"Nachtrag zur Abhandlung über 'Elektromagnetische Grundgleichungen in bivectorieller Behandlung,'" Ann. d. Phys. **329** (1907), 783-784.

Addendum to the treatise "Basic electromagnetic equations in bivectorial form"

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In volume 22 of these Annalen (pp. 579-586), I gave the bivectorial representation of Maxwell's differential equations, as well as the expressions for the electromagnetic energy density and energy flux for *empty space* (or ultimately for a medium with equal dielectric constant and permeability). [1]) In this addendum, the bivectorial formulas will be extended to an arbitrary homogeneous and isotropic dielectric.

If one denotes the electric (magnetic, resp.) force by E_1 (E_2, resp.) then one has:

$$K \frac{\partial E_1}{\partial t} = c \cdot \text{curl}\, E_2 , \qquad \mu \frac{\partial E_2}{\partial t} = - c \cdot \text{curl}\, E_1 .$$

If one introduces the propagation velocity $v = c / \sqrt{K \mu}$ then these equations can be written:

$$\sqrt{K} \frac{\partial E_1}{\partial t} = v \sqrt{\mu}\ \text{curl}\, E_2, \qquad \sqrt{\mu} \frac{\partial E_2}{\partial t} = - v \sqrt{K}\ \text{curl}\, E_1 , \tag{1}$$

where one has to take the positive roots.

Now, if K, μ are ordinary scalar constants then one can set them behind the operation sign, and if one then introduces the *electromagnetic bivector:*

$$\eta = \sqrt{K}\, E_1 + i \sqrt{\mu}\, E_2 \tag{2}$$

then equations (1) combine into the single equation:

$$\frac{\partial \eta}{\partial t} = - i\, v\, \text{curl}\, \eta. \tag{I}$$

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[1]) In the meantime, I have found that in the year 1901 H. Weber (*Die partiellen Differentialgleichungen der math. Physik 2*, pp. 348) had already combined the two Maxwell equations and, in fact, wrote down the equation c curl $(\mathfrak{E} + i\mathfrak{M}) = i\, \partial\, (\mathfrak{E} + i\mathfrak{M}) / \partial t$, without actually going into the details of the conjugate bivector or the representation of the energy density and the energy flux.

If one further introduces the *conjugate electromagnetic bivector* by the definition:

(3) $$\eta' = \sqrt{K}E_1 - i\sqrt{\mu}E_2$$

then one has:

(I′) $$\frac{\partial \eta'}{\partial t} = i\, v \,\mathrm{curl}\, \eta'.$$

This equation naturally says exactly the same thing as (I).

The electromagnetic *energy density* is $e = \frac{1}{2}(KE_1^2 + \mu E_2^2)$; one thus has, from (2), (3), just as one does in empty space:

(II) $$e = \tfrac{1}{2}\eta\,\eta',$$

where one intends the *scalar* product of the mutually conjugate bivectors on the right-hand side.

From the general formula (5) of the previous treatise, the vector product of these bivectors is:

$$V\,\eta\,\eta' = 2i\sqrt{K\mu}\,V\,E_2\,E_1 = 2i\frac{c}{v}V\,E_2\,E_1.$$

The *energy flux* – or *Poynting vector* – $F = c\,V\,E_1\,E_2$ will then take on the expression:

(III) $$F = \frac{i}{2}v\,V\,\eta\,\eta'.$$

If *impressed* forces are also present, and indeed electric E_1' and magnetic E_2' ones then one has to subtract them from the curls in equations (1) for E_1 (E_2, resp.). If one then introduces the *impressed electromagnetic bivector* by the definition:

(4) $$\zeta = \sqrt{K}E_1' + i\sqrt{\mu}E_2'$$

then one has the following replacement for the two relevant differential equations:

(IV) $$\frac{\partial \eta}{\partial t} = i\, v\,\mathrm{curl}\,(\zeta - \eta).$$

I hope to be able to give examples of applications of the latter form of equation in a future paper.

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