the intermediary of retarded potentials.

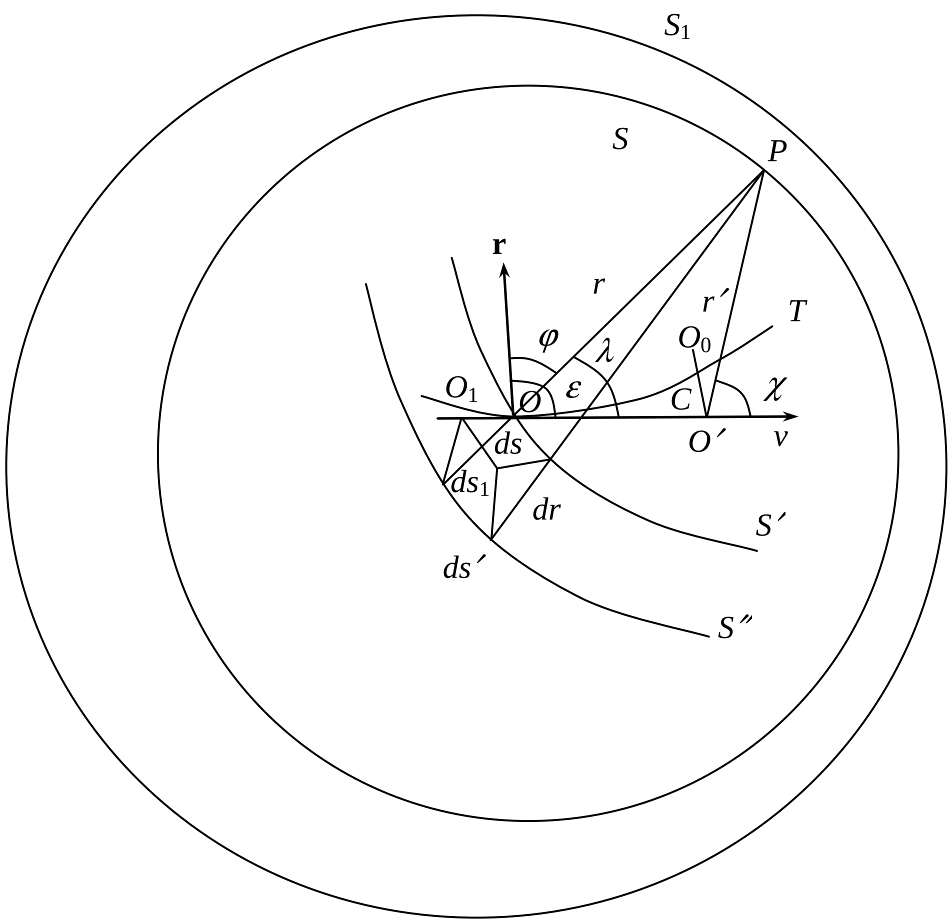

Figure 1.

([1]) H.-A. LORENTZ, *Arch. Néerl.* **25** (1892), pp. 363.

Each given element of charge in motion is determined by its position O and its velocity at the instant $t - \theta$ of the two potentials at the instant t on a sphere S that has its center at the point O and a radius equal to the path $V\theta$ that is traversed by light during the time θ.

In other words, in order to get expressions for the two potentials at a point P of the medium (Fig. 1) whose coordinates are x, y, z at the present instant t, one must suppose that a sphere S' of variable radius $r = V\theta$ starts from the point P in order to duplicate the course of time by sweeping out all of space upon starting at that point. The present potential at the point P depends upon on what one will find on each of those spheres at the corresponding instant $t - \theta$. One must then consider the density ρ of the electric charge and the components ξ', η', ζ' of its velocity upon traversing the ether at a point O of one of those spheres whose coordinates are ξ, η, ζ.

Let $v = \beta V = \sqrt{\xi'^2 + \eta'^2 + \zeta'^2}$ be the absolute value of that velocity, and let λ be the angle that its direction makes with the radius vector OM.

For a point that is linked to these charges during their displacement, the quantities ξ, η, ζ, ξ', η', ζ', and the components ξ'', η'', ζ'' of the acceleration $\Gamma = g\,V$ at the time $t - \theta$ are functions of the latter variable. The trajectory of the charge element that passes through O at that instant has an arbitrary form T that is known along with its motion if one is given ξ, η, ζ as functions of $t - \theta$ for that element.

Once the values of ρ and ξ', η', ζ' have thus been determined at each point in space that surrounds the point P by means of the moving sphere, the potentials will be obtained by taking the following integrals over that space:

$$(1) \qquad \Psi = \int \frac{\rho\,d\tau}{r}, \quad F = \int \frac{\rho\,\xi'\,d\tau}{r}, \qquad G = \int \frac{\rho\,\eta'\,d\tau}{r}, \qquad H = \int \frac{\rho\,\zeta'\,d\tau}{r}.$$

It is essential to remark, with des Coudres and Wiechert ([1]), that $\rho\,d\tau$ *does not represent the electric charge that is contained in the element* $d\tau$ *at a well-defined instant.* Indeed, the different points of that volume element correspond to different values of θ, and in turn, $t - \theta$, which vary with their distance from the point P. For example, consider a volume element $d\tau$ that is contained inside an infinitely thin cone with its summit at P that cuts out a surface ds on the sphere S', and the infinitely-close sphere S'' at a distance of $dr = V\,d\theta$, which corresponds, in turn, to the instant $t - \theta - d\theta$, so:

$$d\tau = V\,d\theta\,ds.$$

We wish to know where one will find the charges that occupy the various points of the element $d\tau$ at the instants that correspond to them *at the same instant;* for example, $t - \theta - d\theta$. The point that is linked with the moving electric charge that is found at O at the instant $t - \theta$ will be found at O_1 at the earlier instant $t - \theta - d\theta$, which is behind O by $v\,d\theta$ in the direction of v.

([1]) E. WIECHERT, *Lorentz Festschrift* [Arch. Néerl. (2) **5** (1900), pp. 349].

All of the points of ds are displaced by quantities that are reasonably equal and are found on an element ds_1, while the other base ds' will correspond completely to the instant $t - \theta - d\theta$. All of the points of $d\tau$ will then be found at *the same instant* in the volume element $d\tau$ that is found between ds' and ds_1.

The elements $d\tau$ and $d\tau_1$ have the same base, but their heights differ by $v \cos \lambda \, d\theta$, which is the projection of $v \, d\theta$ onto the direction that is normal to the common base, so:

$$d\tau_1 = (V - v \cos \lambda) \, d\theta \, ds = d\tau (1 - \beta \cos \lambda).$$

Since the density ρ changes infinitely little during the time $d\theta$, the true electric charge that is present in the medium at the same instant and corresponding to the element $d\tau$ will then be:

$$de = \rho \, d\tau_1 = \rho \, d\tau (1 - \beta \cos \lambda).$$

If one consequently makes the elements of electric charge intervene in the potentials then one must replace $\rho \, d\tau$ with the equal expression $\dfrac{de}{1 - \beta \cos \lambda}$ in the expressions (1).

It once more results that if one would like to calculate the potentials that are produced at the instant t at a point P that is situated *at a distance* $\rho = V \theta$ *that is large* with respect to the dimensions of the electron by an electron of charge e whose center traverses a trajectory T that that passes through the point O (ξ, η, ζ) at the instant $t - \theta$ (Fig. 1) then formulas (1) will become:

$$(2) \quad \Psi = \frac{e}{r(1 - \beta \cos \lambda)}, \quad F = \frac{e\xi'}{r(1 - \beta \cos \lambda)}, \quad G = \frac{e\eta'}{r(1 - \beta \cos \lambda)}, \quad H = \frac{e\zeta'}{r(1 - \beta \cos \lambda)}.$$

III. – The components E_x, E_y, E_z, M_x, M_y, M_z of the two electric E and magnetic M fields at the point P can be deduced from those potentials by the known formulas:

$$E_x = -\frac{\partial \Psi}{\partial x} - \frac{\partial F}{\partial t}, \qquad M_x = \frac{\partial G}{\partial z} - \frac{\partial H}{\partial y}.$$

In order to take the derivatives, it is important to remark how Ψ, F, G, H depend upon x, y, z, and t; one has:

$$r = V \theta = \sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}$$

$$r (1 - \beta \cos \lambda) = r - \frac{1}{V} \sqrt{(x - \xi)\, \xi' + (y - \eta)\, \eta' + (z - \zeta)\, \zeta'}.$$

The potentials then depend upon x, y, z, either directly in the denominator or by the intermediary of r or θ, since the ξ, η, ζ, ξ', η', ζ' are functions of $t - \theta$, and they depend upon t, either by the intermediary of ξ, η, ζ, …, which contain t explicitly, or by the intermediary of θ, which figures in r and in the ξ, η, ζ.

The necessary intermediate derivatives that must be known are:

$$\frac{d\xi}{d\theta} = -\frac{d\xi}{dt} = -\xi',$$

$$\frac{d\xi'}{d\theta} = -\frac{d\xi'}{dt} = -\xi',$$

$$\frac{\partial\theta}{\partial x} = \frac{1}{V}\frac{\partial}{\partial x}\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}$$
$$= \frac{1}{Vr}\left\{(x-\xi) - \left[(x-\xi)\frac{d\xi}{d\theta}+(y-\eta)\frac{d\eta}{d\theta}+(z-\zeta)\frac{d\zeta}{d\theta}\right]\frac{\partial\theta}{\partial x}\right\}$$

$$\frac{\partial\theta}{\partial x}V\left\{r-\frac{1}{V}[(x-\xi)\,\xi'+(y-\eta)\,\eta'+(z-\zeta)\,\zeta']\right\} = x-\xi,$$

$$\frac{1}{V}\frac{\partial r}{\partial x} = \frac{\partial\theta}{\partial x} = \frac{x-\xi}{Vr(1-\beta\cos\lambda)},$$

$$\frac{\partial\xi}{\partial x} = \frac{\partial\xi}{\partial\theta}\frac{\partial\theta}{\partial x} = -\frac{(x-\xi)\,\xi'}{Vr(1-\beta\cos\lambda)}, \qquad \frac{\partial\xi'}{\partial x} = -\frac{(x-\xi)\,\xi'}{Vr(1-\beta\cos\lambda)},$$

so:

$$\frac{\partial\xi}{\partial t} = \frac{d\xi}{dt}+\frac{d\xi}{d\theta}\frac{d\theta}{dt} = \xi'\left(1-\frac{\partial\theta}{\partial t}\right),$$

$$\frac{\partial\theta}{\partial t} = \frac{1}{V}\frac{\partial r}{\partial t} = \frac{1}{Vr}\left[-(x-\xi)\frac{\partial\xi}{\partial t}+(y-\eta)\frac{\partial\eta}{\partial t}+(z-\zeta)\frac{\partial\zeta}{\partial t}\right]$$

$$V\frac{\partial\theta}{\partial t}\left\{r-\frac{1}{V}[(x-\xi)\,\xi'+(y-\eta)\,\eta'+(z-\zeta)\,\zeta']\right\} = -[(x-\xi)\,\xi'+(y-\eta)\,\eta'+(z-\zeta)\,\zeta'],$$

$$\frac{1}{V}\frac{\partial r}{\partial t} = \frac{\partial\theta}{\partial t} = -\frac{\beta\cos\lambda}{1-\beta\cos\lambda}, \quad \frac{\partial\xi}{\partial t} = \frac{\xi'}{1-\beta\cos\lambda}, \qquad \frac{\partial\xi'}{\partial t} = \frac{\xi'}{1-\beta\cos\lambda}.$$

With these intermediate calculations, it is easy to achieve the calculation of the two fields, since one knows the derivatives with respect to *x*, *y*, *z*, and *t* of all the quantities that appear in the expressions for the vector potentials.

Once all of the calculations have been done, the results can be stated in the remarkably simple manner that we shall now occupy ourselves with.

IV. – Each of the two fields E and M can be decomposed into two parts, the first of which, which exists only in the case of a uniform motion of the electron, depends upon only the velocity v that is possessed b the latter at the instant $t - \theta$.

For the electric field, the first part E_1 is directed towards the position O' that the electron occupies at the instant t, if it has continued to move since the instant $t - \theta$ with the velocity $v = \beta V$ that it possessed at that instant, in such a way that $OO' = v\theta$, where O' coincides with the true position of the electron at the present instant t if the motion is uniform and rectilinear. E_1 is given in electrostatic units by:

$$E_1 = \frac{e(1-\beta^2)}{r^3(1-\beta\cos\lambda)^3} \cdot O'P. \tag{3}$$

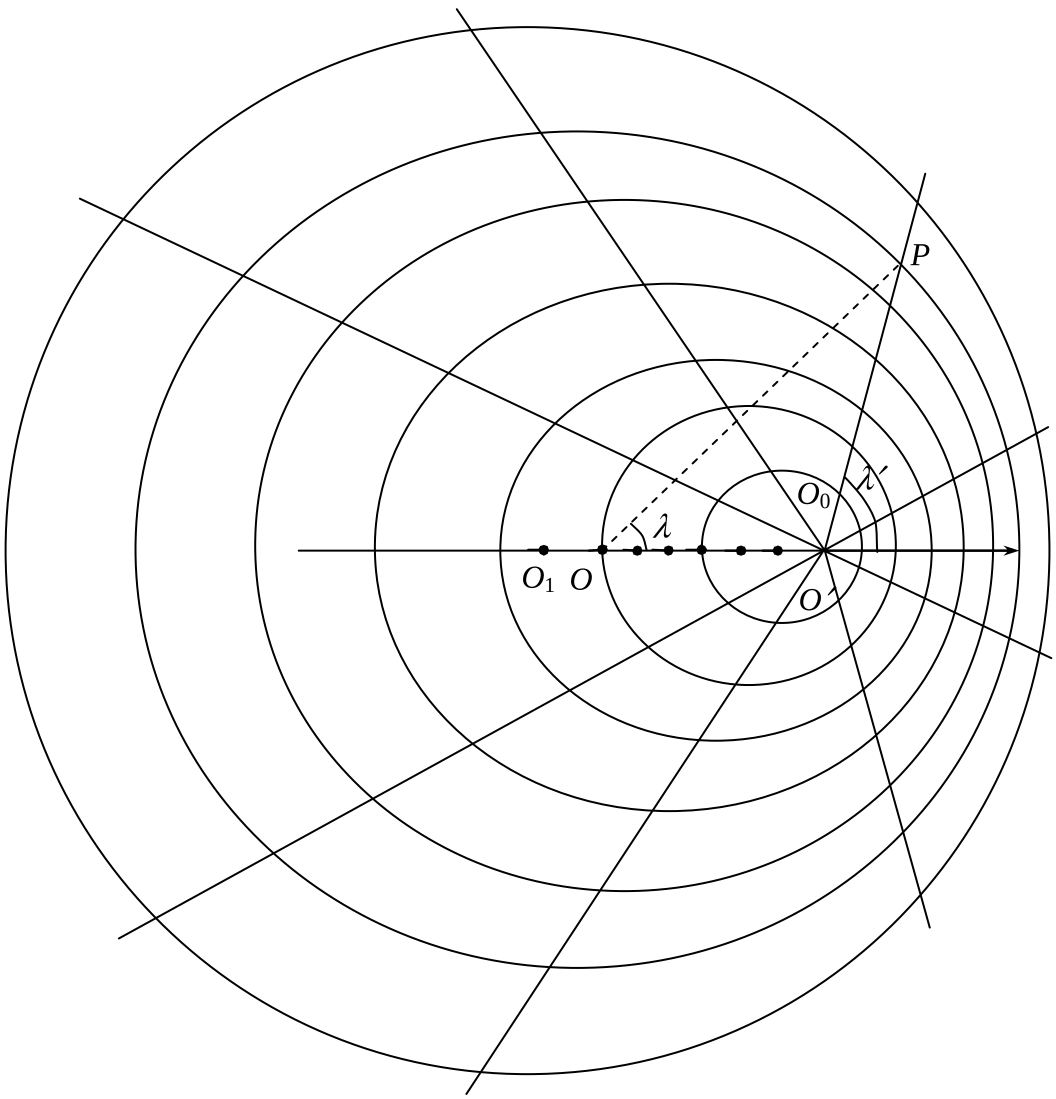

Figure 2.

The corresponding part M_1 of the magnetic field is perpendicular to the plane $OO'P$ of the velocity v and the radius r, and is measured in electromagnetic units to be:

$$M_1 = \beta E_1 \sin \lambda' \tag{4}$$

if λ is the angle between v and $O'P$.

I call that first part of the electromagnetic field the *velocity wave*. The set of those velocity waves that are emitted by the electron at the various instants that precede the current instant t constitute spheres that have their centers at the various positions that are previous to the moving body and mutually envelop it if its velocity never attains the velocity V of light, which is the case to which I shall confine myself here. The set of these waves constitutes what I call the electromagnetic *wake* of the electron, which accompanies it in its displacement. Indeed, we confirm that the velocity wave does not correspond to any radiated energy at a great distance, since the two fields E_1 and M_1 that comprise it have intensities that diminish in inverse proportion to the square of the distance r from the center of emission, while $O'P$ increases like r.

In the particular case of uniform, rectilinear motion (Fig. 2), the point O' is independent of the previous instant $t - \theta$, since it always coincides with the true position O_0 of the moving body at the instant t. The electron that is currently at O_0 will then be accompanied by an invariable wake in its uniform displacement through the ether such that the electric field is directed towards the present position O_0 of the moving body at any point.

The successive velocity waves, which are emitted in this case only at different previous instants, comprise its wake while it propagates, just as the waves that are emitted by the bow of a vessel comprise the wake that follows it. After the instant of its emission, each wave is composed of successive portions that are more and more elongated from the center of the wake, and leave behind it ones that are identical to itself with respect to the moving body and follow it.

Instead of introducing the distance r from the point P of the wake to the retarded position O of the moving body at the moment when the velocity wave that is currently present at P emits it into the values of E_1 and M_1 at P, one can introduce the distance $O'P = r'$ – i.e., the distance from P to the *present* position of the moving body in the case of uniform motion. In the triangle $OO'P$ whose edges are:

$$OP = r, \qquad O'P = r', \qquad OO' = r\theta = v\frac{r}{V} = \beta r,$$

one easily proves that:

$$r\,(1 - \beta \cos \lambda) = r'\sqrt{1-\beta^2 \sin^2 \lambda'} \,.$$

Therefore:

$$E_1 = \frac{e\,(1-\beta^2)}{r'^2 (1-\beta^2 \sin^2 \lambda')^2},$$

$$M_1 = \beta E_1 \sin \lambda'.$$

These are well-known expressions that are obtained in an entirely different way ([1]) in order to represent the wake that accompanies an electrified particle in uniform motion through the ether at a great distance with respect to its dimensions. Since the denominator is a minimum when λ' is a right angle – i.e., in the equatorial plane with respect to the direction of the velocity – it will then result that the field is more intense in the equatorial plane than everywhere else at the same distance r', moreover. The lines of electrical force, which are radial, have a tendency to concentrate more and more in that plane as the velocity increases. Since the lines of magnetic force are given in the direction of M_1, they will be circles that are perpendicular to v and have their common center along OO_1 .

Later on, we shall confirm that the energy that is contained in the wake increases with the velocity, which is the origin of the inertia that the electron presents.

In order to find the wake in the immediate neighborhood of an electron in the case of an arbitrary motion, it is sufficient to decompose the charge that is on its surface or in its interior into elements and to apply the preceding results to each element. The superposition of the elementary wakes permits one to recover, for a spherical electron with a uniform surface charge, the solution that was given by Searle for the wake of a conducting sphere in uniform motion. In the neighborhood of the surface, the lines of electric force will no longer be radial, so the field will no longer be directed towards the center of the sphere, contrary to what happens at a large distance.

V. – If the motion is not uniform and rectilinear then there will exist an acceleration Γ, and the wake will no longer have the simple form that it had in the preceding case. The points such that O' is found on a curve C that joins the trajectory T to the present position O (Fig. 1), and the lines of electrical force of the wake are curves whose tangent at each point, such as P, is directed towards the corresponding point O'. Contrary to what happens in the case of uniform motion that has existed for a very long time, the wake must change here with respect to the moving body from one instant to another, and we shall see that the reorganization – i.e., the necessary change – is produced by the intermediary of the second part of the electromagnetic field that is absent in the case of uniform motion, and that I shall call the *acceleration wave.*

The corresponding electrical field E_2 is the geometric sum of two vectors, one of which is parallel to $O'P$, while the other one is parallel to the acceleration $\Gamma = \gamma V$:

(5) $$E_2 = \frac{e\gamma\cos\varphi}{Vr^2(1-\beta\cos\lambda)^2}\overline{O'P} - \frac{e}{Vr(1-\beta\cos\lambda)^2}\overline{\Gamma},$$

in which φ is the angle between the acceleration Γ and the radius OP. One sees that E_2 is annulled at the same time as γ.

It is easy to verify that the field E_2 is normal to the radius OP – i.e., it is tangent to the spherical wave and transverse to the direction of propagation.

([1]) J.-J. THOMSON, *Recent Researches*, 1892.

Moreover, the second part M_2 of the magnetic field is *perpendicular to* E_3 *and to OP, and therefore transverse, as well,* and will be measured in electromagnetic units as:

$$M_2 = E_2 \,. \tag{6}$$

Thus, this *acceleration wave* has all of the character of a free radiation wave. *It is composed of two transverse electric and magnetic fields that are mutually perpendicular and represent equal energies per unit volume in the medium.*

It truly represents the *radiation* that is emitted by the charge center, since it persists only at a large distance from it as the fields E_2 and M_2 that comprise it vary in inverse proportion to r, which permits it to transport a finite quantity of energy to infinity.

It is remarkable that this spherical acceleration wave presents the character of free radiation *at any distance from the center of emission*, even when it is superposed with the velocity wave for distances r that are insufficient to render the velocity wave negligible. It is truly the elements of radiation that are the basis for the decomposition of an arbitrary complex radiation at any distance from its source. Its production is linked with the acceleration of the electrified center: *Radiation implies acceleration.*

VI. – At the instant t, the electromagnetic perturbation that is emitted by the electron between the instants $t - \theta - d\theta$ and $t - \theta$, and is composed of the set of two velocity and acceleration waves, is found to be comprised between two eccentric spheres S and S_1 (Fig. 1) that have the positions O and O_1 at the instants $t - \theta$, $t - \theta - d\theta$ for their centers and radii of $V\theta$ and $V(\theta + d\theta)$.

The electromagnetic energy that is contained in that layer per unit volume at the point P, namely:

$$\frac{1}{8\pi}(E^2 + M^2),$$

is composed of three parts, since E and M are each the resultant of two vectors, namely, the fields that figure in the velocity and acceleration waves.

The first part:

$$\frac{1}{8\pi}(E_1^2 + M_1^2)$$

corresponds to the velocity wave, which is assumed to be isolated. It is *the energy of the wake*, which is present only in the case of uniform, rectilinear motion. If one calculates its total value in the spherical layer SS_1 by integrating over the sphere S then one will find that:

$$dW_1 = \frac{3+\beta^2}{3(1-\beta^2)}\frac{e^2}{2r^2}V\,d\theta. \tag{7}$$

It then diminishes in inverse proportion to r^2 in such a way that no portion of that *energy of the wake* extends indefinitely from the electrified center; it accompanies it in its displacement.

An application of Poynting's theorem shows that there must be a constant flux of energy in the wake the sense of the motion across a surface that is fixed in the ether if the wake is to accompany its center. In the case of uniform motion, nothing is lost at infinity from the energy that accumulates in the wake. That energy propagates in the sense of motion while following the electron and preserving a fixed distribution in the medium *with respect to the electron*. No external intervention is necessary in order to maintain that wake, so once the electron is launched, it will move with the same velocity indefinitely. An external intervention will be necessary in order to raise or produce energy in the wake, to diminish or augment the velocity, and to produce acceleration. The electron is inertial because it is charged; it implies its wake and is implied by it, and no external cause will modify that.

If the velocity before the instant $t - \theta$ remains constantly βV in magnitude and direction then the wake that is external to the sphere S will be *normal* and contain an amount of energy:

$$W_1 = \frac{3+\beta^2}{3(1-\beta^2)}\int_0^\infty \frac{e^2}{2r^2} V\, d\theta = \frac{3+\beta^2}{3(1-\beta^2)}\int_r^\infty \frac{e^2}{2r^2}\, dr$$

(8)

$$W_1 = \frac{e^2}{2r^2}\cdot\frac{3(1-\beta^2)}{3+\beta^2} = \frac{e^2}{2r^2}\left[1+\frac{4\beta^2}{3(1-\beta^2)}\right].$$

The first part of W_1 is the electrostatic energy $e^2 / 2r$ that is external to the sphere S in the case of an immobile electron. The other part, $\frac{2e^2}{3r}\frac{\beta^2}{1-\beta^2}$, represents the increase in the *energy of the wake* when the velocity passes from zero to βV; it is the *kinetic energy* of electromagnetic origin that is present outside of the sphere S.

That *kinetic energy* is proportional to β^2 – i.e., the square of the velocity – only in the first approximation when β^2 is negligible compared to unity, so if the velocity v is small with respect to that of light. On the contrary, it will increase indefinitely when v approaches V, so when β tends to 1. The infinite value of the kinetic energy in the case of an electron in motion with the velocity of light would imply, moreover, that this velocity had existed for an infinite length of time ([1]).

In the expression for kinetic energy, the role of mass is played by the factor $4e^2 / 3r$ for weak velocities. In a way, it represents the electromagnetic mass that is present outside of the sphere S of radius r for low velocities; for larger velocities, that mass will become a function of the velocity. The total electromagnetic mass of the electron can be calculated by giving r the value a of the radius of the sphere to which the electron can be assimilated. In the region that is close to it, it will be necessary to superpose the wakes that correspond to the various elements of its charge, and one will recover the known result $2e^2 / 3a$, instead of $4e^2 / 3a$, and $4e^2 / 5a$, if the charge is distributed uniformly in the volume of the sphere.

([1]) Heinrich HERTZ, Phys. Zeit. (1903).

VII. – The second part of the energy, which corresponds to the acceleration wave of radiation, and is assumed to be isolated, has the value per unit volume at the point P:

$$\frac{1}{8\pi}(E_2^2 + M_2^2) = \frac{E_2^2}{4\pi}.$$

In the spherical layer SS_1, it represents *an energy that is independent of the distance r*, which propagates, in turn, to an infinite distance from the electron with no change. It is *the energy that is radiated by the center during the time* $d\theta$, which has the value, when all calculated have been done ([1]):

$$dW_2 = \frac{1-\beta^2 \sin^2 \varepsilon}{(1-\beta^2)^3} \times \frac{2}{3}\frac{e^2\gamma^2}{V}\,d\theta, \tag{9}$$

in which ε is the angle that the acceleration Γ makes with the velocity v.

One then recovers the *fundamental result that an electrified center will radiate a quantity of energy per unit time that is proportional to the square of its acceleration*, and will send nothing to infinity when its acceleration is zero ([2]).

VIII. – Finally, the third part is the energy per unit volume that relates to the two waves, which is equal at P to:

$$\frac{1}{4\pi}(\overline{E_1E_2} + \overline{M_1M_2}),$$

in which the products are taken geometrically; if one integrates in the spherical layer SS_1 then one will get:

$$dW_3 = \frac{1}{(1-\beta^2)^2} \cdot \frac{4e^2}{3r}\,\beta\gamma\cos\varepsilon\,d\theta. \tag{10}$$

Now:

$$\Gamma\cos\varepsilon\,d\theta = \gamma V\cos\varepsilon\,d\theta = dv = V\,d\beta,$$
$$\gamma\cos\varepsilon\,d\theta = d\beta,$$

$$dW_3 = \frac{4e^2}{3r} \cdot \frac{\beta\,d\beta}{(1-\beta^2)^2}. \tag{11}$$

That energy will also tend to zero when r increases indefinitely. It represents what remains at the distance r of the energy that is destined for the reorganization of the wake that the acceleration necessitates, which is the energy that is present in the layer SS_1 thanks to the superposition of the velocity wave and the acceleration wave of radiation. Indeed, it is easy to confirm that dW_3, which is present in SS_1, represents precisely the energy that is necessary to change the energy of the wake W_1 that is external to the sphere

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([1]) Cf., M. ABRAHAM, Ann. Phys. (Leipzig) **10** (1903), pp. 135.
([2]) J. LARMOR, *Aether and Matter*, pp. 227, and Phil. Mag. **44** (1897), pp. 503.

S from the value that corresponds to βV to the one that corresponds to $\beta + d\beta$; i.e., the increase that the *kinetic energy* that is external to S must submit to when one passes from β to $\beta + d\beta$. Indeed, from (7), one will have:

$$W_1 = \frac{e^2}{2r}\left[1+\frac{4\beta^2}{3(1-\beta^2)}\right],$$

$$\delta W_1 = \frac{4e^2}{3r}\frac{\beta\, d\beta}{(1-\beta^2)^2} = dW_3 .$$

I will call the energy dW_3 that is provided by the external cause that produced the acceleration, thanks to the superposition of the radiation with the velocity wave, the *energy of change*. It is then precisely by the intermediary of the acceleration wave that the reorganization of the wake comes about, as I said above, which does not happen in the case of uniform motion. As the radiation advances, one will find the old wake in front of it, while it leaving the new wake behind it while dissipating its change in energy in the form of an increase in the kinetic energy, in order to preserve only *the radiated energy* dW_2 at an infinite distance, when the reorganization is complete and the introduction of a change in energy has been exhausted. One can say that *acceleration implies reorganization – i.e., a change in the wake – and that reorganization implies radiation.*

IX. – Acceleration exists only in the presence of an electromagnetic field that is external to the electron and is capable of exerting Lorentz forces on it, namely, an electric force that is parallel to the external electrostatic field and a magnetic force that is perpendicular to the external magnetic field and the velocity of the electron. The external electromagnetic field must provide the sum of the energies of radiation and change, so that sum must be negative, since dW_2 is essentially positive, but dW_3 has the same sign as $d\beta$.

The latter energy has an expression that is analogous to the work that is done by the forces, since it is proportional to the product of the tangential component $\Gamma \cos \varepsilon$ of the acceleration with the displacement $\beta\, V\, d\theta$. The same thing is not true for radiated energy, which is proportional to the square of the acceleration in such a way that the energy that is provided by the cause that produced the acceleration is equal to the work that is done by the forces in its ordinary form, and the equations of mechanics are applicable only if the radiated energy dW_2 is negligible in comparison to the energy of change dW_3 when it is calculated when the electron departs and represents the change in the total kinetic energy.

An important distinction is introduced here: The radiated energy dW_2, which is independent of distance, does not require one to examine what happens in the immediate neighborhood of the electron; only its charge will be involved. The same thing is not true for the energy of change, which will become infinite for r equal to zero. The total energy that is contributed in that form to the external field, like the energy of the wake that is charged with reorganizing, must involve the form and dimensions of the electron. It remains proportional to $\Gamma \cos \varepsilon \times \beta\, V\, d\theta$, with a coefficient that one can calculate for a given distribution of charge in the electron and which is the longitudinal mass of electromagnetic origin here, which is a function of velocity like the coefficient of proportionality of the kinetic energy to β^2 and which also increases indefinitely when v tends to V. I say "longitudinal mass" because Γ cos ε is the longitudinal component of the acceleration – i.e., in the direction of the velocity. The calculation of the transverse mass, which corresponds to the perpendicular component that is normal to the acceleration, involves other considerations than those of energy.

Therefore, the equations of mechanics must be modified in two distinct ways: First of all, the mass must be imagined to be a function of the velocity, where the discrepancy becomes noticeable only for velocities of the same order as that of light. The experiments of Kaufmann on β rays from radium have confirmed that consequence of the electromagnetic laws while giving one solid reasons to believe in the electromagnetic origin of inertia and the impossibility of founding a satisfying theory of mechanics without taking electrical notion to be fundamental.

That modification – viz., a variable mass that is a function of velocity – respects at least the form of the equations of mechanics. That form itself will become incomplete if the radiated energy is not negligible compared to the energy of change; viz., if the motion is not *quasi-stationary*, according to the expression that is used by Max Abraham, so if the residue that is transmitted to infinity in the form of radiation by the wave that is charged with modifying the wake is not too important.

In other words, the condition for quasi-stationary motion is that in the region that is close to the electrified center in comparison to its dimensions, which is a region in which one finds the greater part of the energy of the wake localized, that energy can be reasonably considered to be determined uniquely by the present velocity of the particle, which is almost what it would be if the present velocity had existed for a long time. In order for that to be the case, it would be necessary for the superposition of the acceleration wave with the wake in that region to be negligible in comparison with the velocity wave, while only the first one would persist at an infinite distance.

The mean value of the radiated energy per unit volume has a maximum value on the sphere of radius r of order:

$$\frac{e^2\gamma^2}{Vr^2(1-\beta^2)^2},$$

and the energy of change has order:

$$\frac{e^2\beta\gamma}{r^3(1-\beta^2)^2}.$$

The ratio:

$$\frac{v\gamma}{V\beta(1-\beta^2)}$$

must be small for values of r that have the same order as the radius a of the sphere to which the electrified particle has been assimilated. The condition for stationary motion will be that the quantity:

$$\rho = \frac{a\gamma}{V\beta(1-\beta^2)}$$

must be very small with respect to unity. It is easy to assure that this condition is always satisfied in the experimental cases, since a is of order ([1]) 10^{-13} when the inertia of the electron is completely electromagnetic, and β attained a maximum of 0.95 in Kaufmann's experiments on the deflectable rays from radium.

An interesting case of an exception to the ordinary laws of mechanics is the one in which the moving body is at absolute rest, so β will be zero, and ρ will then become infinite. However, the motion can be once more considered to be quasi-stationary after an exceptionally short length of time. Indeed, if one starts with $\beta = 0$ then ρ will once again become less than unity for a velocity that is greater than:

$$v = \beta V = a\,\gamma.$$

That limit is attained thanks to the acceleration γV at a time:

$$t = \frac{\beta V}{\gamma V} = \frac{a}{V} < 10^{-23} \text{ second.}$$

These exceptions, which are different from the ones that correspond to the variation of the electromagnetic mass with the velocity, then seem less accessible to experiment, and it hardly seems possible to use them in order to exhibit absolute motion.

One can be sure that the condition of quasi-stationary motion will be satisfied in any experimental case, so the radiated energy is never just a tiny portion of the mean energy of change, and that, in turn, the energy that is provided to the electrified center by the external field that produces its acceleration will always be of the form $m\,\gamma ds \cos \varepsilon$, which is comparable to that of mechanical work. The phenomenon of radiation, which is measurable only at some distance from the electrified center, will not modify its laws of motion appreciably, because it is negligible in its immediate neighborhood.

X. – Therefore, the accelerations of the electrons are perceived at a great distance only by the intermediary of the radiation that is determined by the fields E_2, M_2 ; the expressions (5) and (6) of E_2 and M_2 will simplify in the case where β is small, so the velocity v is small with respect to that of light; one will have:

$$E_2 = \frac{c\gamma}{Vr} \sin \varphi,$$

so this electric field will be perpendicular to OP in the plane that contains OP and Γ. The magnetic field M_2 always has the same measure in electromagnetic units that E_2 has in electrostatic units, in such a manner as to represent the same energy per unit volume, and it is directed perpendicular to OP and E_2 .

It then results that the radiated electromagnetic field is distributed symmetrically around the direction of the acceleration Γ, so its intensity will be a maximum for $\sin \varphi =$

([1]) P. LANGEVIN, Ann. Chim. Phys. **28** (1903), pp. 357.

1 – i.e., in the equatorial plane with respect to Γ – and will be zero in that direction itself. One easily deduces from this that the plane of polarization of the radiation that is emitted by the electron towards P is the plane that passes through the light ray OP and is perpendicular to the plane $OP\Gamma$, which contains the acceleration. The direction relations are a little more complicated and are determined by the general expression (5) for E_2 when β ceases to be very small.

The nature of the radiation that passes through the point P depends upon the motion that the electrified center takes and in the manner by which its acceleration varies in time. If it takes a considerable value during a very short time interval then one will be dealing with an impulse that the electron is subjected to, such as, for example, in the case of a cathode corpuscle that is launched with a velocity that is equal to 50,000 kilometers per second, and stops sharply at the moment that it collides against an anti-cathode then the radiation will consist of a pulsation without any periodic character whose thickness will be the distance between the spheres that were radiated at the beginning and the end of the collision; i.e., the product of the velocity of light with the time duration of the collision.

These pulsations seem to provide the best explanation for the Röntgen rays that are emitted by the region of the anti-cathode that is struck by the beam of cathode rays. They are connected with the acceleration that the cathode rays are subject to at the moment when the anti-cathode stops them. From Barkla, those Röntgen rays manifest a polarization that conforms to the one that the preceding interpretation allows one to predict. The secondary rays that are emitted when a metallic surface has been struck vary in intensity when one changes the position of the plane of incidence around the primary beam. Moreover, it is not necessary that one must observe a similar polarization, as well as a variation in intensity around the direction of the cathode beam, for the preceding theory to remain admissible, because it is less probable that a corpuscle will stop completely at the first collision. It must be reflected in the various directions, as evidenced by the emission of secondary cathode rays, and is subject to new acceleration of the same order as the first one, but in different directions. That first acceleration can itself have a direction that varies with the deflection that the cathode particle is subject to.

If the motion of the electron is periodic, as in the case of circulation around a closed orbit, then the acceleration itself will be periodic, as well as the radiation, and one will obtain a phenomenon that is comparable to the emission of light of well-defined wavelength.

If the orbit is circular then the acceleration, which is always centripetal for a uniform rotation, will rotate with the moving body.

In a direction that is perpendicular to the plane of the orbit and at a distance that is very large compared to its dimensions, the plane of polarization will rotate around the direction of the ray without changing in intensity, $\sin \varphi$ will always be equal to 1, and one will have circularly-polarized light. In the plane of the orbit, the light will be rectilinear, with a plane of polarization that is perpendicular to the plane of the orbit. The polarization will be elliptical for an oblique direction.

One finds that this includes the radiation that is involved with the explanation for the Zeeman effect. However, here we are pursuing the very close sort of link between that radiation and the electron in accelerated motion that gave rise to it. The intensity of the radiation can be calculated as a function of the charge and the acceleration of the electron.

XI. – The main points, upon which I believe it is useful to insist, are the following ones:

1. The electromagnetic perturbation that is produced in the medium by an electrified particle on motion is composed of two parts that propagate with the speed of light after they start at the center of emission.

The first part – or *velocity wave* – which exists only in the case of a uniform, rectilinear motion, depends upon only the velocity of the moving body. It contributes to the formation of a *wake* around it whose energy varies with the velocity, and which then contains the kinetic energy that is coupled with the electrified center and which accompanies it in its displacement and modifies it if the motion is accelerated.

2. That modification is produced by the intermediary of the second part of the perturbation – viz., the *acceleration wave* – possesses the character of transversality and the equality of the electric and magnetic energies that correspond to the free radiation *at any distance* from the point of emission.

That *acceleration wave* will transport a finite energy to a great distance where the wave velocity becomes negligible, and that energy will be proportional to the square of the acceleration and will increase indefinitely with the velocity when it approaches that of light. The polarization character of that wave is particularly simple when the velocity is small.

The velocity wave does not transport any energy to a large distance. The *energy of the wake* that corresponds to it will follow the center in its displacement.

3. The energy that relates to the two velocity and acceleration waves – viz., the *energy of change* – represents precisely the energy that must necessarily be provided in order to reorganize the wake and make it correspond to the new velocity. In any case that is accessible to experiment, it will be enormous with respect to the *radiated energy*, which represents the intrinsic energy of the acceleration wave, which is the residue that is necessary for the reorganization of the wake.

4. The energy of change, which is provided by the external field that produced the acceleration, can, by its very expression, be assimilated with the work that is performed by the external forces, and is the only exchange of energy that enters into the ordinary equations of mechanics.

The *radiated energy* that the external field must likewise provide, and which has a different form, is not contained in the laws of dynamics and will oblige us to modify them if the smallness of the radiated energy with respect to the energy of change does not render that correction inappreciable in all of the experimental cases.

The preceding considerations seem to cast some light on the internal mechanism of the phenomena of inertia and radiation.