"Die Wechselwirkungskräfte in der Elektrodynamik und in der Feldtheorie der Kernkräfte (Teil II and III)," Helv. Phys. Acta **11** (1938), 299-328.

Forces of interaction in electrodynamics and in the field theory of nuclear forces. (Parts II and III)

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(6 April 38)

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Abstract

Part II. The method for calculating the interaction between two charges that was given in Part I will be generalized to a four-potential. A positive-definite field energy can also be generated by an auxiliary condition for a field whose particle does not possess a rest mass. The general form of the interaction between two spinor particles that is mediated by that field will be given.

Part III. The equation of motion of the nuclear force field and the spinor field of matter will be derived quantum-mechanically from a Hamiltonian operator. It shows that operators satisfy the continuity equation. If one demands the conservation of electric charge and the conservation of the density of heavy particles then essentially four different fields will be possible. Their particles are: charged and uncharged light particles, with a mass whose Compton wave length corresponds to the range of the forces between massive particles, and charged and uncharged heavy particles whose mass is larger than that of the proton (neutron, resp.).

The empirical form of the force between the neutron and the proton will be implied only when one also assumes the existence of two types of particles (anti-particles) for the uncharged light particles. By contrast, the conjecture that a theory with no anti-neutrino, in the Majorana sense, is possible is confirmed.

Part II

7. Generalization of the theory to a four-potential.

In Part I [**12**], it was shown that the mutual perturbations between two material particles in the first approximation could be calculated from a Hamilton function in which one part of the field-matter interaction could be replaced with a certain matter-matter interaction term. That term has the following form: Operator of the retarded charge of the one particle at the position of the other one times the charge of the other particle.

In the derivation, we restricted ourselves to the scalar case.

However, such a scalar field will give an interaction between the nuclear constituents (viz., protons and neutrons) that has the wrong sign and the wrong spin dependency: The scalar part of (4.22) is positive, and thus gives a repulsion.

The field of a four-potential shall then be treated as a generalization of that.

In the present Part II, we shall then first discuss the question of the sign of the field energy, and then we shall calculate the retarded potential.

Formally, that generalization comes about simply when one appends an index i (i = 0, 1, 2, 3) to the quantities A (potential), J (charge), and S_k (polarization): A_i, J_i, S_{ik}, resp.

The formulas of Part I will once more be true, word-for-word, when one replaces the terms that are bilinear in A, P, J, and S with corresponding sums over i (from 0 to 3).

Thus, for example, one replaces:

$$A^*A \text{ with } \sum_i \varepsilon_i A_i^* A_i = (A, A)$$

(and analogously for P^*P)

$$PS_0 \text{ with } \sum_i \varepsilon_i P_i S_{i0}$$

$$\left(\frac{\partial A^*}{\partial \mathbf{x}}, \frac{\partial A}{\partial \mathbf{x}}\right) \text{ with } \sum_{k=1}^{3}\sum_{i=0}^{3} \varepsilon_i \frac{\partial A_i^*}{\partial x_k}\frac{\partial A_i}{\partial x_k}, \text{ etc.}$$

ε_i has the value + 1 for i = 1, 2, 3 and the value – 1 for i = 0.

The commutation relations (3.2) are replaced with:

$$[P_i(\mathbf{x}), A_{i'}(\mathbf{x})] = \delta_{ii'}(h/i)\,\delta(\mathbf{x}-\mathbf{x}'). \tag{7.1}$$

Due to the appearance of ε_i, equations (2.3) [(2.7) and (2.8), resp.] are replaced with:

$$\dot{A}_i = \frac{\delta H}{\delta P_i} = \varepsilon_i (8\pi c^2 P_i^* - 4\pi c\, S_{i0}), \tag{7.2}$$

$$\dot{P}_i^* = -\frac{\delta H}{\delta A_i^*} = \varepsilon_i \left[\frac{1}{8\pi}(\Delta - l^2)A_i + \tfrac{1}{2}\left(J_i - \sum_{k=1}^{3}\frac{\partial S_{ik}}{\partial x_k}\right)\right]. \tag{7.3}$$

For that reason, formula (3.17) will likewise take on the factor ε_i in the second term $[P(\mathbf{x})^* -]$. That will have the consequence that the commutation relations for the explicitly time-dependent operator $A_i(x)$ take on the form:

$$[A_i(x)^*, A_k(y)] = -2\,\frac{hc}{i}\,\varepsilon_i\,\delta_{ik}\,D(x-y). \tag{7.4}$$

The ultimate forms for the interaction operators (4.22), (4.23), and (4.24) change only insofar as they are replaced with a sum over i (with ε_i).

8. Generating a positive-definite energy density by an auxiliary condition.

We likewise convert the energy density operator of the radiation field (2.5) by the unitary transformation (3.14). That means that in (2.5), the A_i (**x**) are replaced with the explicitly time-dependent operators A_i (x), and the P_i (**x**), with the time derivatives of the A_i (x). One can then write the energy density as the sum of the energy densities of the individual potential components:

$$\mathfrak{W} = \sum_i \varepsilon_i \, \mathfrak{W} \, (A_i), \tag{8.1}$$

with:

$$\mathfrak{W} \, (A) = \frac{1}{8\pi} \left(\sum_k \frac{\partial A^*}{\partial x_k} \frac{\partial A}{\partial x_k} + l^2 A^* A \right). \tag{8.2}$$

The expression (8.2) is always positive, since the factor ε_k *does not* appear (the sum over k *is not* a scalar product then). By contrast, the energy density (8.1) contains a negative summand for $i = 0$. In electrodynamics, the positive-definite energy can be generated by the homogeneous auxiliary condition (6.1). By contrast, in the case of $l = 0$, that auxiliary condition will no longer commute with its complex conjugate. However, if we introduce a scalar component B that likewise satisfies a wave equation (1.1) with the same l, along with the four potential components A_i , then the auxiliary condition (6.2) at the position y will commute with its complex conjugate at the position x. One finds that:

$$\begin{aligned} [(6.2)^*, (6.2)] \; &= \sum_i \varepsilon_i \frac{\partial}{\partial x_i} \frac{\partial}{\partial y} D \, (x - y) + l^2 \, D \, (x - y) \\ &= - \, (\Box - l^2) \, D \, (x - y) = 0. \end{aligned}$$

The final equation results because the D-function owes its provenance to the homogeneous wave equation (viz., the difference of the advanced and retarded potential).

The auxiliary condition can also be written in the following form:

$$\frac{\partial A_0}{\partial x_0} y = (- \operatorname{div} \mathbf{A} - l \, B) \; \psi. \tag{8.3}$$

If one applies the operator $\partial A_0 / \partial x_0$ and considers that it commutes with the operator on the right-hand side then the identity will follow from (8.3) and the complex conjugate condition (6.2) that:

$$- \frac{\partial A_0^*}{\partial x_0} \frac{\partial A_0}{\partial x_0} \psi = [- \operatorname{div} \mathbf{A}^* \operatorname{div} \mathbf{A} - l \, (B^* \operatorname{div} \mathbf{A} + \operatorname{div} \mathbf{A}^* \cdot B) - l^2 \, B^* B] \; \psi, \tag{8.4}$$

which will eliminate one of the negative terms in (8.1).

We write $- f + 2f$ for the term $-$ grad A_0^* grad $A_0 - l^2 \, A_0^* A_0 = f$ and convert the term $2f$ by partial integration into:

$$\int d\mathbf{x}^3\, 2f = \int d\mathbf{x}^3\, [\, A_0^*\,(\Delta - l^2)\, A_0 - A_0^*\,(\Delta - l^2)\, A_0].$$

If one considers that A_0 satisfies the homogeneous wave equation then it will follow from the auxiliary condition that:

$$A_0^*\,(\Delta - l^2)\, A_0\ \psi = A_0^*\,\frac{\partial^2 A_0}{\partial x_0^2}\,\psi = -\,A_0^*\left(\operatorname{div}\frac{\partial \mathbf{A}}{\partial x_0} + \frac{\partial B}{\partial x_0}\right)\psi = 0.$$

We convert the term that contains div linearly by partial integration. The integral of the energy density with the integrand $\mathfrak{W}'$ can then be written:

$$\int d\mathbf{x}^3 \left(\sum_i \varepsilon_i\, \mathfrak{W}(A_i) + \mathfrak{W}(B)\right)\psi = \int d\mathbf{x}^3\, \mathfrak{W}'\ \psi,$$

$$\mathfrak{W}' = \frac{1}{8\pi}\left[(\operatorname{rot}\mathbf{A}^*, \operatorname{rot}\mathbf{A}) + \left(\operatorname{grad} A_0^* + \frac{\partial \mathbf{A}^*}{\partial x_0}, \operatorname{grad} A_0 + \frac{\partial \mathbf{A}}{\partial x_0}\right)\right.$$
$$\left. + \left(lA_0^* - \frac{\partial B^*}{\partial x_0}\right)\left(lA_0 - \frac{\partial B}{\partial x_0}\right) + (l\,\mathbf{A}^* + \operatorname{grad} B^*, l\,\mathbf{A} + \operatorname{grad} B)\right].$$

The auxiliary condition (6.2) [or (8.3)] will always yield a positive energy density then.

If we now introduce the new four-vector of the potential:

$$\Phi_i = A_i + \varepsilon_i\, l^{-1}\,\frac{\partial B}{\partial x_i} \tag{8.6}$$

and the anti-symmetric field strength tensor:

$$F_{ik} = \varepsilon_i\,\frac{\partial \Phi_k}{\partial x_i} - \varepsilon_k\,\frac{\partial \Phi_i}{\partial x_k} = \varepsilon_i\,\frac{\partial A_k}{\partial x_i} - \varepsilon_k\,\frac{\partial A_i}{\partial x_k} \tag{8.7}$$

then when we employ the three-dimensional vector:

$$\mathbf{F} \quad \text{with the components } (F_{01}, F_{02}, F_{03})$$

and the three-dimensional pseudo-vector:

$$\vec{\mathbf{F}} \quad \text{with the components } (F_{23}, F_{31}, F_{12}),$$

the energy density can be brought into the form:

$$\mathfrak{W}' = \frac{1}{8\pi}[(\bar{\mathbf{F}}^*, \bar{\mathbf{F}}) + (\mathbf{F}^*, \mathbf{F}) + l^2(\bar{\mathbf{\Phi}}, \mathbf{\Phi}) + l^2\Phi_0^*\Phi_0]. \tag{8.8}$$

The energy density is then positive-definite, and for $l = 0$ (i.e., B vanishes identically) and real field strengths, it will go over to the energy expression of Maxwellian electrodynamics (*).

9. The auxiliary condition in the absence of charges.

In the absence of charges, the function ψ must satisfy not only the auxiliary condition, but also the Schrödinger equation (3.13) (we shall always write K for K'' in what follows):

$$\left(K + \frac{h}{i}\frac{\partial}{\partial t}\right)\psi = 0. \tag{9.1}$$

Thus, K is the Hamilton operator for matter here, in which the time-dependent potential appears explicitly.

If one writes K in the form of an integral over $d\mathbf{x}^3$ [the field-matter interaction term has the form of, for example, the integral over the expression (3.22)], and introduces the Schrödinger time $x_0 = ct$, then one will compute from the commutation relations (7.4) that:

$$\left[K + \frac{hc}{i}\frac{\partial}{\partial x_0}, A_i(y)\right] = -2\,\frac{hc}{i}\int d\mathbf{x}^3 \left(\frac{\delta K}{\delta A_i^*}D(x-y) + \frac{\delta K}{\delta \frac{\partial A_i^*}{\partial x_0}}\frac{\partial D(x-y)}{\partial x_0}\right)$$

[Naturally, $\partial / \partial x_0$ commutes with $A_i(y)$.] An analogous expression follows for B. The charge and polarization quantities [$\mathfrak{K}$ is the integrand of K and has the form (3.22), for example] will be defined as follows:

$$-2\,\frac{\partial \mathfrak{K}}{\partial B^*} = J^B, \qquad -2\,\frac{\partial \mathfrak{K}}{\partial \frac{\partial B^*}{\partial x_i}} = S_i^B, \tag{9.2}$$

(*) $\mathfrak{W}$ is also, in fact, the 0-0 component of the tensor:

$$\frac{1}{8\pi}\left(\sum_m \varepsilon_m F_{im}^* F_{km} + l^2\Phi_i^*\Phi_k + \text{conj.}\right) - \varepsilon_i\,\delta_{ik}\,\mathfrak{L},$$

in which $\mathfrak{L}$ means Proca's Lagrange density function (11.8).

$$-2\,\frac{\partial \mathfrak{K}}{\partial A_i^*} = J_i\,\varepsilon_i\,, \qquad\qquad -2\,\frac{\partial \mathfrak{K}}{\partial \dfrac{\partial A_i^*}{\partial x_k}} = S_{ik}\,\varepsilon_i\,.$$

We obtain the following commutation relation:

$$\left[K+\frac{hc}{i}\frac{\partial}{\partial x_0},\left(\frac{\partial}{\partial y},A\right)+l\,B\right]$$
$$=\frac{hc}{i}\int d\mathbf{x}^3\left\{[\operatorname{div}(\mathbf{J}-l\,\mathbf{S}^B)+l\,(J^B-l\,T)]\,D(x-y)-(J_0-l\,S_0^B)\frac{\partial D(x-y)}{\partial x_0}\right\}. \quad (9.3)$$

The argument of the charge and polarization operators is **x**. One makes the restricting assumption:

$$S_{ik}=\varepsilon_i\,\delta_{ik}\,T+\,S'_{ik}\,, \qquad S'_{ik}=-S'_{ki} \quad (9.4)$$

for the tensor S_{ik} .

In order for the functional ψ to simultaneously fulfill the auxiliary equation and the Schrödinger equation, the two operators "auxiliary condition and $K + h\,\partial / i\,\partial t$ " must commute. However, according to (9.3), that is not the case.

We then add an inhomogeneous term to the auxiliary condition; i.e., we write:

$$\left[\left(\frac{\partial}{\partial y},A\right)+l\,B+\int d\mathbf{x}^3 J'_0(\mathbf{x})\,D(x-y)\right]\psi=0, \quad (9.5)$$

where J'_0 is the 0-component of a combined charge vector:

$$J'_i\,=J_i-l\;S_i^B\,. \quad (9.6)$$

One then has:

$$\left[K+\frac{hc}{i}\frac{\partial}{\partial x_0},\int d\mathbf{x}^3 J'_0\,D(x-y)\right]=\frac{hc}{i}\int d\mathbf{x}^3\left[(-\operatorname{div}\mathbf{J}'+R')D(x-y)+J'_0\frac{\partial D(x-y)}{\partial x_0}\right]. \quad (9.7)$$

The scalar R' is defined to be the four-divergence of J'_i:

$$\frac{i}{hc}[K,J'_0]+\operatorname{div}\mathbf{J}'\,=R'=\frac{1}{c}\dot{J}'_0\,+\operatorname{div}\mathbf{J}'. \quad (9.8)$$

A comparison of (9.3) and (9.7) shows that the inhomogeneous auxiliary condition (9.5) will be preserved in the course of time when the operator identity:

$$\dot{J}'_0\,+c\,\operatorname{div}\mathbf{J}'=l\,(J^B-l\,T) \quad (9.9)$$

is fulfilled identically.

Furthermore, J_0' must commute with the potential operator and with $J_0'^*$ in order for the inhomogeneous auxiliary condition to remain compatible with itself and its complex conjugate.

B and *l* vanish in electrodynamics. Moreover, the four-divergence of the electric current also vanishes. (9.9.) is then fulfilled, and the auxiliary condition (9.5) is possible. As is known, it leads to Maxwell's equations.

It will be shown that J_0' does not commute with $J_0'^*$ for the nuclear force. An auxiliary condition in inhomogeneous form is not possible then. The single solution that fulfills (9.9) consists of the one that makes the four-vector J_i' vanish, and therefore, its four-divergence, as well, and $J^B = l\,T$.

It then follows from the identical vanishing of the two sides of equation (9.9) and the definition of the charge and polarization quantities (9.2), (9.4), (9.6), and (9.8) that $\mathfrak{K}$ can depend upon only the following combinations of the scalar potential *B* and the four-potential A_i :

1. The scalar:
$$\left(\frac{\partial}{\partial x}, A\right) + l\,B = \left(\frac{\partial}{\partial x}, \Phi\right).$$

2. The potentials Φ_i that were defined in (8.6).

3. The field strengths F_{ik} , due to the antisymmetry of the tensor (9.4).

The scalar dependency that was mentioned in 1 vanishes because of the auxiliary condition, which is now homogeneous and commutes with *K*. (Naturally, a further dependency upon another scalar field *C* that employs *B*, independently of the generation of Φ_i , can be introduced with no contradictions.)

We then write the commutation relations for these new quantities:

$$\begin{aligned}
[\,\Phi_i^*(x)\,,\,\Phi_k(y)] &= -2\,\varepsilon_k \frac{hc}{i}\left(\delta_{ik} - \varepsilon_k \frac{1}{l^2}\frac{\partial^2}{\partial x_i\,\partial x_k}\right) D\,(x-y), \\
[\,F_{ik}^*(x)\,,\,\Phi_l(y)] &= -2\,\frac{hc}{i}\varepsilon_i\varepsilon_k\left(\delta_{kl}\frac{\partial}{\partial x_i} - \delta_{il}\frac{\partial}{\partial x_k}\right) D\,(x-y).
\end{aligned} \tag{9.10}$$

As before, the starred quantities commute with the un-starred ones.

Due to the definition of the Φ_i , the auxiliary condition will assume the form that one recalls from vacuum electrodynamics:

$$\left(\frac{\partial}{\partial x}, \Phi\right)\psi = 0. \tag{9.11}$$

Since the Φ_i, like the A_i, and the B satisfy the homogeneous wave equation:

$$(\Box - l^2)\,\Phi_i = 0, \tag{9.12}$$

Proca's equation [**10**]:

$$\left(\sum_k \frac{\partial F_{ki}}{\partial x_k} - l^2 \Phi_i\right) \psi = 0 \tag{9.13}$$

will follow for the field strengths that were defined in (8.7), due to the auxiliary condition (9.11).

It will go to Maxwell's equations for the vacuum when $l = 0$.

We can consider one of equations (9.13) to be an auxiliary condition with precisely the same right by which we treated (9.11) as an auxiliary condition and derived the four equations (9.13), and develop the other three equations [and equations (9.11)].

In electrodynamics, one can introduce the Coulomb interaction by *eliminating the auxiliary condition.* The field will have only two transversal components then. Such an elimination will be impossible for a non-vanishing rest mass ($l \neq 0$). By contrast, the auxiliary condition can be satisfied identically by a *definition of the operators:*

One chooses equation (9.13) for $i = 0$ to be the auxiliary condition and considers Φ_1, Φ_2, and Φ_3 to be *independent operators* that satisfy the commutation relations (9.10), and the F_{ik} (i, k = 1, 2, 3) to be *operators that were derived* from them. On the other hand, one regards the operators:

$$\Pi_i = \frac{1}{8\pi c} F^*_{i0} \qquad (i = 1, 2, 3) \tag{9.14}$$

as further *independent quantities.* From (9.10), one will have for $x_0 = y_0$ that:

$$[\Pi_i(x), \Phi_k(y)] = \frac{h}{i}\,\delta_{ik}\,\delta(\mathbf{x} - \mathbf{y}) \qquad (x_0 = y_0). \tag{9.15}$$

The operators F_{k0} can then be expressed in terms of the Π^*_i now, by using $(9.14)^*$. If one now likewise defines Φ_0 to be the *derived operator* in the form:

$$\Phi_0 = 8\pi c\, l^{-2}\, \mathrm{div}\, \mathbf{\Pi}^* \tag{9.16}$$

then the last equation ($i = 0$) of (9.13) will be, in fact, fulfilled identically when one considers it to be the auxiliary condition.

With the use of the independent operators $\mathbf{\Phi}$ and $\mathbf{\Pi}$, the energy density (8.8) is written:

$$\mathfrak{W}' = \frac{1}{8\pi}\,[l^2\,(\mathbf{\Pi}^*, \Phi) + (\mathrm{rot}\,\mathbf{\Pi}^*, \mathrm{rot}\,\mathbf{\Pi}) + 8\pi c^2\,[(\mathbf{\Pi}^*, \mathbf{\Pi}) + l^{-2}\,\mathrm{div}\,\mathbf{\Pi}^* \cdot \mathrm{div}\,\mathbf{\Pi}]. \tag{9.17}$$

We can go down two paths in order to derive the field equations:

1. Go over to a "one-sided" theory; i.e., reverse the formalism that led to equation (3.5). The Hamilton function in (3.5) will then once more contain a field component. That is nothing but (9.17), except that now all of the operators $F(x)$ [$= F''(\mathbf{x}, x_0)$] that depend upon time explicitly and are coupled by means of the transformation (3.14) will be replaced with operators $F(\mathbf{x})$ that are not explicitly time-dependent. That can come about formally quite simply when one sets $x_0 = 0$ everywhere. The three Φ_i and their derivatives all commute with each other. The same thing is true for the Π_i^* and their derivatives. By contrast, Π_i and Φ_i obey the relation (9.15); i.e., they are canonically conjugate. The matter part K remains the same, except that here the field quantities $F(x)$ are replaced with $F(\mathbf{x})$. All of this corresponds to KEMMER's formalism [**11**] precisely. More details on this will be given in paragraph 11.

2. The present multi-time theory, on the basis of the relation:

$$\frac{1}{c}\dot{F}(y) = \left(\frac{\partial F(y)}{\partial y_0} + \frac{i}{hc}[K, F(y)]\right)_{y_0 = x_0} . \tag{9.18}$$

We can consider the auxiliary condition in its original form; i.e., consider *all four* Φ_i *to be independent operators.* In place of Φ_0, one must then define a $\bar{\Phi}_0$:

$$\bar{\Phi}_0 = A_0 - \frac{1}{c}\dot{B} . \tag{9.18a}$$

The auxiliary condition in terms of $\bar{\Phi}_0$ then reads:

$$\left[\operatorname{div}\boldsymbol{\Phi} + \frac{1}{c}\dot{\bar{\Phi}}_0 - 4\pi l^{-2}\left(\operatorname{div}\mathbf{J} + \frac{1}{c}\dot{J}_0\right)\right]\psi = 0. \tag{9.19}$$

After applying the rule (9.18) twice, the field equations for the components Φ_1, Φ_2, and Φ_3 will read:

$$(\Delta^2 - l^2)\,\Phi_i - \frac{1}{c^2}\ddot{\Phi}_i = -4\pi\left[J_i - \sum_{k=1}^{3}\frac{\partial S'_{ik}}{\partial x_k} - \frac{1}{c}\dot{S}'_{i0} - \frac{\partial}{\partial x_k} l^{-2}\left(\operatorname{div}\mathbf{J} + \frac{1}{2}\dot{J}_0\right)\right]. \tag{9.20}$$

One obtains an entirely analogous equation for the $\bar{\Phi}_0$ that was defined in (9.18a), except that $\partial / \partial x_0$ must be replaced with $-1/c$ times the time derivative (.) of the following quantities.

If one then introduces the corresponding field strengths:

$$F_{ik} = \frac{\partial \Phi_k}{\partial x_i} - \frac{\partial \Phi_i}{\partial x_k} \qquad (i, k = 1, 2, 3), \tag{9.21}$$

$$\bar{F}_{i0} = \frac{\partial \bar{\Phi}_0}{\partial x_i} + \frac{1}{c}\dot{\Phi}_i ,$$

then if one consider the auxiliary condition (9.19), one can write (9.20) as:

$$\left[\sum_{k=1}^{3} \frac{\partial \bar{F}_{ki}}{\partial x_k} + \frac{1}{c}\dot{\bar{F}}_{0i} - l^2 \bar{\Phi}_i + 4\pi \left(J_i - \sum_{k=1}^{3} \frac{\partial S'_{ki}}{\partial x_k} - \frac{1}{c}\dot{S}'_{i0} \right) \right] \psi = 0. \tag{9.22}$$

For $l = 0$, this will correspond to Maxwell's equations in the absence of charge.

10. The interaction term of the four-potential.

In order to derive the interaction terms, one can either employ the method of paragraph 4 (Part I) explicitly, or recall that the Φ_i can be expressed in terms of the A_i and B (8.6). Since the repeatedly-mentioned expression "retarded potential of the first particle at the location of the second one times the charge of the second one" is true for the latter, it will also be true for the Φ_i . The expressions on the right-hand side of the field equations (9.20) are then generally chosen to be the charges.

We would like to restrict ourselves to the static case. Let it be defined by the fact that:

1. One neglects all quantities $\partial / \partial x_0$ [or the (.) in (9.20)] (neglect the retardation).

2. One likewise neglects the J_i ($i \neq 0$) and the S'_{0k} (neglect the motion).

According to (0.2) and (9.20):

$$\begin{aligned} \Phi^r(x)_0 &= \int d\mathbf{y}^3 J_0^r(\mathbf{y})\, v(\mathbf{x}-\mathbf{y}), \qquad v(\mathbf{x}) = \frac{e^{-l|\mathbf{x}|}}{|\mathbf{x}|}, \\ \Phi^r(x)_i &= \int d\mathbf{y}^3 \sum_{k=1}^{3} S_{ik}^r \frac{\partial}{\partial y_k} v(|\mathbf{x}-\mathbf{y}|) \end{aligned} \tag{10.1}$$

is then the four-vector of the potential that the r^{th} particle generates at the position $\mathbf{x}$ at the Schrödinger $t = x_0 / c$. We shall write S_{ik} for S'_{ik} in what follows. (Note that in this approximation the potentials A_i will be identical with the Φ_i .)

According to (4.22) (one also considers the remark), the expression for the interaction will be:

$$U^{rs} + U^{sr} = \tfrac{1}{2} \int d\mathbf{x}^3\, d\mathbf{y}^3 \left\{ \left(J_0^{2*}(\mathbf{x}) J_0^r(\mathbf{y}) + \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} S_{ik}^{s*}(\mathbf{x})\, S_{ik}^r \frac{\partial^2}{\partial x_m\, \partial x_k} \right) v(\mathbf{x}-\mathbf{y}) + \text{conj.} \right\}.$$

With the introduction of the three-dimensional pseudo-vector $\bar{S}$ (S_{23}, S_{31}, S_{12}) and the vector operator $\vec{\nabla}$, the last term can be converted into the form:

$$(\bar{S}^s \times \vec{\nabla}, \bar{S}^r \times \vec{\nabla}) = (\bar{S}^s, \bar{S}^r)\Delta - (\bar{S}^s, \vec{\nabla})(\bar{S}^r, \vec{\nabla}). \tag{10.2}$$

If one describes the charges (i.e., matter) by Dirac's theory then:

$$J_i^r(\mathbf{x}) = f\, e\, \tau^r \alpha_i^r\, \delta(\mathbf{x} - \mathbf{q}^r), \tag{10.3}$$

in which f is a constant with the dimensions of a number, e is the elementary electric charge, α_i^r are Dirac's velocity operators (i.e., matrices) of the r^{th} particle, τ^r are certain (generally non-Hermitian) matrix operators (viz., isotopic spin) that commute with the α_i^r, and $\mathbf{q}^r$ is the position vector of the r^{th} particle.

Correspondingly, the antisymmetric tensor will be:

$$\begin{aligned} &\text{For } i,\ k \neq 0: \quad S_{ik}^r(\mathbf{x}) = +\, i\, g\, e \frac{1}{l}\, \tau^r \beta^r \alpha_i^r\, \alpha_k^r\, \delta(\mathbf{x} - \mathbf{q}^r), \\ &\text{For } i,\ = 0: \quad S_{0k}^r(\mathbf{x}) = -\, i\, g\, e \frac{1}{l}\, \tau^r \beta^r \alpha_k^r\, \delta(\mathbf{x} - \mathbf{q}). \end{aligned} \tag{10.4}$$

g is likewise a constant with the dimensions of a number in this. β^r is the Dirac β-matrix of the r^{th} particle. If one chooses the spinors such that $\alpha_0 = 1$ and:

$$\beta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

then one will see that only α_0 and $\beta\, \alpha_i\, \alpha_k$ are "diagonal" matrices. Under the reduction to the "large components" of the Dirac function, the non-diagonal matrices will first contribute in the approximation where "kinetic energy equals rest mass times c^2." Thus, the things that were neglected in 2 will be justified. One can then replace β with 1 (for positive energies) and introduce the matrices σ_{ik} for $i\, \beta\, \alpha_i\, \alpha_k$. The interaction will then be (*):

$$\begin{aligned} U^{rs} + U^{sr} = \frac{e^2}{2} &(\tau^r \tau^{s*} + \tau^{r*} \tau^s)\, (|f|^2 + |g|^2\, (\vec{\sigma}^r, \vec{\sigma}^s) \\ &- |g|^2\, (\vec{\sigma}^r, \vec{\sigma}^s)\, (\vec{\sigma}^r, \vec{\nabla})(\vec{\sigma}^s, \vec{\nabla})\, v\, (|\mathbf{q}^r - \mathbf{q}^s|). \end{aligned} \tag{10.5}$$

Use was made of the relation:

(*) In *both* factors, $\vec{\nabla}$ means the taking the gradient with respect to $\mathbf{q}^r$ (or with respect to $\mathbf{q}^s$ both times).

$$(\Delta - l^2)\, v\,(\mathbf{x}) = -\,4\pi\,\delta(\mathbf{x}) \tag{10.6}$$

in this in order to eliminate the operator Δ.

Strictly speaking, a "local action term":

$$-\,4\pi\,|\,g\,|^2\,l^{-2}\,\delta(\mathbf{q}^r - \mathbf{q}^s) \tag{10.7}$$

will then appear in the last bracket, along with the terms (10.5).

This term will always appear (as in electrodynamics) when one completes the conversion (10.2). However, it will be easy to overlook when one first performs the operation $\vec{\nabla}$ on $v\,(\mathbf{x} - \mathbf{y})$; i.e., when one writes:

$$(\bar{S}^s \times \vec{\nabla}, \bar{S}^s \times \vec{\nabla})\, v\,(|\,\mathbf{z}\,|) = (\bar{S}^s \times \mathbf{z}, \bar{S}^r \times \mathbf{z}) \frac{1}{|\,\mathbf{z}\,|} \frac{\partial}{\partial\,|\,\mathbf{z}\,|} \left(\frac{1}{|\,\mathbf{z}\,|} v'(|\,\mathbf{z}\,|) \right) + 2(\bar{S}^r, \bar{S}^s) \frac{1}{|\,\mathbf{z}\,|} v'(|\,\mathbf{z}\,|)\,.$$

In this, $v'\,(|\,\mathbf{z}\,|)$ means the derivative of v with respect to $|\,\mathbf{z}\,|$. If one now converts the vector product term by means of the formula:

$$(\bar{S}^s \times \mathbf{z}, \bar{S}^r \times \mathbf{z}) \;=\; (\bar{S}^s, \bar{S}^r)\,|\,\mathbf{z}\,|^2 - (\bar{S}^r, \mathbf{z})(\bar{S}^s, \mathbf{z}) \tag{10.8}$$

then one will obtain (10.5) precisely *without* the perturbing expression (10.7). However, that is based upon the fact that we are counting a term of order $|\,\mathbf{z}\,|^2 / |\,\mathbf{z}\,|^5$ that will be singular for $\mathbf{z} = 0$ in the conversion of the vector product.

The same term also appears in the calculation of the spin-spin interaction of two electrons:

Namely, we proceed in the usual way: Calculate the Breit interaction terms by developing Møller's interaction in $1 / c^2$, and reduce the Dirac equation to the "large components," so the spin interaction will, in fact, appear in the form of $(\boldsymbol{\sigma}^s \times \vec{\nabla}, \boldsymbol{\sigma}^r \times \vec{\nabla})\,|\,\mathbf{z}\,|^{-1}$. Now, for the sake of simplicity, in the literature, the conversion (10.8) will be employed at that point at the latest, such that the supplementary term can be forgotten.

If one recalls the fact that the entirety of the interaction terms thus-obtained (with the exception of the Coulomb term (*)) can be employed only as *first-order perturbations* then the supplementary term will appear only as a small further splitting between singlet and triplet that is proportional to e^4 in the phenomena. However, if one would like to direct one's attention to the rigorous solution then it would lead to infinitely-deep terms in the attractive case.

We must therefore always remain aware of the fact that in the application of the interaction terms that are calculated in that way, we can consider them to be only *perturbations*, as opposed to the Coulomb term that appears in electrodynamics.

In fact, we need the *rigorous interaction* for the solution of the nuclear problem; i.e., finding its stationary states. If we would then like to compare the empirical laws of

(*) See page 8.

interaction with the results that are obtained here then we must preserve these additional local action terms in all cases.

Formula (10.5) then has, in fact, the correct sign and the correct spin dependency for the force between the neutron and proton, up to the grad term. The fact that the “isotopic spin” factor also can be given the desired form shall be shown in § 12 (Part III).

PART III

11. Equation of motion and Hamilton operator.

The nuclear force field will be described by several four-vectors Φ_i^s. Here, in contrast to the foregoing paragraphs, the upper index *s* no longer distinguishes the individual particles, but a number of different Proca fields whose operators commute with each other. Naturally, the six-vectors F_{ik}^s that are derived from them and the Φ_i^s themselves correspond to the overbarred quantities that follow from equation (9.18).

We describe the spinor field of matter in the form that is preferred by the author as a 16-component spinor field φ_μ^ν [**2**], [**13**], where each of the two indices goes from 1 to 4. The matrices α_i , β of Dirac's theory and the matrices σ_{ik} that originate in the "Pauli terms" shall act upon the lower index μ, while the matrices τ (and μ) that were introduced in paragraph 10 shall represent linear operators that act upon the upper index *v*. They then commute with the Dirac operator. (In the previously-cited papers, they were denoted by Ω and Δ.)

The equations of motion of the field will then read:

$$\sum_k \frac{\partial F_{ki}^s}{\partial x_k} - l_s^2 \Phi_i^s + 4\pi \left(J_i^s - \sum_k \frac{\partial S_{ki}^s}{\partial x_k} \right) = 0. \tag{11.1}$$

If one performs the operation ∂ / ∂x_i on the equations and adds them then it will follow that:

$$\left(\frac{\partial}{\partial x}, \Phi^s \right) - 4\pi l^{-2} \left(\frac{\partial}{\partial x}, J^s \right) = 0. \tag{11.2}$$

The equations of motion for matter read:

$$\left\{ -ihc \left(\alpha, \frac{\partial}{\partial x} \right) + mc^2 \beta\mu - \sum_r \frac{1}{2} [(j^{*r}, \Phi^r) + (s^{*r}, F^r) + \text{conj.}] \right\} \varphi = 0. \tag{11.3}$$

In this, J^r and S^r are abbreviations for the following vectors and tensors that one constructs from φ:

$$\begin{aligned} J_i^r &= \varphi^* j_i^r \varphi, \\ S_{ik}^r &= \varphi^* s_{ik}^r \varphi. \end{aligned} \tag{11.4}$$

The quantities j^r and s^r are constructed from the numerical factors f^r and g^r, the elementary quantum of electricity *e*, and the matrices that act upon the spinor indices in the following way:

$$\begin{aligned} j_i^r &= f^r e \, \alpha_i \, \tau^r, \qquad \alpha_0 = 1, \\ s_{ik}^r &= g^r e \frac{1}{l_r} \sigma_{ik} \, \tau^r, \end{aligned} \tag{11.5}$$

$$\sigma_{ik} = i\,\beta\,\alpha_i\,\alpha_k\,, \qquad \sigma_{0k} = -\,i\,\beta\,\alpha_k\,.$$

(s^{r*}, F^{r}) is the scalar product of the two six-vectors (i.e., = $\frac{1}{2}\sum_i\sum_k \varepsilon_i\varepsilon_k$ …), and μ is a matrix whose eigenvalues are the masses of the electron, neutrino, proton, and neutron (measured as multiples of the electron mass m). l_r are reciprocal lengths (= mass of the particle that is assigned to the field r times c / h), which the characteristic of each field.

In a classical field theory, one obtains the field equations for the nuclear field and matter from requiring that the space-time integral of a Lagrange density function $\mathfrak{L}$ must be an extremum. Its matter part has the form:

$$\mathfrak{L}\,(\varphi) = -\,\varphi^{*} \text{ times the expression (11.3).} \tag{11.6}$$

For the field part, one can write either:

$$\mathfrak{L}\,(\Phi) = \sum_s\sum_i \varepsilon_i\,\mathfrak{L}(A_i^s)_s + \mathfrak{L}(B^s)_s \tag{11.7}$$

(with $\Phi_i^s = A_i^s + l_s^{-1}\varepsilon_i\,\partial B/\partial x_i$), where the summands represent expressions of the form (2.1) [with indices i and s appended and with no matter part, which is already in (11.6)], or else the Proca expression:

$$\mathfrak{L}\,(\Phi^s)_s = -\,\frac{1}{8\pi}\,[(F^{s*}, F^{s}) + l_s^2\,(\Phi^{s*}, \Phi^{s})]. \tag{11.8}$$

One must consider equation (11.2) to be an auxiliary condition in the use of (11.7).

The transition to the Hamilton function comes about in the usual way (cf., e.g., paragraph 2). Generally, only the form (11.7) can be employed, since the time derivatives of Φ_0 do not appear in (11.8). If one employs (11.7) then the A_i^r, B^r, and their conjugate momenta will appear in the Hamilton function.

The formalism that was developed in paragraphs 8 and 9 (naturally, the "two-sided formulation" is not essential) allows one to write a Hamiltonian operator [according to formula (9.17) and the following remark 1] that depends upon only the three field quantities Φ_1^r, Φ_2^r, Φ_3^r, and their conjugate impulses, as well as upon their complex conjugate operators.

The Hamiltonian density reads:

$$\begin{aligned}\mathfrak{H} = {} & \frac{1}{8\pi}\sum_r\Big[l_r^2(\boldsymbol{\Phi}^{r*},\boldsymbol{\Phi}^{r}) + (\mathrm{rot}\,\boldsymbol{\Phi}^{r*},\mathrm{rot}\,\boldsymbol{\Phi}^{r})\Big] \\ & + 8\pi c^2\sum_r\Big[(\boldsymbol{\Pi}^{r*},\boldsymbol{\Pi}^{r}) + l_r^{-2}\mathrm{div}\,\boldsymbol{\Pi}^{r*}\mathrm{div}\,\boldsymbol{\Pi}^{r}\Big] \\ & + \varphi^{*}\left[-ihc\left(\boldsymbol{\alpha},\frac{\partial}{\partial\mathbf{x}}\right) + mc^2\beta\right]\varphi\end{aligned}$$

$$+\tfrac{1}{2}\sum_r\Big[-(\mathbf{J}^r,\mathbf{\Phi}^{r*})+J_0^r\,8\pi c\,l_r^{-2}\,\mathbf{div}\,\Pi^r+\text{conj.}$$
$$-\sum_{i=1}^{3}\sum_{k=1}^{3}S_{ik}^r\frac{\partial\Phi_i^{r*}}{\partial x_k}+\sum_k S_{0k}^r\,8\pi c\,\Pi_k^r+\text{conj.}\Big]$$
$$+\pi\sum_r\Big[l_r^{-2}(J_0^{r*}J_0^r+J_0^rJ_0^{r*})+\sum_k(S_{0k}^{r*}S_{0k}^r+S_{0k}^rS_{0k}^{r*})\Big]. \qquad (11.9)$$

Naturally, according to (11.6), the impulses that are conjugate to the φ_μ^ν are the complex conjugates $\varphi_\mu^{\nu*}$ times *ih*. The Hamilton function is bilinear in the nuclear and matter field quantities up to the last line, which contains the (symmetrized) terms in the remark (2.6a). Those terms are bi-quadratic in the φ.

One obtains the equations of motion classically and quantum-theoretically from the canonical equation:

$$\frac{\partial\Phi_i^r}{\partial x_0}=\frac{1}{c}\dot{\Phi}_i^r=\frac{1}{c}\frac{\delta H}{\delta\Pi_i^r}=\frac{i}{hc}[H,\Pi_i^r],\qquad i=1,2,3, \qquad (11.10)$$

and an analogous equation in which Φ_i^r is switched with Π_i^r, and in which a – appears in the third term of the equation. $\delta H/\delta\Pi_i^r$ means functional differentiation of the functional H (= volume integral of $\mathfrak{H}$) with respect to the function Π_i^r. The analogous relationship is true for φ:

$$-i\,h\,c\,\frac{\partial\varphi}{\partial x_0}=-i\,h\,\dot{\varphi}=-\frac{\delta H}{\delta\varphi^*}=[H,\varphi]. \qquad (11.11)$$

One must pay attention to the sequence of the terms in differentiation in the last equality in (11.11) since J_0^{r*} does not commute with J_0^r.

The last identities (11.10) and (11.11), which express the correspondence principle, are true when the nuclear field is quantized symmetrically:

$$[\Pi_i^r(\mathbf{x}),\Phi_k^s(\mathbf{y})]=\frac{h}{i}\,\delta_{rs}\,\delta_{ik}\,\delta(\mathbf{x}-\mathbf{y}), \qquad (11.12)$$

and when the symmetric (–) or anti-symmetric (+) quantization is true for the matter field:

$$\begin{aligned}&\varphi_\mu^\nu(\mathbf{x})\varphi_\lambda^\rho(\mathbf{y})\pm\varphi_\lambda^\rho(\mathbf{y})\varphi_\mu^\nu(\mathbf{x})=0,\\&\varphi_\mu^{\nu*}(\mathbf{x})\varphi_\lambda^\rho(\mathbf{y})\pm\varphi_\lambda^\rho(\mathbf{y})\varphi_\mu^{\nu*}(\mathbf{x})=\delta_{\mu\lambda}\delta_{\nu\rho}\delta(\mathbf{x}-\mathbf{y}).\end{aligned} \qquad (11.13)$$

All other operators commute with each other. Since Φ_0^r does not appear, the quantum-theoretic derivation of the field equations must be briefly sketched out:

1. Differentiating (11.10) with respect to time and eliminating $\dot{\Pi}_i^r$ from the canonically-conjugate equation leads to equations (9.20) for $i = 1, 2, 3$.

2. If one *defines* the operator:

$$\Phi_0^r = 8\pi c\, l_r^{-2} \operatorname{div} \mathbf{\Pi}^{r*} + 4\pi\, l_r^{-2} J_0 \tag{11.14}$$

then the fourth equation in (9.20) will follow from time-differentiating the canonically-conjugate equation (11.10) for $\dot{\mathbf{\Pi}}^{r*}$ [and eliminating $\dot{\Phi}_s^r$ by using (11.10)].

3. One obtains the relation (11.2) from the equation that is canonically-conjugate to (11.10) by taking the divergence and employing the definition (11.14).

4. One eliminates the four-divergence of the current on the right-hand side of equations (9.20) with the help of (11.2) that is obtained in that way and obtains the field equations in the form (11.1).

We make the following remarks about that:
The operators:

$$\Pi_i^r,\ \Phi_i^r,\ F_{ik}^r = \frac{\partial \Phi_k^r}{\partial x_i} - \frac{\partial \Phi_i^r}{\partial x_k} \qquad (i, k = 1, 2, 3)$$

are *pure field operators*, and therefore commute with the matter operators φ.
The operators Φ_0^r [defined by (11.14)] and the operators $F_{i0}^r = \partial \Phi_0^r / \partial x_i + \dot{\Phi}_i^r / c$ are mixed operators. They *do not commute* with the matter operators φ. The relation:

$$F_{i0}^r = 8\pi c\ \Pi_i^{r*} + 4\pi S_{0i} \tag{11.15}$$

follows directly from equations (11.10) as *the definition of* F_{i0}^r, in analogy with (11.14).
The equations of motion for the matter that follow from (11.11) have the following form:

$$\begin{aligned}
&\left\{ -ihc\left(\alpha, \frac{\partial}{\partial x}\right) + mc^2 \beta\mu + \tfrac{1}{2}\sum_r \Big[-(\mathbf{j}^{r*}, \mathbf{\Phi}^r) + j_0^{r*} 8\pi c^2 l_r^{-2} \operatorname{div} \mathbf{\Pi}^{r*} \right. \\
&\left. - \sum_{i=1}^{3}\sum_{k=1}^{3} s_{ik}^{r*} \frac{\partial \Phi_i^r}{\partial x_k} + \sum_{k=1}^{3} s_{0k}^{r*} 8\pi c\, \Pi_k^{r*} + \text{conj.} \Big] \right\} \varphi \\
&+ \sum_\nu \left[j_0^{r*} 2\pi\, l_r^{-2} (J_0^r \varphi + \varphi J_0^r) + \sum_k s_{0k}^{r*}\, 2\pi (S_{0k}^r \varphi + \varphi S_{0k}^r) + \text{conj.} \right] = 0.
\end{aligned} \tag{11.16}$$

Thus, if the J_0^r and S_{0k}^r were to commute with φ then (11.16) would, in fact, be identical with the classical equation of motion (11.3) after substituting the definitions (11.14) and (11.15). (11.16) is a Dirac equation that is nonlinear in φ. The nonlinearity

originates in the appearance of the derivative of the potential in the Lagrange function when one regards the A_i^r and B^r as the primary quantities [see the rem., Part I, formula (2.6a)].

12. The equation of continuity for electric and heavy charges and the explicit form for the forces of interaction in the nucleus.

Electrodynamics is contained in the general formalism of Part II when one sets $l_0 = 0$ for one of the fields (e.g., $r = 0$). No B_0 will then exist, and one will have $\Phi_i^0 = A_i^0$ and $J_i' = J_i$ in (9.9). Other than the trivial case $J_i^0 = 0$, the only possibility that is still open is that J_0^0 commutes with J_0^{0*}. If one now decomposes things into real and imaginary parts then the description will split into two independent real fields that interact with an independent real current. Both current components must individually satisfy the continuity equation. The continuity equation and the reality of the field are thus consequences of $l_0 = 0$.

By contrast, the formalism of the foregoing paragraph is still generally sufficient for one to describe electrodynamics: Namely, the current (11.4) and (11.5) that is constructed from the φ with the help of real τ^0 does not satisfy the continuity equation in the absence of other fields Φ^r for which τ^r does not commute with τ^0. One must then add a four-vector that is constructed from the Φ^r to the expression for the current; the fields Φ^r must be charge carriers.

In addition to this *law of conservation of electric charge*, which is required by Maxwell's theory, there is, however, obviously yet another conservation law: For all of the observed transformations of matter, no transformations of heavy particles (e.g., neutrons and protons) into light particles (e.g., electrons and neutrinos) have been observed. We would therefore like to demand that there should be a *law of conservation of heavy charge.*

The matrices ($\tau^0 = \lambda$):

$$\lambda = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{and} \qquad \lambda' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{12.1}$$

which act upon the upper index of φ, allow the electric (heavy, resp.) charge density that is carried by the spinor field to be written in the form $\varphi^* \alpha_i \lambda \varphi$. If λ^ν are the diagonal elements of the matrix λ then the 0-component will have the form $\sum_\nu \lambda^\nu \varphi^{\nu *} \varphi^\nu$. When one employs hole theory and the anti-symmetric quantization (cf., also Majorana, *loc. cit.* [**12**]), the eigenvalues of the volume integral of $\varphi^{\nu *} \varphi^\nu$ are whole positive or negative numbers. $\lambda^\nu = 0$ or 1 is then the charge of the particle of the ν^{th} spinor field. Anti-particles have charge $-\lambda^\nu$.

We now calculate the four-divergence of the current that is constructed from the matrices λ:

In order to do that, we multiply (11.16) by $\varphi^* \lambda$ on the left and the complex-conjugate equation by $\lambda \varphi$ on the right, and subtract the two equations from each other. Now, the four-divergence does not generally vanish, but will be a relatively complicated expression. It simplifies considerably when the matrix λ satisfies the following commutation relations:

$$\begin{aligned} [\lambda, \mu] &= 0, \\ [\lambda, \tau^r] &= \Lambda^r \tau^r, \\ [\lambda, \tau^{r*}] &= -\Lambda^{r*} \tau^{r*}, \\ \Lambda^r &= \text{multiples of the identity matrix.} \end{aligned} \tag{12.2}$$

It follows from this that λ must be Hermitian, and Λ^r must be a real number. The divergence equation will then assume the form:

$$\sum_i \frac{\partial}{\partial x_i}(\varphi^* \alpha_i \lambda \varphi) = \frac{i}{2hc} \sum_r \Lambda^r \Big[(\mathbf{\Phi}^{r*}, \mathbf{J}^r) - 8\pi c\, l^{-2} \operatorname{div} \Pi^r \cdot J_0^r + \sum_{i=1}^{3} \sum_{k=1}^{3} \frac{\partial \Phi_i^{r*}}{\partial x_k} S_{ik}^r - \sum_{k=1}^{3} 8\pi c\, \Pi_k^r S_{0k}^r - \text{conj.} \Big]. \tag{12.3}$$

The fact that the fourth-order terms in φ drop out follows from relations (11.13) and the relation:

$$[\lambda \tau^r, \tau^{r*}] + [\tau^{r*} \lambda, \tau^r] = 0,$$

which follows from (12.2).

In order to show that the right-hand side is the divergence of another four-current, we multiply the field equation (11.1) with $\varepsilon_i \Phi_i^{s*}$ on the left and the complex-conjugate equation, likewise on the left, by $\varepsilon_i \Phi_i^s$, and add the sums of the two equations for *i* from 0 to 3.

If one considers the non-commutation of some things then one will obtain the following four-divergence:

$$\begin{aligned} \sum_i \frac{\partial}{\partial x_i} \Big[\frac{i}{8\pi hc} \sum_k \varepsilon_k (\Phi_k^{r*} F_{ik}^r - \Phi_k^r F_{ik}^{r*}) \Big] = \\ - \frac{i}{2hc} \Big[(\mathbf{\Phi}^{r*}, \mathbf{J}^r) - (\mathbf{J}^{r*}, \mathbf{\Phi}^r) - \Phi_0^{r*} J_0^r + \Phi_0^r J_0^{r*} - \frac{l^2}{4\pi} [\Phi_0^{r*}, \Phi_0^r] \\ - \Phi_i^{r*} \sum_k \frac{\partial S_{ik}^r}{\partial x_k} - \Phi_i^{r*} \sum_k \frac{\partial S_{ik}^r}{\partial x_k} - \frac{1}{4\pi} \sum_k [F_{k0}^{r*}, F_{k0}^r] \Big]. \end{aligned} \tag{12.4}$$

Due to the definition of Φ_0^{r*}, (11.14) will yield the third, fourth, and fifth term in the bracket on the right-hand side directly:

$$(\ldots - 8\pi c\, l^{-2}\, \mathrm{div}\, \mathbf{\Pi}^r \cdot J_0^r + \mathrm{conj}\ldots) \tag{12.5}$$

Due to the definition of the F_{k0}^r (11.15), the last term in the right-hand side will assume the form:

$$\left(\cdots - \sum_{k=1}^{3} 4\pi\, S_{0k}^{r*}\, S_{0k}^{r} + \sum_{k=1}^{3} 4\pi\, S_{0k}^{r}\, S_{0k}^{r*} \right). \tag{12.6}$$

If one then multiplies (12.4) by the number Λ^r and sums over r then when one considers the definition (11.15), the sum of (12.3) and (12.4) can be written as the continuity equation:

$$\left(\frac{\partial}{\partial x}, \rho \right) = 0$$

with the components:

$$\rho_i = \sum_{\nu} \lambda^{\nu} (\varphi^{\nu *} \alpha_i \varphi^{\nu}) + \sum_{r} \Lambda^r \left[\frac{i}{8\pi hc} \sum_{k} \Phi_k^{r*} (F_{ik}^r - 4\pi S_{ki}^r) - \Phi_k^r (F_{ik}^{r*} - 4\pi S_{ki}^{r*}) \right]. \tag{12.7}$$

The charged part of the r^{th} field is then the r^{th} summand in the second sum, just as the ν^{th} summand of the first sum represents the charged part of the ν^{th} matter field.

In particular, the charged part of the r^{th} field vanishes when the field is real. MAJORANA [**9**] has shown that an analogous theorem exists for spinor fields.

The 0-component is the actual charge density. When one employs the definition (11.15), it will read:

$$\rho_0 = \sum_{\nu} \lambda^{\nu} (\varphi^{\nu *} \varphi^{\nu}) + \sum_{r} \Lambda^r \sum_{k} \frac{i}{h} (\Pi_k^r \Phi_k^r - \Pi_k^{r*} \Phi_k^{r*}). \tag{12.8}$$

As we have already remarked, the eigenvalues of any individual summand in the first sum will be positive and negative multiples of λ^{ν}, when one considers Dirac's hole theory. From the Pauli-Weisskopf theory [**6**], the same thing will be true for every individual summand in the second double sum. The numbers λ^{ν} and Λ^r thus represent the charge of the particle of the ν^{th} matter (spinor) field and the r^{th} nuclear force (tensor) field. Each of the fields Φ_i^r (with the exception of the real fields) has a particle and an anti-particle. The latter have the opposite sign for the charge (*).

Equations (12.2), which represent the necessary conditions *for the existence of a continuity equation* (12.7), permit one to determine the possible matrices τ^r.

It initially follows from the first equation that λ must be a diagonal matrix, since the eigenvalues of μ are all different. The second and third equations demand the Hermiticity of λ, and thus determine λ^{ν} and Λ^r as real numbers. The matrices in equation (12.1) – viz., the electric and heavy charges – obviously obey that requirement.

In order to determine the form of the four-rowed matrices τ^r, we decompose the most general four-rowed matrix into a sum of direct products of two-rowed matrices. Let the

(*) The sum over k from 1 to 3 means that the particles have three possible spin orientations.

four states – namely, electron, neutrino, proton, and neutron (corresponding to the four possible values of the upper index ν of φ) – be enumerated by 11, 21, 12, and 22, resp. Let $\tau_0 = 1$, τ_1, τ_2, τ_3 be the unit matrix and the Pauli matrices, resp., which act upon the first index of 11, 21, etc. Likewise, let τ'_i (i = 0, 1, 2, 3) be the corresponding matrices that act upon the second index.

Naturally, the primed and unprimed matrices commute with each other. Moreover, one has the known rule for both matrices:

$$\tau_i \tau_k = -\tau_k \tau_i = \tfrac{1}{2}[\tau_i, \tau_k] = i \tau_l, \qquad ikl = \text{cycl. perm.} \tag{12.9}$$

The most general four-rowed matrix then reads:

$$\tau^r = \sum_i \sum_k a^r_{ik} \tau_i \tau'_k, \tag{12.10}$$

and the special matrices (12.1) have the form:

$$\lambda = \tfrac{1}{2}(\tau_0 + \tau_3)\, \tau'_0; \qquad \lambda' = \tfrac{1}{2} \tau_0 (\tau'_0 - \tau'_3). \tag{12.11}$$

Substituting these developments into the second equation (12.2) and comparing the coefficients on both sides of the equation will give the following relations between the a^r_{ik}:

$$\begin{aligned} &\Lambda^r a^r_{0k} = 0, \qquad \Lambda^r a^r_{2k} = +i\, a^r_{1k}, \\ &\Lambda^r a^r_{3k} = 0, \qquad \Lambda^r a^r_{1k} = -i\, a^r_{2k}. \end{aligned} \tag{12.12}$$

An analogous equation for the second index follows for the conservation of heavy charge, except that the quantity $-\Lambda'^r$ occurs everywhere in place of Λ^r.

The solutions of (12.12) are:

$$\begin{aligned} &\Lambda^r = 0 \quad \text{with} \quad a^r_{1k} = a^r_{2k} = 0, \\ &\text{or} \\ &\Lambda^r = \pm 1 \quad \text{with} \quad a^r_{0k} = a^r_{3k} = 0, \quad \text{and} \quad a^r_{2k} = \pm i\, a^r_{1k}, \end{aligned} \tag{12.13}$$

and analogous equations in Λ'^r for the second index.

The following cases are then possible according to that:

1. *Field with no electric or heavy charge:*

$$\begin{aligned} &\Lambda^1 = \Lambda'^1 = 0, \\ &\text{and} \\ &\tau^1 = a^1_{00} + a^1_{10} \tau_1 + a^1_{01} \tau'_1 + a^1_{11} \tau'_1. \end{aligned} \tag{12.14}$$

The electromagnetic field is obviously such a field. In particular, these fields can be real, since the matrices τ are Hermitian, and the constants can be chosen to be real.

The interaction between two material particles in configuration space that is produced by that field follows by substituting (12.14) in (10.5). If we restrict ourselves to heavy particles then we can set $\tau_3'\psi = 1\psi$, and the factor in (10.5) that contains τ will read simply (*):

$$| a |^2 + | b |^2 \tau_3^r \tau_3^s + \tfrac{1}{2}(ab^* + a^*b)\ (\tau_3^r + \tau_3^s), \tag{12.15}$$

in which *a* and *b* are arbitrary complex numbers. In particular, if they are real then the field will be real.

2. *Fields with electric charge, but no heavy charge.*

$$\Lambda^2 = -1, \quad \Lambda'^2 = 0,$$
$$\text{and} \tag{12.16}$$
$$\tau^2 = (\tau_1 - i\tau_2)(a_{10}^2 + a_{13}^2 \tau_3').$$

The interaction between two heavy particles is once more formula (10.5), where the factor that contains *t* reads:

$$| a' |^2 (\tau_1^r \tau_1^s + \tau_2^r \tau_2^s). \tag{12.17}$$

In particular, if one would like to have forces between neutrons and protons that are independent of the charge and depend upon only the symmetry character of the wave function in configuration space of the heavy particles then the interaction must be capable of being written in the form (10.5) with a τ factor ("isotopic spin factor" [**14**]):

$$\left(| a |^2 + | b |^2 \sum_{i=1}^{3} \tau_i^r \tau_i^s \right). \tag{12.18}$$

If one takes the sum of (12.15) and (12.17) when one takes the same constants *f* and *g* in (10.5) for both fields 1 and 2, then one will, in fact, obtain (12.18) when one sets $a' = a$ and $b = ia$. That generally leads to the unattractive feature that field 1 (viz., the field with no electric or heavy charge) is complex, so it will contain two types of particles (viz., anti-particles). Since the τ matrices of the following fields contain only τ_1' and τ_2', they will no longer carry the interaction between heavy particles. The case of $\Lambda^2 = +1$, $\Lambda^1 = 0$ is identical with the one that was treated (when one switches particles and anti-particles).

(*) Naturally, the indices *r* and *s* in equations (12.15), (12.17), and (12.18) do not refer to different fields, but to two different heavy particles, according to the configuration space description in paragraph 10.

13. Continuation of the discussion of the possible fields.

Whereas the representation by four-vectors Φ_i^r was necessary for the field types that were discussed at the end of the foregoing paragraphs in order to find agreement with experiments (e.g., attraction of deuterons in the ground state, etc.), that will no longer be necessary for the other fields. For example, those fields can also have a scalar character. However, one easily convinces oneself that analogous viewpoints are also valid for them, and that, in particular, the relations (12.2) are true, as well as the relations (12.12) and (12.13) that are derived from them.

If we denote the spinor particles – electron, neutrino, proton, and neutron – by (*) e (1, 0), n (0, 0), P (1, 1), and N (0, 1), resp., the neutral particle of field 1 [at least two of which must exist, due to the complex constants in (12.15)] by $\mathfrak{n}$ (0, 0), and the charged particle of field 2 by $\mathfrak{e}$ (1, 0) then the matrices (12.14) and (12.16) will give rise to the following transition:

Field 1.

$$\text{spinor particle} \rightarrow \text{the same spinor particle} + \mathfrak{n}\ (0, 0). \tag{13.1}$$

Naturally, up to now, only reactions with heavy spinor particles have been "observed"; i.e., their existence must be required in order to explain the nuclear forces between the same particles.

Field 2.

$$P\ (1, 1) \rightleftharpoons N\ (0, 1) + \mathfrak{e}\ (1, 0) \tag{13.2}$$

$$e\ (1, 0) \rightleftharpoons n\ (0, 0) + \mathfrak{e}\ (1, 0). \tag{13.3}$$

All of the symbols are considered to be algebraic quantities. (Negative symbols mean the corresponding anti-particle.) For example, it follows from (13.2) and (13.3) that:

$$[-\mathfrak{e}\ (1, 0)] \rightleftharpoons [-e\ (1, 0) + n\ (0, 0)]. \tag{13.3'}$$

In other words, a (negatively-charged) anti-$\mathfrak{e}$-particle ($-\mathfrak{e}$) can be decomposed into a negative electron ($-e$) and a neutrino (n).

Furthermore, for the time being, only the reactions (13.2) have been "observed," since the exchange forces between protons and neutrons would result from them.

However, since the $\mathfrak{e}$-particles must obviously occur quite rarely, (13.3′) will explain their finite lifetimes.

Moreover, (13.3) will give a theory of β-decay:

(*) The two indices in the bracket that follows the symbol refer to the electric and heavy charge of the particles, resp.

A neutron converts into a proton + an anti-$\mathfrak{e}$-particle according to the algebraic description of (13.2):

$$N\,(0,\,1) \rightarrow P\,(1,\,1) + (-\,\mathfrak{e}\,(1,0)). \tag{13.2′}$$

Only the reaction (13.3′) enters into this.

According to the formalism of Part II, that can be interpreted as follows: Under the influence of the retarded potential of a heavy particle that converts a neutron into a proton, a positive electron in a state of negative energy will jump into a neutrino state of positive energy. Since the motion of the heavy particles results slowly, according to paragraph 10, the retardation can be neglected, and an interaction of the form (10.5) can be substituted in the Hamilton function.

Since the range of the $\mathfrak{e}$-field seems to be small in comparison to the wavelength of the de Broglie waves of electrons and neutrinos, from heuristic arguments, the "action at a distance" in equation (10.5) can be replaced with a local action, and what follows will be one of Fierz's generalizations [**5**] of Fermi's theory [**16**] of β-decay.

However, this generalization will still have the disadvantage that it yields a weak asymmetry to the energy distribution of the continuous β-spectrum.

We will see that the other possible field types provide an alternative description of β-decay, and according to the calculations of WENTZEL [**17**], a better one.

The non-occurrence of the reaction (13.3) (the relatively-improbable occurrence of it, resp.) implies an infinite length (a length of the kind that was proposed by BHABHA [**2**], resp.) for the lifetime of the $\mathfrak{e}$-particle. The finite lifetime would then be required *only* by the collisions between neutrons that occur in atomic nuclei (by anti-$\mathfrak{e}$-particles colliding with protons, resp.) [**21**].

We continue our discussion of the field types:

3. *Field with no electric charge, but with a heavy charge.*

$$\Lambda^3 = 0, \qquad \Lambda'^3 = -\,1,$$

which implies, analogous to (12.16) (13.4)

$$\tau^3 = (a_{01}^3 + a_{31}^3\,\tau_3)(\tau_1' + i\tau_2')\,.$$

We denote the particles by $\mathfrak{N}\,(0,\,1)$. They give rise to the following reaction:

$$N\,(1,\,0) \rightleftharpoons n\,(1,\,0) + \mathfrak{N}\,(0,\,1), \tag{13.5}$$

$$P\,(1,\,1) \rightleftharpoons e\,(1,\,0) + \mathfrak{N}\,(0,\,1). \tag{13.6}$$

Since the proton is certainly a stable particle, it will follow from (13.6) that the mass of the $\mathfrak{N}$-particle is large than the difference between the proton and electron masses. However, since it also emerges from measurements of nuclear spin and nuclear statistics

that only neutrons and protons, but no particles with whole-number spins, exist in the nuclear, it is likely that the $\mathfrak{N}$-particle also possesses a larger mass than the neutrons, and is therefore unstable.

According to the arguments about the retarded potentials, this field 3 will give rise to exchange forces between light and heavy particles of very short range (e.g., the Compton wavelength of the proton). These exchange forces permit an alternative explanation for β-decay:

From (13.5), an $\mathfrak{N}$-particle and a neutrino are generated. The $\mathfrak{N}$-particle splits according to the algebraically-depicted equation (13.6):

$$\mathfrak{N}\ (0,\ 1) \rightarrow P\ (1,\ 1) + (-\ e\ (1,\ 0)). \tag{13.6′}$$

Otherwise expressed, that reads: A spinor particle goes from the state "neutron bound to the nucleus" to the state "free neutrino." The retarded or advanced potential of field 3 that is generated by that transition will induce the quantum jump of another particle from the state "negative-energy electron" to the state "proton bound to the nucleus."

Naturally, it will no longer be possible to neglect the retardation. As WENTZEL [**17**] has shown, one will get a stronger asymmetry than in Fermi's theory when the $\mathfrak{N}$-field is affected by the nucleus (i.e., when "intermediate states" exist with bound $\mathfrak{N}$-particles).

4. *Field with electric and heavy charges of equal sign.*

$$\Lambda^4 = \Lambda'^4 = -\ 1,$$

which implies: (13.7)

$$\tau^4 = a_{11}^4(\tau_1 - i\tau_2)(\tau_1' + i\tau_2').$$

The particles that are denoted by $\mathfrak{P}$ (1, 1) give rise to only the reaction:

$$P\ (1,\ 1) \rightleftharpoons n\ (0,\ 0) + \mathfrak{P}\ (1,\ 1). \tag{13.8}$$

In order for the proton to be stable, the $\mathfrak{P}$-particle must have a larger mass than the proton.

5. *Field with electric and heavy charges of different signs.*

$$\Lambda^5 = +\ 1,\ \Lambda'^5 = -\ 1,$$

which implies: (13.19)

$$\tau^5 = a_{11}^5(\tau_1 + i\tau_2)(\tau_1' + i\tau_2'),$$

and likewise only the single reaction:

$$N\ (0,\ 1) \rightleftharpoons e\ (1,\ 0) + \mathfrak{P}\ (-\ 1,\ 1). \tag{13.10}$$

14. Extension of the concept of current J_i^r.

The definitions (11.4) of the current quantities that appear in Proca's equations (11.1) are capable of yet another extension that is likewise bilinear in φ. If one adds the terms:

$$K_i^r = \varphi\, k_i^r\, \varphi\,; \qquad R_{ik}^r = \varphi\, r_{ik}^r\, \varphi\,, \tag{14.1}$$

with the matrix operators:

$$k_i^r = f'^r\, e\, \delta\alpha_i\, \kappa^r\,; \qquad r_{ik}^r = g'^r\, e\, \frac{1}{l_r}\, \delta\sigma_{ik}\, \kappa^r, \tag{14.2}$$

to it, in which δ means the matrix that FERMI introduced [**16**] (cf., also PAULI [**18**]), and κ^r once more represents the operators that act on the upper index ν of φ_μ^ν, then nothing will change in the equations of motion of the field (11.1) and the divergence equation of the field (12.4), except that J_i^r is replaced with $J_i^r + K_i^r$. In order to obtain the equations of motion for φ from the Hamilton function, we must add the terms:

$$2\pi \sum_r [l_r^{-2} (J_0^{r*} K_0^r + K_0^{r*} J_0^r + K_0^{r*} K_0^r) + \text{corr. terms in } S_{0k}^r \text{ and } R_{0k}^r) \tag{14.3}$$

(in addition to the aforementioned replacements). Note that these terms are indeed Hermitian, but, in contrast to the terms in the last line of (11.9), they are *not symmetric*. A continuity equation is possible only with these terms.

For symmetric quantization of φ, one will get the classical wave equations (11.3), but which are extended by terms in φ^*. For anti-symmetric quantization (*), a characteristic sign difference between that term and the one in the classical equations will appear. The divergence equation (12.3) will keep its form when the matrices κ satisfy the anti-commutation relations:

$$\lambda\, \kappa^r + \kappa^r \lambda = -\Lambda^r\, \kappa^r\,. \tag{14.4}$$

If one develops the matrices κ once more according to (12.10) with constants b_{ik} then the relations will follow:

$$\begin{aligned} b_{1k}^r &= -\Lambda^r b_{1k}^r, & b_{2k}^r &= -\Lambda^r b_{2k}^r, \\ b_{0k}^r + b_{3k}^r &= -\Lambda^r b_{0k}^r, & b_{3k}^r + b_{0k}^r &= -\Lambda^r b_{3k}^r, \end{aligned} \tag{14.5}$$

in analogy with (12.12) and (12.13).

The solutions for Λ^r now read 0, – 1, – 2, instead of 0, – 1, + 1, resp., as in paragraph 12. We would like to exclude the solution – 2 (viz., doubly-charged elementary particle).

The field types that were discussed in the foregoing paragraphs will then give rise to the following additional possible reactions:

(*) (11.11) is then true only in the form $[H, \varphi]$, and no longer $\delta H / \delta\varphi$.

1. *Field with no electric or heavy charge.*

$$\kappa^1 = b^1_{00}(\tau_0 - \tau_3)(\tau'_0 + \tau'_3)\,; \tag{14.6}$$

i.e., the reaction:

$$\mathfrak{n}\ (0\ 0) \rightleftharpoons 2n\ (0,\ 0). \tag{14.7}$$

2. *Field with electric, but no heavy charge.*

$$\kappa^2 = (b^2_{00}\,\tau_1 + b^2_{20}\tau_2)(\tau'_0 + \tau'_3)\,, \tag{14.8}$$

or, when written in the reaction notation:

$$e\ (1,\ 0) \rightleftharpoons (-\,n\ (0,\ 0)) + \mathfrak{e}\ (1,\ 0). \tag{14.8}[sic]$$

3. *Field with no electric charge, but with heavy charge.*

$$\kappa^3 = (\tau_0 - \tau_3)(b^3_{01}\,\tau'_1 + b^3_{02}\,\tau'_2)\,, \tag{14.9}$$

or:

$$N\ (0,\ 0) \rightleftharpoons (-\,n\ (0,\ 0)) + \mathfrak{N}\ (0,\ 1). \tag{14.10}$$

4. *Field with electric and heavy charges with the same sign.*

$$\kappa^4 = (b^4_{11}\,\tau_1 + b^4_{21}\,\tau_2)\,\tau'_1 + (b^4_{12}\,\tau_1 + b^4_{22}\,\tau_2)\,\tau'_2\,, \tag{14.11}$$

with the reactions:

$$P\ (1,\ 1) \rightleftharpoons (-\,n\ (0,\ 0)) + \mathfrak{P}\ (1,\ 1), \tag{14.12}$$

$$N\ (0,\ 1) \rightleftharpoons (-\,e\ (1,\ 0)) + \mathfrak{P}\ (1,\ 1). \tag{14.13}$$

The field 5 with heavy and electric charges of different signs gives only the matrix $\kappa^5 = 0$.

In addition to the last reaction (14.13), all new reactions will be the same as the ones with τ matrices, except that the anti-neutrino plays the role of the neutrino everywhere.

The field 4 gives another possibility for β-decay: A neutron will become a $\mathfrak{P}$ (1, 1) particle (14.13) and emit a negative electron. According to the algebraic representation of (14.12), the $\mathfrak{P}$ (1, 1) particle will split into a proton and a neutrino:

$$\mathfrak{P}\ (1,\ 1) \to P\ (1,\ 1) + n\ (0,\ 0). \tag{14.12′}$$

What was said about (13.6′) will be true for (12.12′) [**17**]. The appearance of a neutrino and an anti-neutrino will have the following deeper meaning:

a) Neutrinos and anti-neutrinos are different particles.

One then distinguishes between them by way of the so-called *neutrino charge.* If one demands the *conservation of neutrino charge* then one must also endow the neutron with a neutrino charge, as well. The matrix:

$$\lambda'' = 1 - \lambda \tag{14.12}$$

will then allow the neutrino charge density that is carried by the spinor particles to form. It will then follow from (12.2) that:

$$[\lambda'', \tau^r] = \Lambda^r \tau^r = -\Lambda^r \tau^r ; \tag{14.13}$$

i.e., the particles of the nuclear field have both electric and neutrino charges of opposite signs. However, it follows from (14.14) that:

$$\lambda'' \kappa^r + \kappa^r \lambda'' = -\Lambda''^r \kappa^r = (2 + \Lambda^r)\, \kappa^r. \tag{14.14}$$

(14.14) and (14.13) are compatible with each other only when either κ^r or τ^r vanishes; i.e., for a well-defined field Φ^r, only the reaction of paragraph 13 or the reactions of this paragraph will appear.

b) There exists no difference between neutrinos and anti-neutrinos.

The matrices κ can then be chosen in such a way that the spinor of the neutrino field φ^2 will appear in the field-matter interaction terms in the Hamilton function only in the combination:

$$\bar{\varphi}^2 = \varphi^2 + \delta^* \varphi^{2*}. \tag{14.15}$$

If one chooses the matrices α_i and β in such a form that the α_i are pure real and β is pure imaginary then the matrix δ will be equal to the identity matrix, and one will have $\bar{\varphi}^2 = \bar{\varphi}^{*2}$. MAJORANA [**9**] has shown that one can also write the component of the free spinor particle in the Hamilton function by employing only the real function $\bar{\varphi}^2$. The real spinor field, just like the real tensor field, then knows no anti-particle; i.e., it consists of only one kind of particle (cf., also RACAH [**19**] on this).

15. Concluding remarks.

Once the existence of a continuity equation in the absence of electric fields has been shown, the introduction of the interaction "electric field with electrically-charged Φ^r and φ^r fields" will create no serious difficulties. The PROCA case [**10**] has already been classically.

One must then treat, *inter alia*, the following interesting properties of the new properties (cf., also BHABHA [**2**] on this):

1. Brehmsstrahlung, Compton effect, and pair-creation of $\mathfrak{e}$-particles. (For spinless particles, the formula that is analogous to that of Bethe-Heitler was already computed by PAULI and WEISSKOPF [**6**].)

2. Absorption (and emission) of an $\mathfrak{e}$-particle (or an uncharged $\mathfrak{n}$-particle) by a heavy particle in the atomic nucleus (= atomic disintegration by $\mathfrak{e}$ or $\mathfrak{n}$ particles, since the rest energy of those particles is already sufficient for a nuclear component to lose its binding energy)[**21**].

3. Creation of pairs of $\mathfrak{e}$ or $\mathfrak{n}$ particles by recombination of a proton and a neutron with an anti-proton and an anti-neutron. Radiation of one or more $\mathfrak{e}$ and $\mathfrak{n}$ particles by the braking of fast neutrons and protons.

In addition to pair creation by a primary photon ray, the generation of the obviously unstable $\mathfrak{e}$-particle (on its instability, cf., also the recently-published observations of BLACKETT [**20**]) can then be possibly interpreted by this recombination of a primary cosmic ray that consists of heavy anti-particles with the nuclear constituents.

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