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Determination of the molecular magnetic moment by Planck's theory of quanta

By Stefan Procopiu

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Translated by D. H. Delphenich

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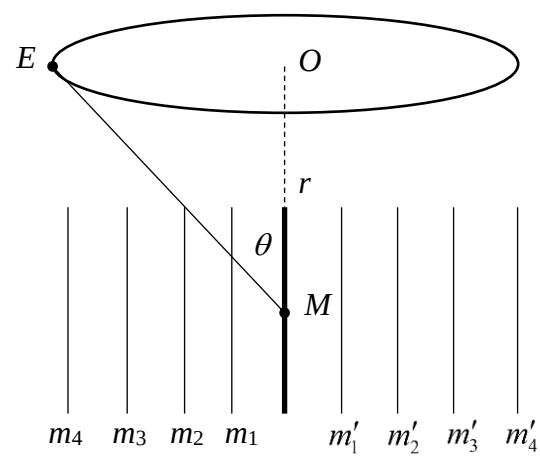
1. – **W. Ritz** explained line spectra ([1]) by considering electrons that oscillate in a molecular magnetic field that is composed of small magnets that are arranged along a line, equal to each other, and identical for all the bodies. Those elementary magnets, upon which **Ritz** did not insist, correspond to the magneton of **P. Weiss**, which is an elementary quantity of magnetism, or rather a sub-multiple that is common to all known molecular magnetic moments.

Upon comparing those attempts by **Ritz** and **Weiss** with **Langevin**'s explanation for magnetism, one can imagine those magnetons as being electrons that orbit around the molecule, just as the Earth orbits around the Sun.



E, the electron. *O*, the trajectory. m_1, m_2, m_3, m_4, m_1', etc., the magnetons. *M*, the resultant moment. θ, the angle between *E*

2. – Suppose that one has a sequence of those magnetons that are mutually parallel, and above them an electron vibrates with a certain frequency. The vibration of the electron will produce a characteristic line in the spectrum, and if the number of magnetons varies then the frequency of the electron and the line in the spectrum will vary.

Let the magnetons be m_1, m_2, m_3, m_4, m_1', etc., which all influence the motion of the electron *E* along the curve *O*. The parallel magnetons give a resultant, namely, the magnetic moment $M = nm$, if m is the value of the magneton and n is the number of them. Let r be the distance from the

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([1]) Annalen der Physik **25** (1908), pp. 660.

center of the magneton to the center of the trajectory of R, and let θ be the constant angle between E and the direction of the resultant magnetic moment M.

3. – The rapid oscillation of the electrons produces an energy of radiation, and one must apply **Max Planck**'s discontinuity relation ([1]) for the energy of radiation to it:

$$\varepsilon = h\ \nu, \tag{1}$$

in which ε is the energy that is absorbed or emitted by the oscillator, n is the observed frequency of the line that is produced in the spectrum, and h is a constant that is equal to 6.55×10^{-27} erg-s.

Thus, the energy of the oscillators must be proportional to the frequency of the spectral line that is produced.

In order to explain the various spectral lines, we consider the number of magnetons for the molecules of the same body to be variable. One can then explain the large number of lines that are emitted by the same body (5000 for iron). The observed molecular magnetic moment will then be a mean moment.

We shall calculate e and n in order to introduce them into the relation (1).

4. – We calculate the energy of the electron when it vibrates in the total magnetic field M, which amounts to calculating the potential of the magnet M at the electron E at a distance of r from that magnet. An elementary electrostatic relation tells us that if E has the charge e (viz., the value of the charge of the electron) then the potential will be:

$$-\frac{Me}{r^2}\cos\theta. \tag{2}$$

If θ remains invariable for the same molecule then one can suppose that its value varies over the large number of molecules that have the same magnetic moment M. Therefore, one must sum over all of those molecules when θ takes all possible values. Kinetic theory permits one to find a *mean value* ([2]).

Here is the argument:

One has dn molecules in the solid angle $d\omega$, which are electrons and make an angle with the moment M that is between θ and $\theta + d\theta$.

If the potential energy is given by the relation (2) and the kinetic energy is αT (α is a constant and T is the absolute temperature) then *Maxwell's equipartition* equation can be written:

([1]) Ann. Phys. (Leipzig) **22** (1907), pp. 180, and *ibid*. **31** (1910), pp. 759.

([2]) **Reinganum**, Ann. Phys. (Leipzig) **38** (1912), pp. 649.
Boltzmann, *La théorie des gaz*, t. 1, pp. 46.

$$dn = K\, e^{\frac{eM\cos\theta}{r^2 \alpha T}}\, d\omega\,,$$

K = a constant,
e = the value of the electron,
e = (also) the base for Napierian logarithms (2.71…),
αT = A = the kinetic energy.

If one takes $d\omega$ to be the zone around the magnetic axis whose opening is $d\theta$ then one will have:

$$d\omega = 2\pi \sin\theta\, d\theta,$$

and therefore:

$$dn = 2\pi K\, e^{\frac{eM\cos\theta}{r^2 A}} \sin\theta\, d\theta\,.$$

If one sums over all energies when θ varies from 0 to π, and if one divides by the total number of molecules then one will get the *mean potential energy* $\bar{E}$.

One will then have the following expression for $\bar{E}$:

$$\bar{E} = \frac{-2\pi K \int_0^{\pi} \frac{eM\cos\theta}{r^2} e^{\frac{eM\cos\theta}{r^2 A}} \sin\theta\, d\theta}{2\pi K \int_0^{\pi} e^{\frac{eM\cos\theta}{r^2 A}} \sin\theta\, d\theta}\,.$$

Upon making the substitution:

$$\frac{eM}{r^2 A} = a,$$

one will get:

(3) $$\bar{E} = -\frac{a\,A \int_0^{\pi} e^{a\cos\theta} \cos\theta \sin\theta\, d\theta}{\int_0^{\pi} e^{a\cos\theta} \sin\theta\, d\theta}\,.$$

If one sets:

$$\cos\theta = X$$

then the expression (3) will become:

(3 cont.) $$\bar{E} = -\frac{a\,A \int_{-1}^{+1} e^{ax} x\, dx}{\int_{-1}^{+1} e^{ax} dx}\,.$$

Upon integrating, one will get:

$$\bar{E} = -Aa\left(\frac{\cosh a}{\sinh a} - \frac{1}{a}\right),$$

and if one replaces a with its value then one will finally have:

(4) $$\bar{E} = -\frac{eM}{r^2}\left(\frac{\cosh a}{\sinh a} - \frac{1}{a}\right),$$

in which cosh and sinh signify the hyperbolic sine and cosine, resp.

When a increases indefinitely, the expression in the parenthesis will become equal to unity. (4) will then become:

(5) $$\bar{E} = -\frac{eM}{r^2}.$$

5. - The relation (5) represents the energy of the Planck oscillator. We have only to introduce it into the relation (1) in order to find that:

(6) $$\frac{eM}{r^2} = h\,\nu.$$

If the magnetic moment M is composed of a number n of magnetons of value m, and if one takes into account the relation:

$$\nu = \frac{c}{\lambda},$$

in which c is the speed of light and λ is the wavelength of the spectral line, then the relation (6) will become:

(6 cont.) $$\frac{e\,m\,n}{r^2} = h\frac{c}{\lambda}.$$

Numerical verifications of the relation (6 cont.), while introducing the values of e, M, h, c, λ that are known already: As far as r is concerned – namely, the distance from the electron to the magneton – **Ritz** compared the **Balmer** formula, which gives the distribution of spectral lines for hydrogen, with his own formula, and found that r was equal to twice the length of an elementary magnet, at least for hydrogen. Now, if one gives the length of the magneton the very probable value of 10^{-9} cm that I found (work that I shall publish later) then one will get the value of 2×10^{-9} cm for r.

With those values:

$e = 4.7 \times 10^{-9}$ electrostatic units,
$h = 6.55 \times 10^{-27}$ erg-s,
$c = 3 \times 10^{-10}$ cm/s,
$r = 2 \times 10^{-9}$ cm,

and one will find the value of M when one knows λ and the value of n (viz., the number of magnetons per molecule) when one takes m to be the value of the magneton that was given by **P. Weiss**, namely, 1.64×10^{-21}.

The following table gives the values of M and n (the number of magnetons per molecule) for the four principal lines of hydrogen H_α, H_β, H_γ, H_δ:

$10^7\ \lambda$ (cm)	M	n
656	5.2×10^{-20}	32
486	6.9×10^{-20}	42
434	7.7×10^{-20}	47
410	8.1×10^{-20}	50

One sees from this table that the values of M vary between 0.6×10^{-19} and 0.8×10^{-19}, while the value that was found by **Reinganum** (*loc. cit.*, pp. 664) in a different way was 1.0×10^{-19}.

The agreement is very satisfactory, but more so for the magnetic moments that produce the violet and ultraviolet rays than it is for the ones that produce the red lines.

It results from that work that the constitution of the molecule must be further complicated by the addition of a number of new electrons of a certain type that are called "magnetons." From what results for spectral lines, their number is very large for a molecule, and greatly exceeds the number of the other kinds of electrons.

As far as the electrons that vibrate are concerned, from the value of r, they are probably outside the molecule.

Laboratoire d'électricité de l'Université de Iassy.