"Eine neue Singularitätenmethode für die Elastizitätstheorie," Zeit. f. angew. Math. und Mech. **11** (1931), 488-490.

A new method for singularities in the theory of elasticity ([1]).

BY P. NEMÉNYI in Berlin.

(Of the Institute for Engineering Flow Research, Technische Hochschule Berlin)

Translated by D. H. Delphenich

In the one-dimensional theory of beams, as well as in the theory of frameworks, it is known that all influence lines represent not just the deformation, but also the internal force magnitudes, as bending lines. However, that defines the way that one conceives of the difference between statically-indeterminate and statically-determinate structures up to now. The "line of deflection" of a kinematic chain that is derived from the given system will be determinate for the latter, although in the statically-indeterminate case, the given system will be converted into a less statically-indeterminate system by omitting the terms to be examined in a step-by-step manner and ascertaining the influence lines as bending lines of this system for the case of a completely determined loading.

The objective of the foregoing investigation is first to put the representation of the influence lines of one-dimensional load-bearing structures (*Traggebilde*) on a *unified*, general foundation that will, at the same time, admit an effortless generalization to two and three-dimensional problems. To that end, the following theorem was derived:

The influence line, influence surface, etc. (in full generality, the influence field), of any internal force magnitudes (e.g., stress, stress moment, lateral force, etc.) can be represented as bending lines, bending surfaces (displacement fields, resp.) of a suitably-chosen equilibrium system of external loads (the limiting structure of such an equilibrium system) that is attached to the unvaried structure.

These limiting structures are the newly-introduced *singularities* (viz., load singularities).

In this way, one comes to the association of any notion of internal force (or, more generally, any notion of an elastic action) with a dual notion of singularity.

The generalization of "load" that this gives makes it possible to make a far-reaching generalization of *Maxwell's theorem on the reciprocal nature of displacements.* Namely, let r, s be any sort of concept of elastic action, and let R, S be the singularities that are dually associated with them. There then exists a reciprocal relationship that can be expressed as follows when one appeals to the concept of influence lines (plates of the influence surface, resp.): The line (surface, resp.) of the action s of singularity R that acts

([1]) A more thorough version will appear later in this Zeitschrift.

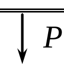
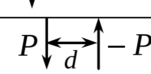
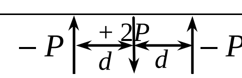
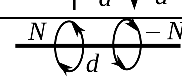
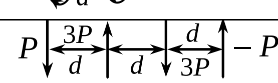
at the location α represent the influence line (influence surface, resp.) for the action *r* at the fixed location α of a mobile singularity *S*, for a suitable unit of measurement.

In many cases, the new singularities lead to be convenient new ways of *actually ascertaining certain influence lines or surfaces*. For example, with the new-found connections, one can ascertain a very simple independent construction of the influence for the bending moment, lateral force, etc., of an arbitrary cross-section of a beam or framework, or test the influence lines that were found in some other way quickly and independently of each other.

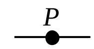
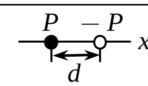
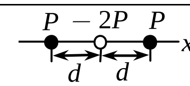
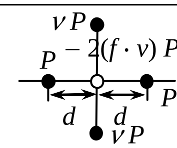
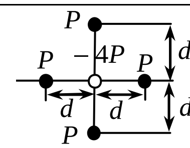
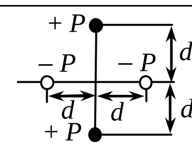
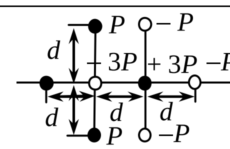
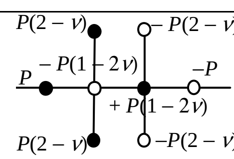
A much more important application of the new conceptual picture is the *simple solution of certain boundary-value problems* in the theory of elasticity in a fundamentally new way. A double moment demands, in fact, a buckling in the influence line of a beam, and a double impulse demands a jump. If one would then like to treat a one-dimensional load-bearing structure with free ends then, in many cases, one could start with one of the other ones to be treated that is included in it, and apply a double moment and a double impulse to the places that one desires to be the free ends whose magnitudes are determined by the condition that the bending moment and lateral force must vanish at these places. The simplest example of the convenience of this method is the conversion of a framework with many equal fields to one with infinitely-many such fields, so one must solve four linear equation in four unknowns in the most general case; the savings in calculation is very considerable in comparison to the known process in this case. Much more numerous are the analogous applications of the new concept to the solution of boundary-value problems for plates and discs with free boundaries. In the most general case, one will be led to two simultaneous integral equations here whose kernels will be defined by the action functions of the higher singularities.

The elastic effects of dually-associated singularities.

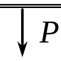
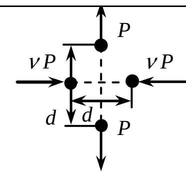
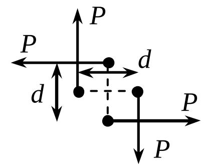
a) For beams.

Order	Action	Singularity	
1	Deflection	Unit load	
2	Tilt	Unit moment	$Pd = M$ 
3	Bending moment	"Double moment"	$Pd^2 = S$ 
	Twisting moment	"Rotation-pair"	$Nd = T$ 
4	Lateral force	"Double impulse"	$Pd^3 = U$ 

b) For plates.

Order	Action	Singularity	
1	Deflection	Unit load	
2	Tilt in the x-direction	Unit moment	$Pd = M$ 
	Curvature in the x-direction	“Double moment of the first kind”	$Pd^2 = S$ 
3	Moment in the x-direction	Overlap of two “Double moments of the first kind”	$Pd^2 = S$ 
	Moment sum	“Central moment”	$Pd^2 = C$ 
	Twisting moment	“Double moment of the second kind”	$Pd^2 = T$ 
4	Lateral force in the x-direction	“Double impulse of the first kind”	$Pd^3 = U$ 
	Boundary support force for a boundary that is perpendicular to “x”	“Double impulse of the second kind”	$Pd^3 = U$ 

c) For a disc.

Order	Action	Singularity	
1	Displacement field	Unit load	
2	Normal stress in the x-direction σ_x	“Tension (pressure) pole”	 $Pd = S$
	Shear stress in the x-direction τ_{xy}	“Shear pole (shear)”	 $Pd = T$