"Über die Analogie zwischen absoluter Temperatur und elektrischen Potential (Erwiderung an F. W. Adler)," Ann. d. Phys. **23** (1907), 994-996.

On the analogy between absolute temperature and electric potential (dedicated to F. W. Adler)

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1. F. W. Adler published an article in volume 22 of these Annals (1) in which he challenged the analogy between absolute temperature and electric potential that was proposed by E. Mach and later by myself.

Adler said (2) "the Mach-Lippmann analogy exhibits a remarkable asymmetry, which seems to me to demand an explanation. Whereas, according to it, in the second main theorem, the temperature corresponds to the potential:

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}, \qquad \frac{W_1}{W_2} = \frac{V_1}{V_2},$$

in the energy variation of the first main theorem the temperature T corresponds to the *square* of the potential V:

$$cT, \qquad CV^2."$$

I wish to be permitted to respond here to the objection above.

The asymmetry that emerges between the products cT and CV^2 is only apparent, because the asymmetry that exists between T and V^2 will be compensated by the asymmetry that exists between the coefficients c and C, on the other hand, which was not observed by Adler.

One cannot, in fact, be tempted by the equality of the names (i.e., capacity) to consider c and C as corresponding quantities. c is equal to the differential quotient $\partial Q / \partial T$, while C is equal to the differential quotient $\partial m / \partial V$. However, Q refers to energy, while m (quantity of electricity) does not refer to energy; rather, m must first be multiplied by V in order to give energy. If one brings the hidden factor V into consideration then the symmetry of the formulas becomes obvious.

The thermal and electric energies Q and W are corresponding quantities. The quotient $Q/T = S$ (viz., entropy) corresponds to the quotient $W/V = m$ (quantity of electricity). As a result, the differential quotient $\partial m / \partial V = C$ corresponds to the differential quotient $\partial S / \partial T = \gamma$. They carry no special names in physics, but have a well-defined meaning; moreover, one has $\gamma = c/T$.

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(1) F. W. Adler, Ann. d. Phys. **22** (1907), 587.
(2) F. W. Adler, *loc. cit.*, pp 588.

Within the limits where one can regard the coefficients C and γ as constants, the increases in energy are:

$$\tfrac{1}{2}\gamma T^2, \qquad \tfrac{1}{2}CV^2, \text{ resp.}$$

2. The analogy between thermal and electric processes is indeed a purely formal one; however, the remarks that were made above have a particular utility, in that they draw attention to a theorem that plays the same role in the theory of electricity as the Clausius-Carnot theorem does in the theory of heat. As is known, the latter is expressed by the equation:

$$\int dS = 0$$

for a closed, reversible, cyclic process; as a result, dS must be a complete differential.

For electricity, this corresponds to the theorem:

$$\int dm = 0$$

for a closed, reversible, cyclic process; instead of the theorem of the conservation of entropy, one must pose the theorem of the conservation of quantity of electricity. In any special problem, one must express the integrability conditions for dm. Therefore, a complete parallelism exists between the two main theorems for the theory of heat and any theorem on the conservation of electricity, with entirely analogous applications.

In general, this analysis has the same uses as it does for heat: When one is given a reversible process, it allows one to derive a second process that is reciprocal to the first one from its existence and also the quantity. Thus, one can calculate beforehand, e.g., the variations of the capillary stresses in quicksilver by polarizing the current formation under dilatation of the quicksilver surface.

Permit me to recall a second example. One thinks of a condenser whose dielectric is not made of glass or air, but from a quartz or tourmaline plate that is cut perpendicular to the axis so it becomes a piezoelectric substance. Such a condenser works as not only a condenser, but also as a source of electricity, since under varying loads – and indeed ones that will already vary through the mutual attraction of the metal plates – it will develop piezoelectricity. It is therefore not absolutely self-explanatory that $\int dm$ should be equal to zero when one has not proposed this theorem as a basic principle.

If one does this then one arrives at the following conclusions: The dimensions of a crystal change in an electric field. One finds an extension in the direction of the axis in the case where the end of the axis that would be positively-charged under compression of the crystal stands next to the positive plate. In the opposite case, one finds contraction. The quantity of contraction can be calculated, moreover. It is well-known that P. Curie arrived at these conclusions as a result of his research into verifying them qualitatively and quantitatively.

These and other examples ([1]) seem to me to have proved that the contested analogy of Adler does not exist in isolation, but that, whether or not it is purely formal, it leads to conclusions that confer a certain significance to it.

([1]) Annales de Physique et Chimie. 1881.