"Über die Ablenkung eines Lichtstrahls von seiner geradlinigen Bewegung durch die Attraktion eines Weltkörpers, an welchem er nahe vorbeigeht," Ann. Phys. (Leipzig) **370** (1921), 593-604.

On the deflection of a light ray from its rectilinear motion by the attraction of a celestial body that it passes close to.

By **J. Soldner** (1801)

With preliminary remarks by **P. Lenard.**

Translated by D. H. Delphenich

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1. – Preliminary remarks.

The calculation of the German mathematician and geodesist **Soldner**, which is now 120 years old, and which I will communicate in the following as excerpts from the currently less-accessible original ([1]), yielded a deflection of 0.85″ for a fixed star that is observed when it is close to the solar limb. That deflection was apparently confirmed by observations of the solar eclipses up to now within the limits of precision of those observations ([2]).

Soldner's calculation is based upon merely the very simple and clearly-stated assumption of mass and the gravity of light, which is proportional to it. 110 years later, **Einstein** derived the same deflection by a different way that was by no means simpler, or even free from objections, namely, by means of his well-known revolutionary theory of relativity that is conceived in space and time,

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([1]) Berliner Astronomisches Jahrb. (1804), pp. 161. I would like to thank **M. Wolf** (Heidelberg) for first bringing that old article to my attention, which he learned about from a message that **Näbauer** (Karlsruhe) had sent to him on 20 April 1921. A recent (25 June) letter by **Seeliger** (Munich) on the occasion of my publication in the Astron. Nachr. (which cites only **Libotzky**) likewise made me aware of **Soldner**, with the addition: "When I performed the small calculation nine years ago (i.e., 1912), I remarked that it had already been carried out by **Soldner** (following citation), but generally in a pointlessly complicated way." At the same time, I would like to thank **Seeliger** for the suggestion that **Libotzky** [Phys. Zeit. **22** (1921), pp. 69] did not explain the result of his calculation that he performed on the same basis as **Soldner** in manner that agreed correctly with the theory of relativity of 1915 that he cited (eq. I, *loc. cit.*, pp. 70, following **Einstein**, is correct, but missing a factor of 2). **Libotzky**'s result, like that of **Soldner**, then agrees with only the somewhat-older theory of relativity of 1911.

([2]) Cf. the discussion of **E. Wiechert**, Ann. Phys. (Leipzig) **63** (1920), pp. 318, *et seq.*, and also **L. C. Glaser**, Annalen für Gewerbe und Bauwesen **87** (1920), pp. 30 *et seq.* If one considers the refraction of rays by the solar atmosphere, which is undoubtedly present and acts in the same sense, then one would expect the deflection to be somewhat larger than 0.85″, and that is also the mean result of the observations. The refraction of rays and **Soldner**'s deflection both decrease with increasing distance from the solar limb, but according to different laws, which means that the law for the refraction of rays by itself must be regarded as unknown, since one does not know enough about the distribution of matter and pressure in the solar atmosphere, and indeed one is not even justified in assuming that those distributions are static. Therefore, it would not be easy to eliminate the refraction of rays from future refined observations and to ascertain the purely gravitational effect.

and indeed an extended form of it ([1]). Many observations of the theory of relativity have been made. On the other hand, **Soldner**'s much older achievement also generally deserves to be emphasized all the more because (as is always true in such cases) no one can say to what degree the older achievement was the stimulus and basis for later activity in the same subject, and that alone might already justify the demand of the Annalen for the present communication, apart from other viewpoints that might still follow. Here, one has a case that is similar to that of **Gerber**'s equation for the precession of perihelion of Mercury that also essentially preceded that of **Einstein** ([2]), but generally the one here differs by the fact that **Gerber**'s calculation was not found to be free of objection ([3]), while it would hardly be possible to object to anything essential in **Soldner**.

Soldner's starting point was his opinion that light that came from matter at high temperature might probably be itself subject to matter, and therefore gravitation ([4]). At that time, one could probably share that opinion, but hardly later on after **Fresnel**'s work was fully developed. That probably explains the fact that **Soldner**'s work was apparently forgotten. Things are different now. Ever since **Hasenöhrl**'s investigations of light pressure ([5]), which were later continued by **Einstein** in a somewhat-different form, the inertia of energy, in particular, that of light rays, has been considered to be assured, since **Maxwell**'s equations, and light pressure in particular, have been well-confirmed experimentally. On the other hand, the general proportionality of inertia and gravity was likewise tested sufficiently by the progressively-refined investigations of **Galilei**, **Newton**, **Bessel**, and **Eötvös**. The coupling of both facts implies the gravity of energy, and therefore also **Soldner**'s gravity of light, as a by no means arbitrary, but very natural, consequence as long as one makes no distinction between the inertia of material masses and that of energy, but no verifiable reason for that exists, either. One can also regard the deviations of the atomic weights from whole numbers as a confirmation of that consequence of the gravity of energy, insofar as isotopes do not suffice as an explanation. The weights of the different amounts of energy content of the whole-number dynamids that the atoms are constructed from are what leads one to expect atomic weights to be close to, but not precisely, whole-numbers. As far as the gravity of light is concerned, that result will even become directly accessible to intuition when one adds **Planck**'s discovery that light rays are composed of light quanta. Each such light quantum has its own energy content, and therefore its own mass, as well, which can also be ascribed to the associated gravity, and that was just the concept that **Soldner** based his calculation upon (if also more obscurely at the time).

([1]) **A. Einstein**, Ann. Phys. (Leipzig) **35** (1911), pp. 908. Later on, **Einstein** derived twice the deflection using the so-called general theory of relativity [Berl. Akad. (1915), pp. 834], but that cannot be regarded as confirmed up to now, since the observations yield barely twice the deflection, whereas they must yield *more* than twice the deflection as a result of the refraction of rays (cf., previous note).

([2]) See **E. Gehrcke** in Ann. Phys. (Leipzig) **51** (1916), pp. 119 and *ibid.* **52** (1917), pp. 415.

([3]) Cf., especially, **H. Seeliger**, Ann. Phys. (Leipzig) **53** (1917), pp. 31; also **M. Laue**, *ibidem* **53** (1917), pp. 214.

([4]) He developed that opinion thoroughly in **Gilbert**'s Ann. Phys. (Leipzig) **39** (1811), pp. 231, at the same time as a communication on the logarithmic integral, which is a function that he first gave that still-useful name to at the time while investigating it more closely and calculating a table for it. On the life of **Soldner**, one should see "Johann Georg von Soldner" by F. J. Müller (Dissertation, Munich, Oct. 1914, with a portrait). The first proposal for a tau-point hygrometer also goes back to **Soldner**, and which was subsequently put to use under the names of **Daniell** and **Regnault** [Ann. Phys. (Leipzig) **32** (1809), pp. 213]. **Soldner** was Bavarian, the son of a farmer. In addition, he had the advantage of not having attended too many schools.

([5]) **F. Hasenöhrl**, Wien. Akad. **113** (1904), pp. 1039; Ann. Phys. (Leipzig) **15** (1904), pp. 344; see also **Stark**'s Jahrb. **6** (1909), pp. 485.

It is necessary here to emphasize an erroneous notion that is presently becoming almost the established one, namely, that none of those facts – viz., the inertia of energy, the gravity of light rays, the curvature of light rays near the Sun – can be derived from well-established facts of experiment and principles that are accessible to ordinary intuition without any assistance from the theory of relativity, and especially without any assistance from the space and time representation of that theory. I have shown in a different place ([1]) how simple it is to derive the inertia of energy, including the equation $M = E / c^2$. It was also emphasized there ([2]) already that one is naturally free to ascribe experimentally a corresponding gravity to the light quantum to its thus-verified mass, and that one will be led to conjecture a curvature of light rays near the Sun in that way completely independently of the theory of relativity, and with no less certainty than with that theory, and the magnitude of that curvature can also be likewise calculated in the simplest-possible way entirely without the theory of relativity, in such a way that one will be clear about the basis for the calculations with no further analysis. Namely, the light ray will then have, quite simply, the trajectory of any body that is launched with the speed of light. It was just the latter trajectory that **Soldner** calculated. The verification of a curvature of a light ray with the amount that **Soldner** first calculated for the Sun would then be a confirmation of **Soldner**'s simple idea, and not perhaps a confirmation of the theory of relativity, as one generally assumes without justification. That is because a complicated theory with very far-reaching assertions that are not at all necessary for the derivation of a result can never be confirmed by the fact that the result is true. If the result is confirmed then one can say only that either the content of the theory of relativity (of 1911) must be identical with the simple assumptions that were actually necessary for us to derive the result (and that is not the case for the theory of relativity, with its well-known far-reaching assertions) or it is only artificially and apparently interwoven with the result (which must then be assumed). One cannot declare that the validity of the weight of something has been proven when one lays it on the weighing pan with a previously-confirmed weight but removes it again before the end of the weighing, or when it is part of a collection of doubtful weights while one is weighing it.

However, let it be likewise emphasized on this occasion that things have also been completely the same for all other experimental confirmations up to now that one finds to be asserted repeatedly with great emphasis. I likewise showed that thoroughly ([3]). Therefore, one ultimately arrives at the dependency of mass on velocity ([4]), namely, to the known relation $m = m_0 / \sqrt{1 - v^2 / c^2}$. That

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([1]) For more details, see: *Über Relativitätsprinzip, Äther, Gravitation*, Hirzel, Leipzig, 1921, pp. 27 and "Über Äther und Uräther," **Stark**'s Jahrb. **17** (1921), pp. 321 (also Hirzel, 1921).

([2]) *Loc. cit.*, **Stark**'s Jahrb. **17** (1921), pp. 307. However, it is generally true that the relation $M = E / c^2$ has not been found, as Einstein proposed it. As our derivation shows, that will be true with certainty only in matter-free spaces, so for example, for light rays and cathode rays *in vacuo*, which are the particular cases that one generally deals with in the applications of the relation up to now.

([3]) "Über Äther und Uräther." ([1]) At the same time, a new way around the difficulties that are, in fact, present in the physics of the ether was given there that makes the single way around them that seems to exist (viz., the theory of relativity) irrelevant. The new alternative employs the ordinary representation of space and time but makes special assumptions in regard to the medium that fills up space, which one has called the ether, as such, up to now. The absence of contradictions in the alternative theory, which I have verified by comparing it with all experiments that come under consideration that I know of, shows the expendability of the theory of relativity, not just in its details, but also in its general understanding of nature.

([4]) In addition, one can include **Einstein**'s theorem for the addition of velocities, which has been confirmed by the observed dragging coefficient of light. However, there is also something peculiar about that confirmation. Namely, relation can be derived in the simplest-possible way with no use of the neologisms that the theory of relativity that were introduced into physics from **Hasenöhrl**'s inertia of energy and the tried and tested known principles ([1]). Calling the relation "relativity-theoretic," as is often done today, is therefore not only especially unjustified, but also very misleading, and should therefore be avoided ([2]). The fact that the relation seems to have been confirmed by the observations with cathode rays is therefore not evidence for the theory of relativity either but has nothing to do with it. The same thing is true for the confirmation of the relation by its application to the derivation of the fine structure of spectra from the atomic models ([3]). As far as the precession of perihelion of Mercury is concerned, from the stated relation, the dependency of mass on velocity gives only 7″ per century ([4]). For the rest of the precession of perihelion that is thus still unexplained, given the state of affairs that was described and from everything that is otherwise known up to now, it seems quite reasonable to ascribe it to those masses whose presence near the Sun is likely from the outset and is even exhibited by the zodiacal light ([5]). On cannot reject that explanation for the precession of perihelion as long as no knowledge of the behavior of mass in the neighborhood of the Sun

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the dragging coefficient agrees, within the limits of precision of the observations just as well with the prediction of the usual theory of dispersion as it does with the statement of theory of relativity, which is entirely different from the latter. The theory of relativity and its addition theorem is not merely refuted by the observations of the optical dragging. The confirmation of the special statement of the theory of relativity that would deviate from the results of the usual theory of dispersion is still pending. **Zeeman**'s most recent experimental contributions to the question [Kon. Akad. Amsterdam **22** (1919), pp. 462 and 512] have not remotely been able to reach a decision with the required precision either.

([1]) *Loc. cit.* (Stark's Jahrb.), pp. 341, *et seq.*

([2]) In order to allude to the origin of the conceptual development in question, one might call the mass that depends upon velocity the *mass of energy* or **Hasenöhrl** mass.

([3]) One finds the theory of relativity called into play in an especially conspicuous way in **Sommerfeld**'s book *Atombau und Spektrallinien*. The derivation of the dependency of mass upon velocity is carried out there (pp. 321, *et seq.*) with the express assistance of the "Lorentz contraction" of lengths and the "Einstein dilatation" of times, and the validity of the results is cited as the *experimentum crucis* of the theory of relativity. From the above, all of that entirely arbitrary, and every appearance of relativity-theoretic weapons in the book, as well as in many other places in the current literature, is likewise as irrelevant as perhaps appealing to quaternions once was (which was once a fad), if not considerably more persistent.

([4]) See **A. Sommerfeld**, *Atombau und Spektrallinien*, 1919.

([5]) For this topic, cf., the thorough investigations of **Seeliger**, Sitz.-Ber. der Münchener Akad. d. Wiss. **36** (1906), pp. 595. From the explanation above, the validity of **Einstein**'s derivation of the total observed precession of perihelion from the "general principle of relativity" would be as fortuitous as the validity of **Gerber**'s flawed derivation, whereby one must also observe that the very small precession of perihelion can be established by the observation only with limited accuracy [cf., especially, **Ernst Grossmann**, Zeit. Phys. **5** (1921), pp. 280, in which a new critical examination on the basis of all of the present associated observations gave a precession of perihelion of only 29″ to 38″, in place of the 43″ that **Einstein** calculated]. There is a certain step in the calculations of **Gerber**'s derivation whose justification is not obvious. In the general theory of relativity, the stumbling block seems to resolve into a cloud of finer particles, so to speak. For example, it is not clear why the "general theory of relativity" should be true for Mercury, while only the "special theory of relativity" (in reality, it is the **Hasenöhrl** mass) would yield correct results for the electron orbits in atoms (viz., the fine structure of the spectra). Plus, one has the general lack of clarity in the general theory of relativity that I have previously emphasized sufficiently. (*Über Relativitätsprinzip, Äther, Gravitation*, 3rd ed., 1921) One should also confer the continually-repeated discussion of the "clock paradox" [**E. Gehrcke**, Münch. Akad. d. Wiss. (1912), pp. 220 and in the lecture "Die Rel. Th. eine wissenschaftliche Massensuggestion," Verlag Köhler, Leipzig, 1920, and more recently, **E. Gehrcke** and **H. Thirring** in the weekly publication Naturwissenschaft **9** (1921), pp. 550.] If the objection at the basis for the argument, as one poses it in the general theory of relativity, demands entire treatises in which it is still not resolved then that theory must still leave much to be desired in terms of clarity.

exists that is more precise or no otherwise-new knowledge comes to our aid. To call upon an especially radical theory like the theory of relativity for that, which (as was shown) finds no reliable support in experiment, might seem only arbitrary, and therefore outlandish, up to now. The fact that surprises might still be expected in regard to gravitation is still self-explanatory then. However, one might expect that the insights might be of a simpler kind, as all great insights into the behavior of inanimate nature have been up to now, unlike the theory of relativity.

The redshift of the spectral lines of large celestial bodies remains as an experimentally-testable prediction of the theory of relativity, since it still seems particularly inherent to it up to now. However, that prediction is known to not have been confirmed with any certainty until today. Rather, one can say that it is contradicted by the best measurements of it that currently exist ([1]).

2. – Excerpts from Soldner's treatise from March 1801.

> "With the current, so very complete, state of astronomical practice, it is always necessary to develop all circumstances that might influence the true or mean position of a celestial body from the theory, that is, from the general properties and interactions of matter, in order to be able to infer the uses that it might have from a good observation,"
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> "It is certainly true that one will already become aware of considerable deviations from an assumed rule by observations and chance, as was case with, e.g., the aberration of light. However, there can be deviations that are so small that it is difficult to decide whether they are actual deviations or errors in observation. There can also be deviations that are indeed considerable but combined with quantities that even the most practiced observer can overlook, since one will still not arrive at pure truth from their mean."
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> "The deflection of a light ray from a straight line when it passes close to a celestial body, and therefore the attraction that it exhibits, can probably be included in the latter category. That is because since one easily sees that this deflection must be greatest when the light ray arrives in a horizontal direction, as seen from the surface of the attracting body, and zero when it arrives perpendicular to it, the magnitude of the deflection must be a function of the elevation. However, since the refraction of rays is a function of the elevation, those two magnitudes will combine with each other, and it would then be possible that the maximum deflection might amount to several seconds without it having been hitherto possible to average it over observations."
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> "Those are roughly the considerations that moved me to ponder the perturbation of light rays in more detail, which no one has investigated yet, to my knowledge."
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> Before I go on to that investigation itself, I would like to make a few general remarks that will lighten the calculations. Since I would first like to determine only

([1]) See the citations and remarks that are compiled in *Über Relativitätsprinzip, Äther, Gravitation*, 1921 (pp. 43) and "Über Äther und Uräther" (**Stark**'s Jahrb., pp. 355).

the maximum of such a deflection, I will let the light ray pass horizontally to the surface of the attracting body at the location of the observer, or I shall assume that the heavenly body that emits it appears to be on the rise. For the sake of convenience in the examination, I will assume that the light ray does not arrive at the location of observation but goes from it. One will easily see that this is entirely independent of the determination of the figure of the path. Furthermore, if a light ray arrives at a point on the surface of the attraction body in a horizontal direction and then its course, which once more initially horizontal, continues further then one will easily note that it will describe the same curved line by that further continuation that it had followed up to that point. If one then lays a straight line through the position of the observation and the center of the attracting body then that line will be the principal axis of curvature for the path of the light, since two completely congruent branches of the curved line will be described below that line and above it." (*loc. cit.*, pp. 161-163)

The calculation then follows, which is supported by a simple construction whose repetition can be passed over, since it is identical with the calculation of the path of any unperturbed comet with a given velocity of perihelion. I have not found anything missing from that calculation. Its final result is the theorem (pp. 167):

"Thus, when a light ray passes close to a celestial body, the attraction of that body will require the ray to describe a hyperbola whose concave side points to the attracting body, instead of a straight line. One then has the equation:

$$\tan \varpi = \frac{2g}{v\sqrt{v^2 - 4g}} \text{"} \qquad \text{(pp. 169),}$$

in which (pp. 163) ϖ is one-half the angle of the total deflection of the curvature of the ray, $2g / r^2$ is the force with which a unit mass at a distance r from the center of the heavenly body will be attracted, and v is the speed of light. The radius of the heavenly body is taken to be the unit of length throughout that (pp. 164). ([1]) If one introduces the usual unit of length that is independent of the size of the heavenly body and one then denotes is radius by R, its mass by M, the gravitational constant by K, and the speed of light ([2]) by c then **Soldner**'s equation above will assume the form:

([1]) The meanings of the symbols that are given in that way are not convenient to a transparent numerical evaluation of the final result. Perhaps that is explained by the fact that the numbers that **Soldner** gave for the Earth and Sun (see what follows above) do not seem to be entirely correct.

([2]) More precisely, **Soldner** introduced the speed of light that exists very close to M, i.e., as his own calculation implies, a speed of light that is increased by a ratio of $1 : \sqrt{1 + 2KM / c^2 R}$. Should c mean the unaltered vacuum speed of light then the – sign inside the square root in the formula for tan ϖ must be replaced with a +. However, the ratio of the two speeds of light differs from 1, e.g., at the solar surface, by only 0.000 002.

$$\tan \omega = \frac{K\,M}{c\,R\sqrt{c^2 - 2KMR}}\,,$$

or to a very good approximation, $\omega = KM / c^2 R$, which will make the total deflection $\beta = 2\omega$ of the ray ([1]):

$$\beta = \frac{2KM}{c^2 R}\,.$$

(**Einstein** gave the same value for β using the theory of relativity of 1911 and twice that value using the "general" theory of relativity.)

Soldner further calculated the deflection for Earth, the Moon, and the Sun from the formula that he obtained and found it to be very small. For the Sun, $\omega = 0.84''$ (pp. 170) (whereas, in reality, from his formula $2\omega = \beta = 0.84''$, which also seems to agree with experiments, to the extent that they suggest today, as was explained in the preliminary remarks). He then said (pp. 171):

> "It is thus revealed that one does not need to take into account the perturbation of the light ray by attracting celestial bodies, at least with the present state of astronomical practice."

In conclusion, he remarked (pp. 171, 172):

> "Hopefully, no one will find it conceivable that I am treating a light ray as a virtual gravitating body. That is because one sees that light rays possess the absolute properties of matter from the phenomenon of aberration, which is possible only because the light rays are actually material, and furthermore, one can think of nothing that exists and acts upon our senses that does not have the property of matter." ([2])
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> "Moreover, I do not believe that is necessary to excuse me to for the fact that I am making the present treatise known, since the result within it is that all perturbations are unnoticeable. That is because it should be almost as important for us to know what exists in theory, but has no noticeable influence in practice, as it is to be interested in the things that actually do have some influence in practice. Our insights will be expanded by both of them to the same extent."

Heidelberg, Radiological Institute, 20 July 1921.

([1]) As is quite obvious, the equation is true not only when R means the radius of the mass M, but also for an arbitrarily small distance R of the passing light ray from the center of the mass M.

([2]) One probably cannot agree with **Soldner** in that statement. In fact, he himself found it advisable to appeal to an ancient classical poet that he cited (Lucretius, *de rer. nat*. I, pp. 431), which is always a sign that something is not entirely correct. Today, we have **Hasenöhrl** as a better witness for the alleged gravity of light.