Pre-metric Electromagnetism

and

Emergent Gravity

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How gravity "emerges" from pre-metric electromagnetism

- Gravity is a manifestation of the curvature of spacetime due to the presence of a Lorentzian metric g.

- The "light cones" of *gravity* actually originate in the way that spacetime supports the propagation of *electromagnetic* waves.

  - Suggests that EM structure is more fundamental and that gravity is a consequence of it.

- Real issue is the *dispersion law* for EM waves that follows from the constitutive properties of the medium by way of the field equations.

  - General dispersion law defined by a quartic polynomial in $k = k_\mu dx^\mu$: $P[k] = 0$.

  - Often factorizes into a product of quadratics:

$$g(k,k)\bar{g}(k,k) = 0$$

    "bi-metric structure"

  - Lorentzian metric is the degenerate case $g = \bar{g}$

$$P[k] = [g(k, k)]^2$$

Quantum electrodynamical considerations

- Effective Lagrangians for QED suggest that vacuum polarization makes the components of the dispersion polynomial take the form:

$$(\eta^{\kappa\lambda} + aT^{\kappa\lambda})(\eta^{\mu\nu} + bT^{\mu\nu})$$

$\eta^{\mu\nu}$ = diag[+1, –1, –1, –1],

$T^{\mu\nu}$ = Faraday tensor for EM field strength 2-form

$$F = 1/2\, F_{\mu\nu}\, dx^{\mu} \wedge dx^{\nu}$$

- Charge renormalization says that the cloud of polarized vacuum that surrounds an elementary charge is more "physical" than the "bare" charge.

- This suggests that the true scale at which general relativity breaks down is not the Planck scale of quantum gravity, but the scale of vacuum polarization surrounding elementary charges.

Compton wavelength:	3.862×10^{-11} cm.
Classical electron radius:	2.818×10^{-13} cm.
Planck scale:	1.6×10^{-33} cm.

- Physical meaning of the Planck scale becomes moot, since you never get there, if this is true.

Pre-metric electromagnetism

- The only place where the Lorentzian metric g affects the Maxwell equations is by way of the Hodge * isomorphism:

$$dF = 0, \qquad dH = \#\mathbf{J}, \quad H = *F, \qquad d\#\mathbf{J} = 0,$$

in which:

$$F, H \in \Lambda^2(M), \quad \mathbf{J} \in \Lambda_1(M)$$

$$\#: \Lambda_k \to \Lambda^{4-k}, \qquad \mathbf{a} \mapsto i_{\mathbf{a}}\mathcal{V},$$

$$\mathcal{V} = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 = \frac{1}{4!}\varepsilon_{\kappa\lambda\mu\nu}\, dx^\kappa \wedge dx^\lambda \wedge dx^\mu \wedge dx^\nu ,$$

$$(\#\mathbf{J})_{\lambda\mu\nu} = \varepsilon_{\kappa\lambda\mu\nu} J^\kappa,$$

$$* = \# \cdot (g \wedge g)$$

$$(g \wedge g)^{\kappa\lambda\mu\nu} = \frac{1}{2}(g^{\kappa\mu} g^{\lambda\nu} - g^{\kappa\lambda} g^{\mu\nu})$$

- The pre-metric approach to EM replaces * with the EM constitutive law for the medium:

$$\mathfrak{h} = \kappa(F), \qquad\qquad \kappa: \Lambda^2 \to \Lambda_2,$$

- If:

$$F = dt \wedge E - \#(\partial_t \wedge \mathbf{B}), \qquad \mathfrak{h} = \partial_t \wedge \mathbf{D} + \#(dt \wedge H)$$

then a linear constitutive law can be expressed in matrix form:

$$\begin{bmatrix} D^i \\ \hline H_i \end{bmatrix} = \left[\begin{array}{c|c} \varepsilon^{ij} & \alpha^i_j \\ \hline \bar{\alpha}^j_i & \tilde{\mu}_{ij} \end{array}\right] \begin{bmatrix} E_j \\ \hline B^j \end{bmatrix}$$

- Pre-metric form of Maxwell's equations is then:

$$dF = 0, \qquad \delta\mathfrak{h} = \mathbf{J}, \qquad \mathfrak{h} = \kappa(F), \qquad \delta\mathbf{J} = 0$$

in which:

$$\delta = \#^{-1} d\, \#$$

is the divergence operator that takes *k*-vector fields to (*k* – 1)- vector fields, and is adjoint to the exterior derivative operator *d*.

- Local component form of the equations is:

$$F_{\mu\nu,\lambda} + F_{\nu\lambda,\mu} + F_{\lambda\mu,\nu} = 0, \qquad \partial_\mu \mathfrak{h}^{\mu\nu} = J^\nu,$$

$$\mathfrak{h}^{\mu\nu} = \kappa^{\mu\nu\kappa\lambda} F_{\kappa\lambda}, \qquad \partial_\mu J^\mu = 0.$$

Dispersion laws for EM waves

- If one introduces an EM potential 1-form A:

$$F = dA$$

then one can combine three of the field equations into one:

$$\Box_\kappa A = \mathbf{J}, \qquad \Box_\kappa = \delta \cdot \kappa \cdot d$$

- The symbol of $\Box_\kappa$ defines a linear map:

$$\sigma[\Box_\kappa; k]: \quad \Lambda^1 \to \Lambda_1, \qquad i_k \cdot \kappa \cdot e_k$$

- Invertibility depends upon k, so the characteristic 1-forms k must satisfy the *characteristic equation:*

$$\det(\sigma[\Box_\kappa; k]) = 0.$$

- For EM, the lack of longitudinal modes reduces this determinant to a homogeneous quartic polynomial in k:

$$P^{\kappa\lambda\mu\nu} k_\kappa k_\lambda k_\mu k_\nu = 0$$

The $P^{\kappa\lambda\mu\nu}$ are completely symmetric and also define the components of the *Tamm-Rubilar tensor*.

- This generally factorizes into:

$$g^{\kappa\lambda} k_\kappa k_\lambda \, \bar{g}^{\mu\nu} k_\mu k_\nu = 0$$

- One assumes that k has the properties that make this a product of Lorentzian dispersion laws:

$$g(k, k) \, \bar{g}(k, k) = 0$$