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A clarification of the universal electrodynamical meaning of Planck's radiation constant h

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I.

It is known that **Max Planck** proved that only finite amounts of energy and be received and released during the emission and absorption of radiation, and that the elements of energy that appear in that way are proportional to the frequency. The proportionality factor h is a universal constant that has the dimensions of "action," and for that reason, **Planck** called it the element of action. Among the various attempts to interpret the element of action, that of **A. E. Haas** (¹) seems noteworthy to me, which achieved good numerical agreement by appealing to **Thomson**'s atomic model.

One can generally reproach that attempt at explanation by saying that the hypotheses employed, and in part, also the chosen numerical data, seemed to have been chosen *ad hoc*, to some extent. However, **H. A. Lorentz** (²) had also connected the **Haas** hypotheses with the customary intuitions and also proved that certain facts whose theoretical interpretation was given by **Einstein** (³) seems to be very much in the spirit of the **Haas** picture.

However, an important objection to **Haas**'s theory still exists: There is no explanation for the universal character of the elementary quantum of action, but one must be satisfied with confirming a very remarkable numerical agreement. Here, I would like to try to fill that lacuna and give a complete and general explanation. The special hypothesis that this representation is based upon might perhaps seem strange. However, it is no stranger than the assumptions that are generally employed in the theory of electrons, and it is supported by an experimental fact whose significance for the electron hypothesis has still not been considered sufficiently. Furthermore, I have tried, as much as possible, to link my representation with **J. J. Thomson**'s theory of the atom. It might not be difficult to show that despite the aforementioned deviation, the assumptions that are made here can be made to agree with that theory. However, that would lead us to digress too far from the actual topic of this treatise.

⁽¹⁾ A. E. Haas, Wien. Ber. 119, IIa February 1910 and Jahrb. d. Radioakt. 7, 2 Heft, (1910), pp. 261.

^{(&}lt;sup>2</sup>) **H. A. Lorentz**, Phys. Zeit. **11** (1910), pp. 1232.

^{(&}lt;sup>3</sup>) **A. Einstein**, Ann. Phys. **17** (1905), pp. 132.

II.

The starting point for our considerations is defined by the conception of the nature of positive electricity that **J. J. Thomson** (¹) developed. According that well-known theory, positive charges are balls of various sizes in which the spatial density ρ always exhibits the same value. Such a ball defines the atomic nucleus, but it can still be enveloped by a positive atmosphere of low density.

As far as negative charge is concerned, **J. J. Thomson** proposed that the electrons (corpuscles) assumed widely-separated equilibrium positions. However, we would like to make that assumption true for only part of the negative charge and assume that the atomic interior can be defined by a union of several electrons with likewise-spherical nuclei. Indeed, the greater part of the negative charge shall always be combined into a single ball of the same charge density as the electron, but with a correspondingly larger radius, whose equilibrium position coincides with the center of the much-larger ball.

The corpuscles of the outer ring, or the valence corpuscles that are dispersed in the enveloping atmosphere that **J. J. Thomson** employed to explain the periodic properties of the elements, can be excluded from that assumption by restricting oneself to the consideration of balls with large electron numbers, so to atoms of high atomic weight.

The ball considered contains N positive charges and just as many negative ones. The tremendous forces that act upon the internal electron masses (about whose origin we can naturally say nothing) might always keep that negative nucleus in an unvarying state unless the entire ball breaks up into its components. By contrast, in some situations, one or more of the positive balls can leave the outer electrons.

Under the influence of a uniform external field of field strength \mathfrak{E} , the negative charge will displace collectively like a rigid body through a distance of Δr , where:

$$\mathfrak{E}=\frac{4\pi}{3}\rho\,\Delta r\,.$$

In that respect, the system that is considered here behaves completely like Thomson's atom. The electric moment that is created by the displacement is:

$$N e \,\Delta r = \frac{3N \mathfrak{E} e}{4\pi \,\rho}$$

which implies that:

$$N e \Delta r = \frac{3}{4\pi} \mathfrak{E} v$$

in the case of a neutral ball ($N e = \rho v$), where v means the volume of the positive ball. The polarization is therefore equal to that of a conducting ball of volume v.

If a unit volume contains *n* atoms then the dielectric constant will be:

^{(&}lt;sup>1</sup>) J. J. Thomson, "Die Korpusculartheorie der Materie," Die Wissenschaft, Heft 15, 1908.

$$\varepsilon = 1 + 3n v$$
.

If one introduces the index of refraction μ in place of the dielectric constant for infinitely-long waves then one will get the approximate relation:

(1)
$$\mu = 1 + \frac{3}{2}nv$$
,

which shall be employed in what follows in order to calculate the atomic radius.

The negative charge will be put into oscillation under the influence of an incident light wave. The force:

$$K = \frac{4\pi}{3} \rho N e \Delta r = \frac{N^2 e^2}{A^3} \Delta r$$

acts on the total charge Ne under a displacement Δr , where A means the radius of the positive ball.

Now, if *M* is the mass of the negative charge then the frequency of the proper oscillation of the ball will be:

(2)
$$v = \frac{e}{2\pi} \frac{N}{A\sqrt{AM}}.$$

We now assume that the absorption of radiant energy can take place only when an electron is separated from the positive ball under the influence of an external electromagnetic field. Conversely, the emission of radiant energy is possible only when a foreign electron is received.

Thanks to the proposed assumptions, it is very simple to calculate the potential energy that is absorbed by the emission of an electron. If our neutral system loses one electron then it will exhibit a charge surplus of + e. Now, since the negative charges are almost all found at the center of the positive ball, from the outside, it acts like a conducting ball with a charge of + e and surface potential e / A. Therefore, a potential energy of:

(3)
$$\varepsilon = \frac{e^2}{A}$$

is gained by the emission of the electron, and the same energy will be liberated when an electron penetrates the positive ball from the outside.

A ball of positive electricity with a charge number of *N* and a radius *A* that corresponds to that charge number (at constant spatial density ρ) represents a model of the **Planck** resonator. Formula (3) yields the energy element of that resonator, while formula (2) implies its proper oscillation number.

H. A. Lorentz (*loc. cit.*) showed that the value of the energy element that is calculated from (3) for the Argon atom leads to a value of the radiation constant *N* that is roughly correct. I have carried out the calculation for a larger number of atoms with the use of the data that can be derived

from the dispersion measurements of **C**. and **M**. **Cuthbertson** (¹), and I have found a confirmation of the theory in all cases that is just as good as it is for the argon atom. It seems to me to be out of the question that such a striking agreement would be based upon coincidence.

III.

It follows from **Planck**'s theory of radiation (²), in conjunction with **Wien**'s law of displacement (³), that the ratio ε / v is equal to a universal constant that takes on the numerical value:

$$h = 6.5 \times 10^{-37}$$

according to **Planck** (⁴).

We would now like to attempt to show that the foregoing theory of the **Planck** resonator gives an explanation for the constancy of the quantity h and that this theory also yields a numerical value for h that is roughly correct. In general, it cannot achieve a complete numerical agreement, which is possibly due to the fact that the experimental data that was employed does not, in part, agree completely. However, I have distanced myself from making a fundamental revision of the numerical material, because this treatise is supposed to give merely the impetus for further formulations and completions of the theory that is put forth.

It follows from formulas (2) and (3) that:

(4)
$$\frac{\varepsilon}{v} = h = 2\pi e \frac{\sqrt{AM}}{N}$$

We would next like to assume that *M* means the electromagnetic mass of the negative charge. One cannot generally exclude the possibility that the mass of an individual electron that oscillates in the atomic interior is already larger than its electromagnetic rest mass *in vacuo*. Only the latter value is known from cathode ray experiments, but the determinations of it that are based upon observations of the Zeeman effect might not relate to electrons that oscillate in the atomic nucleus. The spectroscopic determinations probably give no information about the processes that take place in the atomic nucleus.

However, one can determine the ratio of charge e to mass m for an individual electron that oscillates in an atom from the data of the dispersion measurements for hydrogen using the method that **Drude** gave when one assumes that the number of electrons in the atom is equal to the atomic weight. With that assumption and from the dispersion data, **Drude** calculated that:

$$\frac{e}{m} = 1.48 \times 10^7$$

for the hydrogen atom, instead of:

^{(&}lt;sup>1</sup>) **C.** and **M. Cuthbertson**, Proc. Roy. Soc. London (A) **83** (1909), pp. 149, 151, and 171.

^{(&}lt;sup>2</sup>) **M. Planck**, *Vorlesungen über die Theorie der Wärmestrahlung*, 1906, §§ 148 and 149.

^{(&}lt;sup>3</sup>) **W. Wien**, Berliner Ber., 1893.

^{(&}lt;sup>4</sup>) M. Planck, Ann. Phys. (Leipzig) 4 (1901), pp. 553, and *Vorlesungen*, § 157.

$$\frac{e}{m_0} = 1.87 \times 10^7,$$

which would follow from the determinations using cathode rays. The more recent dispersion measurement lead to an even smaller value of e / m for the electron in the hydrogen atom.

J. J. Thomson (¹) derived the fact that the number of electrons in the atom is equal to the atomic weight from various facts. One might then regard that assumption as a fact of experiment from which further conclusions can be drawn.

If one calculates the ratio e / m for the electrons in atoms of higher atomic weight using **Drude**'s method then one will find that the ratio decreases rapidly with increasing atomic weight, which might be interpreted as an increase in the specific oscillating mass.

However, since the electron number is proportional to the atomic weight in all cases, one must assume that either the electrons in the atomic interior are loaded with a varying material, not electromagnetic, mass or that the electromagnetic mass of the negative charge grows faster than the charge number. Nevertheless, the growth in specific electromagnetic mass with charge number cannot be easily explained by the assumption that was made above that the negative charge defines a compact spherical nucleus.

Our negative ball is nothing but a large electron of charge N e whose radius a is given by the condition that:

$$\frac{a}{\alpha} = N^{1/3},$$

where α means the radius of the ordinary cathode ray electron.

According to the hypothesis of a constant density of negative electricity, the volume of the negative ball is, in fact, equal to the volume of *N* electrons.

The electromagnetic rest-mass of a homogeneous ball of charge N e and radius a is:

$$M=k\frac{N^2e^2}{a},$$

where k is known from electromagnetic considerations. Now, since the mass of the individual electron is given by:

$$m=k\frac{e^2}{lpha},$$

it will follow that:

(5)
$$M = N^{5/3} m$$

We would now like to introduce the value (5) into equation (4), and in that way, obtain:

^{(&}lt;sup>1</sup>) Cf., J. J. Thomson, *loc. cit.*, Chap. VII.

(6)
$$\frac{\varepsilon}{\nu} = h = 2\pi e \sqrt{\frac{Am}{N^{1/2}}}$$

Since the density of positive charge ρ is the same in all resonators, by assumption, Ne/A^3 , and therefore

 $\frac{N^{1/3}}{A},$

as well, will be constant.

However, it follows from this that h is a universal constant whose value depends upon merely the charge element e, the density of positive charge, and the electromagnetic rest mass of the electron.

IV.

Now, as far as confirming formula (6) by means of observational data is concerned, it should first be pointed out that the consequences that can inferred from the dispersion measurements of gases do not seem to yield a completely constant value for ρ .

However, **J. J. Thomson** (*loc. cit.*, pp. 162) explained that by the assumption that the atomic nuclei, which are always equally dense, are enveloped by a more or less extended positive atmosphere in which the "valence corpuscles" move.

Gas	$(\mu - 1) \ 10^6$	М	$\frac{(\mu-1)10^6}{M}$	$10 \sqrt[6]{\frac{\mu-1}{M}}$
Helium	69.4	4	17.3	1.60
Neon	133	20	6.65	1.37
Argon	559	40	14.0	1.55
Krypton	840	80	10.5	1.48
Xenon	1364	128	10.65	1.48
Oxygen	266	32	8.3	1.42
Nitrogen	295	28	10.5	1.48
Hydrogen	136	2	68.0	2.02
Phosphorus	1165	62	18.8	1.63
Arsenic	1550	150	10.3	1.47
Sulfur	1045	64	16.35	1.57
Mercury	1765	200	8.82	1.44
SO_2	641	64	10.0	1.47
H_2S	624	34	18.3	1.62

Only the density of the positive electricity in the atomic nucleus comes under consideration in the case that was treated here. Moreover, the table above that **C**. and **M**. **Cuthbertson** calculated from the results of dispersion measurements shows that the ratio:

$$\sqrt[6]{\frac{\mu-1}{M}},$$

which is all that matters here, is markedly constant for most atoms (with the exception of the hydrogen atom). Naturally, our theory is not even applicable to atoms of small atomic weight.

For the sake of explaining the preceding numbers, let it be pointed out here once more that according to **J. J. Thomson**'s atomic theory, which agrees completely with the picture that is developed here in that regard, the ratio $\mu - 1 / M$, where μ is the index of refraction for infinitely-large waves, and *M* means the molecular weight of the gas, must be proportional to the density of positive electricity.

For atoms of large atomic weight, $\sqrt[6]{\rho}$ seems noticeably constant in the data, such that our assumption can be regarded as well-founded.

Only atoms of large atomic weight come under consideration for the calculation of the numerical values of h, although the values thus-obtained will be approximately valid for all other atoms (with the exception of hydrogen).

According to **Planck**, the number of monoatomic molecules in a cm^3 of a gas at 0° and atmospheric pressure is:

$$n = 2.8 \times 10^{19}$$
.

That number, along with the value of $\mu - 1$ that was given above for mercury, implies that:

$$A = 2.8 \times 10^{-8}$$
.

If we employ the values for *e* and *m* that correspond to **Planck**'s data:

 $e = 4.7 \times 10^{-10}, \qquad m = 0.84 \times 10^{-27}$

and set:

N = 200 (mercury)

then we will find that:

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h = 5.2 \times 10^{-27}.
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That value strongly approximates the one that **Planck** calculated. Nonetheless, it is too small, so we must assume that the mass M of the oscillating negative charge is larger than the mass that was calculated above.

However, since we do not even know what processes take place in the atomic interior, and we cannot exclude the possibility that the negative charge in the interior of the positive nucleus has a larger electromagnetic mass than it has *in vacuo*, or also carries a non-electromagnetic mass due to its oscillation, we can correct the result that was obtained above by determining the increase in mass of the electron from the data of the dispersion measurements for hydrogen using **Drude**'s method.

If we use the values that C. and M. Cuthbertson gave then we will get:

$$\frac{e}{m} = 1.37 \times 10^7$$
 e.m. units,

from which we calculate the value of *m* to be:

$$m = 1.14 \times 10^{-27}$$
.

When that value is substituted in the formula above, that will give:

$$h = 6.05 \times 10^{-27}$$
.

V.

We have based our calculations upon the assumption that the atom can function as a **Planck** resonator. However, one should not imagine that those resonators are atoms, in general. The proper oscillations of atoms lie far into the ultraviolet part of the spectrum. Even the resonators that respond to visible light possess significantly larger radii than the atom. Thus, for example, from (3), the radius of the optical resonator is roughly three times as large for sodium light ($\nu = 5 \times 10^{14}$) as it is for mercury, and the charge number N grows rather rapidly with increasing wavelength.

The constitution of those resonators must be precisely the same as that of the atom, moreover.

From the hypothesis that was proposed here, the **Planck** resonator will be composed of a ball of positive electricity that more or less large, in whose interior the negative electricity carries out oscillations in the form of a compact ball of significantly smaller volume. The spatial electrical density of the negative ball is equal to that of the electron. That ball must not necessarily contain a whole number of electrons. The density of positive electricity is likewise constant.

The following value of the radiation constant *h* will be derived from those assumptions:

$$h = 2\pi e \sqrt{\frac{Am}{N^{1/3}}} = \text{const.}$$

In it, e means the charge element, A is the radius of the positive ball, N is the number of charges, and m is the mass of the electron that oscillates in the interior of the hydrogen atom.

Using that formula, one calculates the value of h from the results of the dispersion measurements of **C**. and **M**. Cuthbertson to be:

$$h = 6.05 \times 10^{-27}$$
.

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