"Sur l'interprétation des masses de l'électron et du proton dans l'univers à cinq dimensions," C. R. Acad. Sci. Paris **185** (1927), 889-891.

On the interpretation of the masses of the electron and proton in a five-dimensional universe

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L. de Broglie (²) gave a very simple geometric representation of the electric charge of a material point in Kaluza's (³) five-dimensional universe. That representation was based upon the theoretical considerations of O. Klein (⁴), which allowed one to prove that expression:

(1)
$$I^2 = m^2 - \frac{e^2}{16\pi C}$$

is invariant in the five-dimensional universe, where *m* denotes the proper mass of the material point, *e* is its charge, and *C* is Newton's constant of universal gravitation. *I* will then be a vector whose covariant component along the x^0 -direction will be proportional to the electric charge *e* of the material point, while the orthogonal projection of the vector onto a line perpendicular to x^0 , which belongs to the quadri-dimensional multiplicity $x^0 = \text{const.}$ that corresponds to the space-time of physics, is proportional to the proper mass *m* of the point.

Let *ds* be the invariant element of the world-line of the material point and let $d\mathcal{D}$ and *ds* be two differential invariants.

One has:

(2)
$$\left(\frac{d\mathcal{D}}{d\sigma}\right)^2 + \left(\frac{ds}{d\sigma}\right)^2 = 1.$$

In order to obtain the usual equations of motion of the material point in an electric or gravitational field, de Broglie (⁵) set:

^{(&}lt;sup>1</sup>) Session on 24 October 1927.

^{(&}lt;sup>2</sup>) L. DE BROGLIE, "L'univers à cinq dimensions et la mécanique ondulatoire," J. de Phys. 8 (1927), 65-75.

^{(&}lt;sup>3</sup>) TH. KALUZA, Sitzungsber. Akad. Berl. (1921), pp. 966.

^{(&}lt;sup>4</sup>) O. KLEIN, Zeit. Phys. **37** (1926), 895-906.

^{(&}lt;sup>5</sup>) L. DE BROGLIE, *loc. cit.*

(3)
$$I \frac{d\mathcal{D}}{d\sigma} = \frac{e}{\sqrt{-16\pi C}}, \qquad I \frac{ds}{d\sigma} = m.$$

Upon applying those general considerations to the proton and electron, we first confirm that the ratio:

$$\frac{\frac{m^2}{e^2}}{\frac{16\pi C}{1}}$$

is on the order of 4×10^{-35} for the proton and 10^{-41} for the electron. That situation suggests the idea of attributing the same magnitude to the world-vector in both cases. However, since, on the other hand, the covariant components along x^0 are equal and opposite for the proton and electron, while the orthogonal projection onto the direction ds is 1840 times as large for the proton as it is for the electron, that hypothesis must be abandoned if ds^2 is to be an invariant in the five-dimensional universe with full rigor.

If we free ourselves from the restrictions that Klein and de Broglie placed at the basis of their theory by supposing that the coordinate transformations in the five-dimensional universe that leave ds in the plane that is defined by ds and dD are meaningful then the general relation between the three differentials dD, ds, and $d\sigma$ will be:

(4)
$$d\sigma^2 = a \, d\mathcal{D}^2 + b \, d\mathcal{D} \, ds + c \, ds^2,$$

in which *a*, *b*, *c* are functions of the components γ_{ik} (*i*, *k* = 0, 1, 2, 3, 4) of the metric tensor, and will consequently be functions of the coordinates.

Meanwhile, if the relation (4) is applied to the motion of a material point in the absence of any external field then the γ_{ik} , and as a result the coefficients *a*, *b*, *c*, will be universal constants.

Suppose, moreover, that the ratio $\left(\frac{d\mathcal{D}}{d\sigma}\right)^2$ presents the same value for any electric charge of

the material point.

Equation (4) can then be written:

(5)
$$\left(\frac{d\mathcal{D}}{d\sigma}\right)^2 - \beta_1 \sqrt{-1} \frac{ds}{d\mathcal{D}} - \beta_2 = 0,$$

in which β_1 and β_2 are positive coefficients. According to formulas (3) (which are probably not rigorously exact, but which are undoubtedly a sufficient approximation for one to use them here), one has:

(6)
$$\frac{ds}{d\mathcal{D}} = \sqrt{-16\pi G} \frac{m}{e} \; .$$

Equation (5) then expresses the proper mass of the material point as a function of its charge. That will give:

(7)
$$m^2 - \frac{e\beta_1}{\sqrt{-16\pi G}} m - \frac{e^2\beta_2}{16\pi G} = 0.$$

If one introduces the values:

(8)
$$\frac{e\,\beta_1}{\sqrt{-16\,\pi\,G}} = \pm\,M, \qquad \frac{e^2\beta_2}{16\pi\,G} = M\,\mu$$

then equation (7) will provide the mass of the electron μ and that of the proton $M + \mu$, where the + sign refers to the proton. Indeed, from (7) and (8), one will get the solutions:

(9)
$$m = \pm \frac{M}{2} + \frac{M}{2} \sqrt{1 + \frac{4\mu}{M}} .$$

The negative solutions of the second-degree equation have no physical significance. Formulas (8) permit one to calculate the numerical values of the coefficients β_1 and β_2 in equation (7), which has a universal significance, in principle. For e = 0, one finds that m = 0.