"Sur les mouvements des planètes, en supposant l'attraction représentées par l'une des lois électrodynamiques de Gauss ou de Weber," C. R. Acad. Sci. Paris **100** (1890), 313-315.

On the motion of planets when one supposes that the attraction is represented by either the electrodynamical law of Gauss or that of Weber.

Note by **F. Tisserand**

Translated by D. H. Delphenich

One might demand to know what would happen if the motions of the celestial bodies in our planetary systems were governed, not by Newton's law, but by one of the laws that were proposed in electrodynamics by Gauss and Weber:

$$R_g = \frac{fmm'}{r^2}\left[1+\frac{1}{h^2}\left(2u^2 - 3\frac{dr^2}{dt^2}\right)\right],$$

$$R_w = \frac{fmm'}{r^2}\left[1+\frac{1}{h^2}\left(2r\frac{d^2r}{dt^2} - \frac{dr^2}{dt^2}\right)\right].$$

The force that is exerted between two molecules M and M', of masses m and m', is supposed to act along the line MM'; r denotes the distance MM', u is the relative velocity of the two molecules, and R_g or R_w is the intensity of the force. The constant h, which represents a velocity, is very large with respect to u; if it can be regarded as infinite then one will get back to Newton's law.

I have examined the question for R_w on another occasion (C. R. Acad. Sci. Paris, 30 September 1872), and I was led to the following results:

When one gives h values that are comparable to the speed of light, the substitution of Weber's law for that of Newton will produce only negligible periodic inequalities in the elliptic elements of the planets. The longitude of perihelion is an exception to that; it will contain a secular term whose expression is:

$$\delta\varpi_w = \frac{f\mu}{ah^2}nt(1+\tfrac{3}{2}e^2+\cdots) = \frac{(f\mu)^{3/2}}{a^{5/2}h^2}t(1+\tfrac{3}{2}e^2+\cdots);$$

it will become more noticeable as the planet gets closer to the Sun. (One has used a, e, n, m to represent the semi-major axis, eccentricity, mean motion, and the sum of the masses of the planet and the Sun, resp.) If one supposes that h is equal to the speed of light (300000 km/s) then one will find that under the hypothesis of Weber's law, the major axis of the orbit of Mercury will rotate in the direct sense by 14.4″ per century. For Venus, the variation will be only 3.0″, and it can have no appreciable effect during several centuries, due to the slight eccentricity of Venus. The mean longitude of the epoch *e* is also affected with a small secular term that contains e^2 as a factor and as is known, that will have as a consequence, moreover, that the mean motion will be altered very little, or rather, the theoretical value of *a* that shows up in the perturbations.

Upon reading the beautiful book by Bertrand *Sur la Théorie mathématique de l'Électricité*, in which the electrodynamical formulas that were proposed by Gauss and Weber are compared to each other, I was led to make the calculation for the former that I had made previously for the latter, and upon applying the known formulas from the variation of arbitrary constants, I effortlessly found that:

$$\delta\varpi_g = \frac{2f\mu}{ah^2} nt(1+e^2+\cdots) .$$

One sees that the displacement of the perihelion for a given time is roughly twice what it will be under Weber's law. With the value of h that was adopted before, one will then obtain:

$$\delta\varpi_g = +\ 28.2''$$

per century for Mercury.

Le Verrier found that the attraction of the planets must make the perihelion of Mercury rotate in the direct sense by 527″ per century. A discussion of the set of observations of the planet, and above all, its passages across the solar disc, showed him that the real motion is greater by 38″. That excess is very certain, due to the large eccentricity of Mercury; moreover, it was confirmed by the recent research of Newcomb. On the other hand, it is impossible to obtain it by changing the perturbing masses without introducing some intolerable contradictions with the theories of the other planets.

That is why Le Verrier was led to hypothesize the existence of an intra-Mercurial planet, which is a hypothesis that one could have believed to be realized plainly at some point. However, the mass of that planet would have to be relatively large, so one could not fail to see it during total eclipses of the Sun that were observed so carefully during the last twenty years. In the absence of a single planet, one might also assume with full rigor that there exists a ring of corpuscles between Mercury and the Sun that is analogous to the ring of asteroids that is found between Mars and Jupiter. The question remains open, without having been resolved. It is curious to remark that Gauss's law explains the 3/4 in the excess that pertains to it ([1]) without also appreciably disturbing the agreement that is realized by Newton's law in the theory of celestial motions. I shall confine myself to pointing out that coincidence without actually pretending that Gauss's law is the true one.

The element ε has a secular inequality:

$$+\ \frac{4f\mu}{ah^2} nt(1+\tfrac{1}{2}e^2+\cdots)$$

([1]) It will be explained completely when one gives *h* a value that is equal to 6/7 of the speed of light.

that is much more noticeable than the one that is given by Weber's law, but it will nevertheless have no appreciable effect, for the reason that was pointed out already.