Electrodynamic Measurements in Particular Attributing Mechanical Units to Measures of Current Intensity

R. Kohlrausch and W. Weber

Abstract

English translation of Rudolf Kohlrausch and Wilhelm Weber's 1857 paper "Elektrodynamische Maassbestimmungen insbesondere Zurückführung der Stromintensitäts-Messungen auf mechanisches Maass", [KW57]. This paper is the fourth of Weber's eight major Memoirs on "Elektrodynamische Maassbestimmungen", *Electrodynamic Measurements*.

Contents

By Rudolf Kohlrausch and Wilhelm Weber[1,2,3]

1 Section 1

The intensity of an electric current can be usually determined by observing either its *magnetic*, *electrodynamic*, or finally, its *electrolytic* effects. However, those effects can be observed under very different situations, and it is the task of the observer to choose those situations in such a manner as to give his observations the greatest completeness, while appealing to the *electromagnetic*, *electrodynamic*, and *electrolytic laws*, which can *reduce* the effects that are observed in the various situations to each other. That is because it is only by *reducing the observations under the same conditions* that one can achieve a *comparison of the current intensities*. Now, one calls those *common conditions*, to which all observations that were made under different circumstances should be reduced, *normal conditions*, and *the unit of current intensity* will be established by defining those normal conditions according the following rule:

> *The unit of current intensity* is the intensity of the current that will produce one unit of measurable effect *under normal conditions*.

The normal conditions for the observation of the *magnetic effects* of a current are the following: *The current goes through a circular conductor that encircles a unit area and acts upon a magnet that possesses one unit of magnetism*[4] *at an arbitrary, but large, distance equal to R. The center of the magnet lies in the plane of the conductor, and its magnetic axis points towards the center of the circular conductor.* Under those conditions, the *rotational moment D* that is exerted on the magnet by the current will vary with current intensity, as well as with distance R. However, the product $R^3 D$ depends upon merely the current intensity, and is therefore *the measurable effect* of the current under those conditions, from which one will then get

---

[1] [KW57].

[2] Translated by D. H. Delphenich, http://www.neo-classical-physics.info/index.html, e-mail: feedback@neo-classical-physics.info, and edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[3] The Notes by Kohlrausch and Weber are represented by [Note by KW:]; the Notes by H. Weber, the Editor of Volume 3 of Weber's *Werke*, are represented by [Note by HW:]; D. H. Delphenich's Notes are represented by [Note by DHD:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[4] [Note by AKTA:] That is, a magnet that possesses one unit of magnetic moment.

the *unit of the current intensity* from the intensity of the current whose measurable effect under the conditions that were just described will be:

$$R^3 D = 1 \ .$$

That unit of current intensity, which is then obtained from *electromagnetic laws*, is at the same time also the intensity of that current that flows around a planar region of size one unit of area, producing the same effect everywhere at a distance as a magnet that is found at the center of that region that possesses one unit of magnetism and whose magnetic axis is perpendicular to the plane. Alternatively, it is also the intensity of the current *that will equilibrate a tangent galvanometer*[5] *with a single multiplier loop of radius equal to R* when it deviates from the magnetic meridian by:

$$\varphi = \arctan \frac{2\pi}{RT} \ ,$$

if T denotes the horizontal component of the Earth's magnetism.

The normal conditions for the observation of the *electrodynamic effects* of a current are as follows: *The same current goes through two circular conductors, each of which encircles a unit area and lie at an arbitrary, but large, distance equal to R from each other: The line of intersection of the two perpendicular planes of the circles bisects the first circular conductor.* — Under those conditions, the *rotational moment D* that the current in the *first* conductor exerts upon the current that flows in the *second* conductor will vary with the current intensity, as well as with the distance R. However, the product $R^3 D$ depends upon merely the current intensity and is therefore the *measurable effect* of the current under those conditions, from which one will then get the *unit of the current intensity* from the intensity of the current whose measurable effect will be:

$$R^3 D = 1$$

under those conditions.

The normal conditions for the observation of *electrolytic effects* are the following ones: *The current goes through water during a time interval T that can be measured with arbitrary precision without suffering any change in intensity.* — Under those conditions, the mass of water M that is decomposed by the current, when expressed per assumed unit of mass (e.g., milligrams), will vary with current intensity, as well as with the time interval T (expressed in seconds). However, the quotient M/T will depend upon merely the current

---

[5][Note by AKTA:] *Tangentenboussole* in the original.

intensity, and is therefore the *measurable effect* of the current under those conditions, from which one will then get the *unit of the current intensity* from the current whose measurable effect is:

$$\frac{M}{T} = 1$$

under those conditions.

All that remains for one to be able to compare the intensities of all currents whose *magnetic*, *electrodynamic*, or *electrolytic* effects were observed is to relate the three units that were given under the aforementioned normal conditions to each other.

One infers the relationship between the *first two units* from the *fundamental electrodynamic law*, which include the laws of magnetism and electromagnetism, as Ampère exhibited them, namely, as was proved before in "Elektrodynamischen Maassbestimmungen II," p. 261,[6,7] one infers that the

---

[6][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 360.

[7][Note by AKTA: [Web52, p. 360 of Weber's *Werke*].

first unit relates to the *second* one like:[8]

---

[8][Note by KW:] It is therefore interesting to point out that one can exhibit a complete identity between those two units when one, under the aforementioned *normal conditions*, defines the *electrodynamic effect* to be the rotational moment that the current in the *second* circle exerts upon the current in the *first* one, instead of the rotational moment that the *first* one exerts upon the *second*. The reason why that is not done is found merely in the fact that the expression that Ampère gave for the force of repulsion between two current elements would remain unchanged, so if α and α' are the lengths of both elements, i and i' are the current intensities, r is the distance between them, ε is the angle between α and α', ϑ is the angle between α and r, and ϑ' is the angle between α' and the extension of r, then that force will be represented by:

$$-\frac{\alpha\alpha'}{r^2}ii'\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right)\ ,$$

or

$$\frac{1}{2}\cdot\frac{\alpha\alpha'}{r^2}ii'\left(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon\right)\ .$$

However, all that generally follows from Ampère's *fundamental law of electrodynamics* is that that force is *proportional* to that expression, and therefore when one leaves the measure of the current intensity undetermined, the force itself will be represented by the product of that expression with an arbitrary constant, and so by:

$$-C\cdot\frac{\alpha\alpha'}{r^2}ii'\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right)\ ,$$

or by

$$D\cdot\frac{\alpha\alpha'}{r^2}ii'\left(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon\right)\ .$$

in which C or D refer to the aforementioned constant. Now, Ampère assigned the value $C = 1$ to the constant C or the value $D = 1/2$ to the constant D in order to establish a well-defined unit for the current intensity, and in that way, he obtained the aforementioned expression for the force of repulsion between two current elements:

$$-\frac{\alpha\alpha'}{r^2}ii'\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right) = \frac{1}{2}\cdot\frac{\alpha\alpha'}{r^2}ii'\left(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon\right)\ ,$$

which reduces to:

$$-\frac{\alpha\alpha'}{r^2}ii'$$

for two parallel current elements that are perpendicular to r and for which $\varepsilon = 0$ and $\vartheta = \vartheta' = 90°$. However, for the sake of agreement with the electromagnetic measurements, it would be preferable to set $D = 1$ or $C = 2$, which would then make the expression for the force of repulsion between two current elements equal to:

$$\frac{\alpha\alpha'}{r^2}ii'\left(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon\right) = -2\frac{\alpha\alpha'}{r^2}ii'\left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right)\ ,$$

$$\sqrt{2} : 1 \ .$$

The *third* unit will imply the reduction to the *first*, and therefore also the *second* one immediately, by simultaneous observations of the *magnetic* and *electrolytic* effects that are produced by one and the same current. Namely, one will find upon comparing the reduced observations, under the afore-mentioned normal conditions, that the *third* unit of current intensity, or the intensity of the current that will decompose 1 milligram of water in 1 second, is $106\frac{2}{3}$ times larger than the *first* unit, or than the intensity of the current that, when it flows around a plane of size one unit area, will produce the same effects everywhere at great distances as a magnet at the center of that planar region that possesses one unit of magnetism and whose magnetic axis is perpendicular to the plane. See "Resultate aus der Beobachtungen des magnetischen Vereins in Jahre 1840," p. 96,[9],[10] and Casselmann "Über die galvanische Kohlenzinkkette. Marburg 1844," p. 70.[11]

2 Section 2

However, the intensity of an electric current can be determined not only by its *effects*, but also by its *origins*. Nonetheless, the deepest roots of an electric current lie in the mass of neutral electric fluid that is contained in a closed conductor, and in the velocity with which its two components, namely, the masses of the *positive* and *negative* fluids, move simultaneously in opposite directions. On the basis of this *origin, the unit of the current intensity* will be established from the following measurements:

> The unit of the current intensity is the intensity of the current
> that is *produced* by such a velocity for the two electric fluids, for

---

and for two current elements that coincide with r, [that is, for two current elements parallel to one another and pointing along the direction r,] for which $\vartheta = \vartheta' = \varepsilon = 0$, that will reduce to:

$$\frac{\alpha\alpha'}{r^2} ii' \ .$$

The cited *change in the normal conditions for the electrodynamic current effects* will agree with that, and in that way, one will arrive at a complete identity between the *electrodynamic* unit of the current intensity and the *magnetic* one.

[9][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 17.

[10][Note by AKTA:] [Web41, p. 96 of the *Resultate*] and [Web42, p. 17 of Weber's *Werke*].

[11][Note by AKTA:] [Cas43, p. 70].

which the mass of each fluid that flows through the cross-section
of the conductor divided by the time during which it flows through
it is equal to 1.

This unit is the *mechanical unit of the current intensity*, and the problem
that is being addressed in this treatise is to reduce the unit that was described
in the previous Section to this unit, which is most simply based in the essence
of the current, and will therefore have an advantage over the other measures
for the fundamental determination of that current intensity.

3 Section 3. Reducing the Magnetic, Electrodynamic, and Electrolytic Units for the Current Intensity to Mechanical Units

Up to now, no attempt has been made to determine current intensities from
a *mechanical unit*, and even less, to *reduce* the current intensities that were
determined from the other units to the latter. One merely knows that the
amount of electricity that flows in the form of weak currents through the
cross-section of the closed circuit, which can be produced by the most humble
galvanic processes, must also be very large for a very brief time, since the
most powerful electrification machine (whose conductor is coupled with the
site of friction by a conducting wire) will give a much weaker current than
a single galvanic element that is closed by a conducting wire of very large
resistance.

The lack of any way of determining current intensities by *mechanical
measurements* is based upon the difficulties that one finds in their implementation, while the determination of current intensities in the other aforementioned units is very easy to do, and thus allows for a much higher degree of
precision. The last unit will always be the first choice to be applied *in practice*, and one will essentially deal with the fact that a current intensity that
is known in one of the latter units can only *once* be measured as precisely as
possible in order to ascertain the ratio of the magnitude of the *mechanical
unit* to that of the latter unit, and in that way, to find oneself in a position to
reduce all of the determinations that were made in those units *to mechanical
units*.

For such a measurement, one lacks, above all, any knowledge of the
amount of electricity in a closed conductor that carries current, or rather,
since that knowledge while the current is flowing is not at all gained, from
knowing the *amount of electricity that is transferred by the current, and*

which is found to have been accumulated previously — e.g., *in a Leyden jar.* In order to do that, one possesses excellent means and methods for measuring electricity that go back to Coulomb, but which are never used to measure the electricity that is collected in a charged Leyden jar.[12,13]

The question of the *amount of electricity* that is found to be collected in a Leyden jar is often raised: Once it has been answered, and the *amount of electricity* has been determined by the *forces* that it might exert, it is in no way merely a question of curiosity, but is linked with important determinations that are presently still lacking in the theory of electricity, and might lead the way to interesting investigations.[14]

This question in regard to the amount of electricity in a Leyden jar has a special relationship to *electrodynamic units* that deserves a closer look, in any event. A fundamental law of electrical action was presented in the first part of this series of articles on [electrodynamic] measurements that simultaneously included *electrostatics*, *electrodynamics*, and *induction*. According to that fundamental law, the force that the electrical mass e exerts upon the electrical mass e' at a distance r is not merely a function of that *distance*, but at the same time, a function of the relative state of motion of the two electrical masses that is given by the *relative velocity dr/dt* and acceleration d^2r/dt^2

---

[12][Note by KW:] In the *Annalen der Chemie und Physik*, Vol. 86, p. 33, Buff found, with the help of his tangent galvanometer and long conducting wires, that the amount of electricity that would be sufficient to liberate 1 milligram of hydrogen from 9 milligrams of water electrically, when one possesses the means to condense it, was to charge a battery of 45,480 Leyden jars that were each 480 millimeters in height and 160 millimeters in diameter up to a spark gap of 100 millimeter. Buff's determination is the best and most precise one that exists, but it still does not suffice to determine the *amount of electricity* that is included in that jar, for which, from *mechanical principles*, one requires a knowledge of the *force of repulsion* that this *amount of electricity*, when concentrated into a point, would exert upon an equal amount of electricity that is also concentrated into a point at a large distance from the latter. However, knowledge of that *force of repulsion* is still lacking, and up to now, no attempts have been made to measure such forces by the various means and methods that were given by Coulomb and others, or even to gain a better knowledge of it.

[13][Note by AKTA:] As a matter of fact, this paper was published in the *Annalen der Chemie und Pharmacie* and not in the *Annalen der Chemie und Physik*: [Buf53, p. 33].

[14][Note by KW:] When one observes that most of the applications of the laws of nature depend upon determining the values of certain constants, that fact is based upon the determination of the unknown *constants in the theory of electricity*, which depend upon answering the question above. — Moreover, it is very likely that determining the electricity required to *decompose water* by the forces that it might exert could be used to investigate the forces that are active in the decomposition of water, and that in the same way, a determination of the amount of electricity by which a wire would be made to *glow* within a certain period of time from the *forces* that it might exert would lead to a deeper insight into the forces at work during the generation of heat, etc. Some of those applications will be discussed in more detail in Part Two.

with which they pass to the distance r. That fundamental law of the electrical action is:

$$\frac{ee'}{r^2}\left[1 - \frac{1}{c^2}\left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2}\right)\right]$$

in which *the constant c* means *the relative velocity at which the electrical masses would exert no effect at all on each other, as long as it remained unchanged.* In the second article in this series,[15] it was then shown how the determination of the value of that *constant c* might provide the possibility of reducing not merely the units of the electromotive forces, but also the units of the current intensities, to the *units of mechanics*, and that in itself will give the relationship by which the *constant c* will allow one to determine the *amount of electricity* that passes through a cross-section of the conductor in a unit of time in terms of the unit of measurement of current intensity that is based in the magnetic and electrodynamic effects of the current. Conversely, the knowledge of that *amount of electricity* that is acquired in other ways would also lead to a determination of the value of the *constant c*, which is brought to our attention by the fundamental law above. The determination of such a *constant* of nature is a topic that is especially appropriate for a finer measurement. In the foregoing case, that determination can be reduced to the following problem.

4 Section 4. Problem

Determine the *amount of electricity* that passes through the cross-section of a conductor in unit of time for a current whose intensity has a unit of measurement that is based in its *magnetic* or *electrodynamic* or *electrolytic* effects, and indeed, that amount of electricity shall be determined *from the magnitude of the fundamental electrostatic force that it exerts*; or, more especially:

> Let a constant current be given,[16] under which a tangent galvanometer with a simple multiplier circle of radius equal to R would take on a deflection of $\varphi = \arctan 2\pi/RT$ in equilibrium, where T means the intensity of the horizontal component of the Earth's magnetism that directs the compass. It should be determined how the amount of electricity that flows through the cross-section of the conductor in one second under such a current, relates itself to the amount of electricity that is contained on each

---

[15][Note by AKTA:] [Web52].

[16][Note by AKTA:] Current of intensity $= i$.

of two small equally-charged balls that repel each other with a unit force at a unit distance. In that way, the unit of force shall be taken to be the force that would accelerate a mass of one milligram to one unit of velocity in one second.

From our previous determination, the given *current* is one that will exert entirely the same effects at a distance as a magnet that possesses one unit of magnetic moment when it flows around a planar region of magnitude one unit of area; i.e., the current whose strength is ordinarily chosen the be the unit of the strength of all other currents by observing it with the tangent galvanometer, and the *amount of electricity* that is present on each of the small balls is the amount that one is accustomed to assign as the basis for the unit of measurement for electrostatic measurements with the Coulomb torsion balance.

5 Section 5. Plan for Solving the Problem

If the *amount of electricity E that is collected on an isolated conductor* is discharged to Earth through the multiplier of a galvanometer, then it will exert a rotational moment on the magnetic needle of the galvanometer as it flows through it. Now, if one also extends the *discharge time* as much as needed by inserting a column of water into the path of the current in order for no spark to jump between the windings of the multiplier, then that *discharge time* will still define only an extremely small fraction of the *period of oscillation* of the magnetic needle, such that the part of the path that the needle covers during that *discharge time* (that is during the action of the discharge current) will be vanishingly small in comparison to the entire path of the needle; i.e., in comparison to the magnitude of the *elongation* that the needle attains over the course of *one-half an oscillation period.* The effect of the discharge current can then be considered to be an *impulse* that the needle would experience in its rest position, after which the *angular velocity* that the needle acquired could be calculated from known laws of oscillation by *observing the initial elongation of the needle after the discharge* at the moment of the impulse itself.

Furthermore, everything behaves exactly like an *induction impulse*, and also insofar as the nature of the discharge current is entirely indifferent to whether it consists of many separate partial discharges that rapidly follow each other or whether it is continuous with an intensity that decreases to zero rapidly according to some law. *The angular velocity that the needle acquires*

in that way will always depend upon the amount of electricity E.[17]

We can give the needle of the galvanometer a similar *impulse* by means of a *constant current* when we let the current act for only a very short time, and indeed the initial elongation will be the same whether the current has an intensity i during the time t or with the greater intensity ni during the shorter time t/n . Namely, if the duration t of the current is very small compared to the period of oscillation of the needle, then the *angular velocity* will always be found to be the same.[18] However, precisely the same *amount of electricity* will flow through the cross-section of the conductor in time t with an intensity of i that flows through it in time t/n with an intensity of ni.

Hence, when we impart an *impulse* to the needle by a *constant current of short duration*, the *angular velocity* of the needle (and as a result, its *elongation* as well) will also depend *solely upon the amount of electricity that has moved through the cross-section of the multiplier during the duration of the current* in this case.

Now, if we have discharged a known amount E of positive electricity through the same multiplier *in one case* and produced *the same elongation of the magnetic needle* by means of a constant current of very short duration *in the other case*, then we can conclude that *the positive amount of electricity x that flows through the cross-section of the conductor during the short duration of the constant current is:*

$$x = \frac{1}{2}E \ ,$$

which is a result of whose validity one can easily convince oneself, and which one might have to envision in terms of the processes that take place inside of the conductor during the discharge.

---

[17][Note by KW:] One finds that fact confirmed in all experiments. As is shown in the experiment in Appendix II (among other things), the elongation is not only proportional to the *amount of electricity* that is discharging, but it is also independent of the *discharge time*, within wide limits; because it does not matter how long or short the water column that one inserts is, provided the windings of the multiplier are not jumped over or the discharge time is extended in such a way that the effect of the discharge current will still continue when the needle has already moved noticeably from the rest position.

[18][Note by KW:] As long as the direction of its magnetic axis deviates only slightly from the plane of the multiplier windings, the *acceleration* that a needle whose magnetic moment is M and whose moment of inertia is K will acquire from a *constant current* of intensity i will be equal to AMi/K, where A means a constant that depends upon the dimensions of the multiplier and the distribution of magnetism in the needle. It will follow from this that the *angular velocity* that it acquires during time t will be equal to $AMit/K$, whose value will remain unchanged when i is replaced with ni and at the same time t is replaced with t/n.

If one would like to assume of the discharge, e.g., that *all* of the accumulated *positive* amount of electricity E flows through the *entire* multiplier in only the direction of the Earth, or that an *equal* amount of *negative* electricity flows in the opposite direction from the Earth, then the magnetic effect of such a *discharge current* would be precisely the same as the effect of a current for which only *one-half* of that positive amount of electricity flowed through the cross-section of the conductor in the given direction, but at the same time, an equal negative amount of electricity flowed in the opposite direction, which is a process that is assumed to take place at *constant current.* — However, should one be of the opposite opinion, namely, that absolutely none of the amount of electricity E that is collected in the isolated conductor (and just as little of the amount that is found in the Earth) will flow through the total windings of the multipliers, but that it will merely *give rise to* a double current in the wire that will include masses of neutral fluids that are large enough that a very small change in those masses will suffice to supply the isolated conductor with so much negative electricity that the positive electricity E that is collected in it will be neutralized, then one would also arrive at the same result in that way, since the whole discharge wire could be divided into a very large number of small pieces such that the amount of electricity $+\frac{1}{2}E$ would flow from each piece into the *following* one, while $-\frac{1}{2}E$ would flow into the *preceding* one, and as a result, an amount of electricity $+\frac{1}{2}E$ would flow from the last piece into the Earth, which would replace the first piece of the wire with the isolated conductor, while the amount of electricity $-\frac{1}{2}E$ would flow out of the first piece into the isolated conductor and neutralize the electricity that remains in it, but which will replace the last piece of the wire with the Earth. Finally, if one were also required to assume that somewhat more than one-half of the positive amount of electricity E went from the isolated conductor to the wire, while somewhat less than $-\frac{1}{2}E$ of negative electricity went in the opposite direction from the wire to the isolated conductor, then nothing would change in the result, since the magnetic effect will be determined by the sum of the two moving charges.

The impulse that the needle feels when the *accumulated amount of electricity E discharges* through the multiplier will be *just the same* as when a *constant current* goes through the multiplier during a time interval τ such that precisely *one-half* of E goes through the cross-section in the direction of the current *as positive electricity* and just as much goes in the opposite direction *as negative electricity*, assuming that the time interval τ represents only a very small part of the period of oscillation of the needle.

The solution to the problem will then emerge from taking the following *two steps*:

1. Measure the amount of electricity E in the given electrostatic units and observe the elongation of the magnetic needle of a galvanometer under its discharge.

2. Determine the small time τ during which a constant current of intensity equal to 1 (in magnetic units) must go through the multiplier of that galvanometer in order for the needle to acquire the same elongation.

If one then multiplies $\frac{1}{2}E$ by the number that shows how often t is included in one second, then $E/2\tau$ will express the *amount of positive electricity* that passes through the cross-section of the conductor during one second in the direction of a current whose intensity is equal to 1 in magnetic units. In other words:

$$\frac{1}{2\tau} \cdot E : 1$$

is the ratio of the *amount of positive electricity* that passes through that cross-section, to the one whose unit is based upon measuring the accumulated amount of electricity E in the isolated conductor, namely, the amount that must be found on each of two small balls for them to repel each other with a force equal to 1 at a distance equal to 1.

As far as the *second* step is concerned, the determination of τ requires no special experiment, since the value of τ can be determined by calculation from the number and dimensions of the windings of the multiplier, the elongation of the tangent galvanometer that is observed under the discharge, and the intensity of Earth magnetism much more precisely than would be possible by direct experiment, as one will see in Section 13.

However, the *first* step, which is concerned with determining the amount of electricity E, requires a combination of several experiments, which shall be described in Sections 6-12. Namely, it is important that, *first of all*, a still-unknown, but greater, amount of electricity is split into two parts in a *previously*-determined ratio, *and then* that the *greater* part E *is discharged* through the tangent galvanometer in order to observe its magnetic effect, but *finally*, the *smaller* part is measured by the electric force that it exerts upon the Coulomb torsion balance in order for the discharged part E to also get measured by the same measurement.

A *Leyden jar* whose external coating is connected to the Earth in a well-conducting way seems to be most suitable as a vessel for that amount of electricity whose part E should not be insignificant if its discharge were to produce a precisely-measurable effect on the needle of the tangent galvanometer. Hence (Section 6), that would next require the *ratio* by which

the *positive charge in that jar* is divided between it and *a large isolated ball*, the latter of which contacts the knob on the jar. The ratio $n : 1$, by which the charge in the jar *before* contact with the large ball to its charge *afterwards* is determined with the help of the *sine electrometer*, which will yield the ratio $1 : (n-1)$ of the amount of electricity E to the amount that goes over to the ball.

After this *ratio* has been determined precisely by several repetitions, the measurement of the amount of electricity that would go over to the large ball after such a division would be continued, to which end, the large ball, likewise after a charge that results from contact with the Leyden jar, would itself once more contact the 1 inch larger *fixed ball* of a *Coulomb torsion balance* that is equipped with a larger measuring scale. As Poisson and Plana have shown,[19] the ratio by which the electricity is divided between those two balls can be calculated from the ratio of their radii. That will be done in Section 8, from which, the charge that the *large ball* has received from the Leyden jar can be found from the amount of electricity e that is transferred to the *fixed ball of the torsion balance*, and then also the amount that remains in the Leyden jar, which will be employed to find the *discharge current* whose *magnetic effect* is to be observed.

The amount of electricity e would be measured after the *fixed ball* of the Coulomb torsion balance in which it is contained contacts the equally large *moving ball*, and in that way, e would be divided equally between those two balls. Namely (Section 7), from observing the gradual decrease in the torsion that would be necessary in order to keep the two balls at a *well-defined distance* from each other, the torsion would then be calculated that would, on first glance, be required if the charge in it were likewise to be able to go from the large ball through the Leyden jar, the fixed ball through the large one, and the moving one though the fixed ball while one observes the torsion. — In Section 9, one will find the calculation of the *amount of electricity ε* that would exert *a unit rotational moment* on the balance at the same distance when it is divided equally between the two balls of the torsion balance, in which one must take into account the non-uniform distribution of electricity on the surface of the ball. — In Section 10, one will find the determination of the *torsion* that would likewise exert *a unit rotational moment* on the balance from various observations. — With the help of the determinations that are contained in Sections 9 and 10, the amount of electricity e itself can be determined easily from the torsion that was found in Section 7, and then also the amount that remains in the Leyden jar, which will be done in Section 11, where the latter will be denoted by E', in order to distinguish it from

---

[19][Note by AKTA:] [Poi12a], [Poi12b], [Poi13], [Pla45] and [Pla54].

the amount of electricity E that is employed by the discharge current whose magnetic effect is to be determined. — In the brief intervening time between the moment of the division to the moment of the discharge, the electricity that remains in the Leyden jar will change, namely, a small part of the charge in the jar will be lost to the air, and part of it will be lost to a change in the *residue* in the jar, and although that change during such a brief intermediate time of — say — only three seconds would be extremely negligible, from the superb quality of the jar that was selected for that experiment, it will still be included in the calculation in Section 12, from which, one will at least see how the change $E - E'$ would be determined for other jars and longer intermediate times.

Finally, with the help of the determination of τ that is contained Section 13 and mentioned on p. 233,[20,21] the quantity $\frac{1}{2\tau} \cdot E$ will be calculated in Section 14, and with that, the problem that was posed above will be solved. The Section that follows it will include *applications*, for the most part, to which the determination of the *constant c*, which has been mentioned several times, belongs.

The two *Appendixes* include more precise descriptions of the *torsion balance* and the *tangent galvanometer*; for that of the *sine electrometer*, see Poggendorf's *Annalen* 88 (1853).[22]

It can be inferred from the satisfying agreement, without exception, between all published experiments (of which ones that were analyzed in Sections 6 and 7 were the most difficult) that the result can be considered to be accurate to within 1 to 2 percent. The calculation was performed with a precision of an even smaller fraction in order for the determination of the uncertainty in the results to depend upon merely the magnitude of the unavoidable observation error.

---

[20][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 621.
[21][Note by AKTA:] [KW57, p. 621 of Weber's *Werke*].
[22][Note by AKTA:] [Koh53].

6 Section 6. Determining the Conditions under which Electricity will be Divided between the Interior of a Leyden Jar and a Large Ball while the Exterior of the Jar is Connected to the Earth

The following table gives the results of two series of observations that were performed with the *sine electrometer* of the decrease in charge in a Leyden jar by transferring it to a large uncharged ball that contacted the knob on the jar, while the exterior of the jar was connected to the Earth by a good conductor.

The Leyden jar was previously connected to the sine electrometer with a conducting wire whose end was placed in a small indentation into the knob on the jar. Once the position of the sine electrometer had been observed, that end of the conducting wire was raised with a silk thread, and then the large ball contacted the knob of the jar, whereupon the exterior of the jar was connected to the Earth by a conductor. With double, triple and quadruple contacts, the individual contacts followed as quickly in succession as would permit the large ball to discharge completely in between them. If the sine electrometer, which suffered only a negligible loss to the air in between contacts, was then once more connected with the jar by a connecting wire that was kept insulated by a silk thread, then the needle of the electrometer, which was initially at rest, would be deflected only slightly in that way, since the jar had lost relatively little of its charge by contact with the ball, and since that loss was compensated approximately by the relatively smaller loss to the air that the jar suffered in comparison to the sine electrometer, which explained the shortness of the time during which the individual measurements could be performed in comparison to the end of each series of experiments.

Precise time measurements of the moment at which each individual contact was made could not be carried out, and the data that is contained in the following table is based upon mere estimates, which can, however, be considered to be admissible to within 1-2 seconds, which is a precision that suffices completely for this. Both series were made on 2 April 1854 in the Physics Institute at Göttingen.

First Series			
No.	Time	Needle deflection on the sine electrometer	n
1.	$8^h49'54''$	32º36.2′	
2.	$50'0''$	(quadruple contact)	1.0324
3.	$51'25''$	24º13.7′	
4.	$53'46''$	23º31.3′	
5.	$53'52''$	(quadruple contact)	1.0299
6.	$54'42''$	17º45.6′	
7.	$58'56''$	14º49.3′	
8.	$59'2''$	(quadruple contact)	1.0167
9.	$59'55''$	12º47.6′	
10.	$9^h2'7''$	12º34.3′	
11.	$2'13''$	(quadruple contact)	1.0325
12.	$2'50''$	9º41.7′	
13.	$4'12''$	9º41.7′	
14.	$4'18''$	(quadruple contact)	1.0355
15.	$4'53''$	7º21.3′	
16.	$7'22''$	7º30.2′	
17.	$7'28''$	(quadruple contact)	1.0311
18.	$8'9''$	5º51.2′	
19.	$10'7''$	4º48.3′	
20.	$10'13''$	(quadruple contact)	1.0305
21.	$10'51''$	4º32.9′	

	Second Series		
No.	Time	Needle deflection on the sine electrometer	n
1.	$9^h40'7''$	46º30.5'	
2.	$41'57''$	44º9.0'	
3.	$42'0''$	(single contact)	1.0330
4.	$42'23''$	40º23.9'	
5.	$44'0''$	39º10.5'	
6.	$44'3''$	(single contact)	1.0308
7.	$44'23''$	36º15.7'	
8.	$46'24''$	35º11.7'	
9.	$46'27''$	(single contact)	1.0379
10.	$46'51''$	32º24.6'	
11.	$48'24''$	32º46.6'	
12.	$48'27''$	(single contact)	1.0490
13.	$48'51''$	29º21.1'	
14.	$51'41''$	28º31.0'	
15.	$51'44''$	(single contact)	1.0390
16.	$52'9''$	26º14.2'	
17.	$52'52''$	26º14.2'	
18.	$52'55''$	(single contact)	1.0375
19.	$53'25''$	24º14.7'	
20.	$58'30''$	19º41.9'	
21.	$9^h58'33''$	(single contact)	1.0303
22.	$59'1''$	18º27.6'	
23.	$10^h5'52''$	17º42.6'	
24.	$5'56''$	(double contact)	1.0328
25.	$6'28''$	15º30.1'	
26.	$7'14''$	15º30.1'	
27.	$7'19''$	(triple contact)	1.0338
28.	$7'45''$	12º38.7'	
29.	$10'13''$	12º38.7'	
30.	$10'19''$	(quadruple contact)	1.0315
31.	$11'27''$	9º50.0'	
32.	$12'44''$	9º50.0'	
33.	$12'50''$	(quadruple contact)	1.0292
34.	$13'27''$	7º47.8'	

The last column in this table, under n, gives the *ratio* of the charge in the jar before contact with the ball to the charge after contact, which is

always made immediately before and after the moment of contact between the two. The *second* and *third* columns contain observations that are reckoned according to the following rule:

- $q_{''}^2$ and $q_{'}{}^2$ denote the sines of the observed deflections for the two previous times of observation,

- $q^{'2}$ and $q^{''2}$ denote the sines of the observed deflections for the two following times of observation,

- $-t_{''}$, $-t_{'}$, t', t'' are the associated observation times, measured from the moment of contact,

- m is the number of times the contact was repeated.

Hence:[23]

---

[23][Note by KW:] The observations of the deflection of the needle in the third column and the time in the second column will immediately give the values of $q_{''}$, $q_{'}$, $q^{'}$, $q^{''}$ and the associated values of $-t_{''}$, $-t_{'}$, t', t'' at which the values of q_0 and q^0 should be calculated, which are true for the moments immediately before and after the contact. The cited rule will then be implied in the following way:

1) For the brief time duration of the experiment, it suffices to assume that the charge lost to the air over time and the charge at the moment of observation are proportional, from which, one will then get the following four values for the reduced observations at the moment of contact:

$$(1 - \alpha t_{''})\, q_{''}\ , \qquad (1 - \alpha t_{'})\, q_{'}\ , \qquad (1 + \alpha t')\, q^{'}\ , \qquad (1 + \alpha t'')\, q^{''}\ .$$

2) If one adds each of these values to the residue in the jar at the time in question then the first two, which represent the *total charge before contact*, must be equal, and similarly for the last two, which represent the *total charge after contact*. When one denotes the residue at time t by r_t, one will then get the equations:

$$(1 - \alpha t_{''})\, q_{''} + r_{-t_{''}} = (1 - \alpha t_{'})\, q_{'} + r_{-t_{'}} = q_0 + r_0\ ,$$

$$(1 + \alpha t')\, q^{'} + r_{t'} = (1 + \alpha t'')\, q^{''} + r_{t''} = q^0 + r^0\ .$$

However, the residue *before* and *after* contact (see Section 12) can be represented by:

$$r_t = \beta \left(1 - e^{-\gamma(\vartheta + t)^\delta}\right) \cdot (q_0 + r_0)\ , \qquad r_t = \beta \left(1 - e^{-\gamma(\vartheta' + t)^\delta}\right) \cdot \left(q^0 + r^0\right)\ .$$

The residue remains unchanged at the moment of contact, so $r_0 = r^0$. That easily implies that *for small values of t before* and *after* contact, r_t can be set to:

$$r_t = r_0 + at\ , \qquad r_t = r_0 + a't\ ,$$

where a and a' denote two coefficients that are determined from the observations. — By substituting those values in the equations above, in which one might likewise replace $\alpha q_{''}$ and $\alpha q_{'}$ with αq_0, and similarly replace $\alpha q^{'}$ and $\alpha q^{''}$ with αq^0, one will get:

$$q_0 = q_{'} - (a + \alpha q_0)\, t_{'} = q_{''} - (a + \alpha q_0)\, t_{''}\ ,$$

$$q^0 = q^{'} + \left(a' + \alpha q^0\right) t' = q^{''} + \left(a' + \alpha q^0\right) t''\ ;$$

and as a result:

$$q_0 = \frac{t_{''} q_{'} - t_{'} q_{''}}{t_{''} - t_{'}}\ , \qquad q^0 = \frac{t'' q^{'} - t' q^{''}}{t'' - t'}\ ,$$

$$n = \sqrt[m]{\frac{q_0}{q^0}} = \sqrt[m]{\frac{t'' - t'}{t_{''} - t_{'}} \cdot \frac{t_{''} q_{'} - t_{'} q_{''}}{t'' q^{'} - t' q^{''}}}\ .$$

$$n = \sqrt[m]{\frac{t'' - t'}{t'' - t_{\prime}} \cdot \frac{t_{\prime\prime}q_{\prime} - t_{\prime}q_{\prime\prime}}{t''q' - t'q''}} \; .$$

Indeed, some of the observations in these two series of observations are less definitive (which is almost unavoidable when three observers collaborate), and in that way one can find that is permissible to discard some values of n completely: for example, the one that is cited in no. 8 in the first column and the ones in nos. 12, 15, 33 in the second column. However, it will follow that the removal of those values will have no appreciable effect on the determination of the mean value of n, since one finds that the mean values *with* and *without* removal are:

$$n = 1.03282 \; , \qquad n = 1.03297 \; ,$$

respectively.

A similar series of observations with the same jar and ball that was carried out earlier in Marburg yielded the following mean value for the ratio n:

$$n = 1.03263 \; .$$

Hence, the desired ratio will be henceforth assumed to be:

$$n = 1.03276 \; .$$

Finally, this ratio of the charge in the jar *before* and *after* contact with the large ball also yields the *ratio of the distribution of the electricity between the jar and the large ball at the moment of contact; namely it is equal to*:

$$1 : 0.03276 \; .$$

7 Section 7. Corresponding Observations of the Deflection of the Tangent Galvanometer that is Produced by the Amount of Electricity E that Flows Through the Multiplier, and the Torsion in the Coulomb Torsion Balance Through which the Two Balls Charged with the Amount of Electricity e will be Maintained at the Same Distance as the Uncharged Ones

The arrangement of the instruments that were used in the experiment that was mentioned before in Section 5 is depicted in Figure 1, which will serve to make it more intuitive.[24]

---

[24][Note by AKTA:] An improved version of Figure 1 has been prepared by D. H. Desphenich, namely:

Fig. 1.

The *tangent galvanometer* is denoted by m, whose multiplier wire is connected to the Earth at its one end by a conducting wire l that is soldered to a plate E that is buried in wet soil, while the other end of the wire leads through the air to the long U-shaped glass tubes g and g', which are filled with water. m' represents the scale and telescope for observing the needle of the tangent galvanometer, which is provided with a mirror.

d refers to the *Coulomb torsion balance*, which will be described in more detail at the end of this treatise in Appendix I. d' represents the scale and telescope for observing the state of the torsion balance. Namely, a long hanging shellac rod is fixed to the torsion wire under the arm that carries the moving ball, and it carries a mirror at its end, to which the telescope points. — The large ball hangs from the ceiling of the room by a silk thread at k. l' is a fork in the conducting wire l so one can connect the exterior of the jar f to the Earth. — u is a clock, and a is a hole in the ceiling of the room through which a wire from the conductor of an electrification machine that was found in the upper room was led to the small conductor c in order to charge the jar f.

Once the jar f was charged, and a clamping screw was fixed to the wire l', the jar was then contacted by the large ball k. The amount of electricity that remained in the jar after that contact will be denoted by E'. After three seconds, during which E' went to E by losing electricity to the air and the formation of a residue, the knob on the jar f, as is suggested in Figure 1, was contacted by a metal knob that stuck out of the U-shaped tube g, and the observer at the telescope m watched the elongation of the magnetic needle of the tangent galvanometer that was produced by the discharge current of

the amount of electricity E that went through the multiplier.

Immediately after the jar f was discharged, the fixed ball on the Coulomb torsion balance, which had been kept on standby, was charged by the ball k and quickly placed into the torsion balance; however, the ball k itself was likewise discharged in that way.

Thereupon, the torsion was measured several times in brief intermediate times, which was necessary in order to keep the two balls in their positions, in which, the two radii that pointed from the rotational axis to center of the ball would define a right angle. The torsion that would exist at the moment when the large ball k was charged by the jar f (so the two balls in the torsion balance had also been charged and could be inserted) could then be *calculated* from the gradual decrease in that torsion according to Coulomb's law, which says that the charge decreases geometrically when time increases arithmetically.[25] The torsion that was first noticed for each number is *calculated* in that way in the following table. The amount of electricity e that went from the large ball k to the fixed ball of the torsion balance at the moment of contact will be determined from it in Section 11.

The last column of the following table, which is labeled with $A/\sqrt{T}$, contains the quotients that take the form of the deflection of the magnetic needle in the tangent galvanometer, expressed in scale divisions, divided by the square root of the torsion in the torsion balance, expressed in minutes. — The distance from the mirror to the scale of the tangent galvanometer was equal to:

$$6437\frac{1}{2} \text{ scale divisions .}$$

---

[25][Note by KW:] By a series of experiments in which the fixed ball was sometimes found to be outside the case of the torsion balance and sometimes inside of it between the individual determinations of torsion, it was confirmed that the loss of electricity to the air when it was inside the case was the same as the loss to the air when it was outside of it, which might have been expected from the size of the case. If that were not the case then the aforementioned application of Coulomb's law would not be directly applicable, since the fixed ball would be found outside of the case for some moments before it could be placed inside the torsion balance.

No.	Time	Deflection of the tangent galvanometer in scale divisions $= A$	Torsion of the torsion balance in minutes $= T$	$A/\sqrt{T}$
1.	$8^h11'8''$ 16'13'' 21'16'' 26'35'' 32'32''	73.5	175.3' 152.4' 136.1' 118.3' 99.9'	5.55
2.	$8^h37'8''$ 42'4'' 45'14'' 50'10'' 54'40''	80.0	237.1' 208.4' 189.1' 165.3' 148.1'	5.20
3.	$9^h0'37''$ 5'14'' 9'19'' 14'11'' 18'10''	96.5	332.9' 297.5' 270.6' 238.5' 218.3'	5.29
4.	$9^h31'14''$ 35'17'' 41'1'' 47'43'' 55'0''	91.1	265.1' 249.2' 226.2' 201.1' 178.0'	5.59
5.	$10^h1'46''$ 6'24'' 10'54'' 16'31'' 22'4''	97.8	332.4' 306.0' 280.4' 251.1' 228.6'	5.36

8 Section 8. Calculating the Ratio of the Two Amounts of Charge $E' : e$

The radius of the large ball was:

$$a = 159.46 \text{ millimeters },$$

and the radius of the fixed ball in the Coulomb torsion balance was:

$$ba = 11.537 \text{ millimeters }.$$

If one now sets the ratio by which the electricity equal to $0.03276E'$ that is transferred from the jar to the first ball by contact with the latter equal to:

$$(0.03276E' - e) : e = A : b^2 B ,$$

as in Section 6, then, from Plana ("Mémoire sur la distribution de l'électricité à la surface de deux sphères conductrices," Turin, 1845, pages 64, 66):[26]

$$\frac{B}{h} = \frac{1}{1+b} + \frac{1}{(1+b)^2} \left\{ k_2 + \frac{b}{1+b} k_3 + \frac{b^2}{(1+b)^2} k_4 + \frac{b^3}{(1+b)^3} k_5 \ldots \right\} ,$$

and when one sets $b/(1+b) = a$:

$$\frac{A}{h} = \frac{1}{2} + \frac{a^3}{1-a^2} + \frac{\pi a}{2} \cot \pi a + a^3 k_3 + a^5 k_5 + a^7 k_7 \ldots ,$$

where:

$$k_n = \frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{4^n} + \frac{1}{5^n} + \ldots$$

That yields the cited value for the desired ratio:

$$(0.03276E' - e) : e = A : b^2 B = 1 : 0.0079377 ;$$

as a result:

$$E' : e = 3876 : 1 .$$

9 Section 9. Calculating the Amount of Electricity ε with which the Two Balls in the Coulomb Torsion Balance Must be Charged in Order for Their Repulsion to Exert One Unit of Rotational Moment on the Torsion Balance

The radius of the fixed ball on the Coulomb torsion balance was equal to 11.537 millimeters, and the radius of the moving ball was equal to 11.597 millimeters, so one can then assume to no detriment that the mean radius of the two almost-equal balls in the following calculation is:

---

[26][Note by AKTA:] [Pla45].

$$a = 11.567 \text{ millimeters} .$$

Furthermore, the distance from the rotational axis to the center of the fixed ball was equal to 93.53 millimeters, the distance from the rotational axis to the center of the moving ball was equal to 61.7 millimeters, and both centers defined a right angle with the axis of rotation. That yielded the distance between the centers as being equal to:

$$112.05 \text{ millimeters} ,$$

which was also confirmed by direct measurement of that distance.

Now, if each of the two balls contains one-half of the amount of electricity to be determined ε, then if one assumes that *this electricity is distributed uniformly on the surface of each ball* then, from known laws, that:

1. A uniformly distributed amount of electricity on the surface of the ball will act upon all points in external space just as if it were concentrated at the center of the ball.

2. The force of repulsion that the amount of electricity that is concentrated at a point exerts upon another [amount of electricity] concentrated at a point, is equal to the quotient of the product of both amounts of electricity divided by the square of the distance between them,

one would obtain immediately the *force of repulsion between both balls*, namely:

$$\frac{1}{4} \cdot \frac{\varepsilon^2}{112.05^2} = \frac{\varepsilon^2}{50221} .$$

However, if that force of repulsion were to be found *precisely*, then the assumption above would be inadmissible, and one would have to determine the *non-uniformity of the distribution of electricity on the surface of every ball* precisely from the given magnitude and distance and include it in the calculation.

In Poisson's "Mémoire sur la distribution de l'éléctricité à la surface des corps conducteurs" (*Mèmoires de l'Institut. Année* 1811. *Première partie*, page 88),[27] one finds the following expression for the *density z* of the electricity on the surface of a small ball at a great distance from another ball when

---

[27][Note by AKTA:] [Poi12a] and [Poi12b].

the *mean* density on the first ball is given to be equal to B and is equal to A on the latter:

$$z = B - \frac{3a^2 A}{c^2} \cdot \mu_| + \frac{5a^2 bA}{2c^3}\left(1 - 3\mu_|^2\right) \ ,$$

in which b and a are the radii of the two balls, c is the distance between their centers, and $\mu_|$ means the cosine of the angle φ that the radius of the first ball defines with the direction of c at the location in question. — If one wishes to apply that general rule to the foregoing case, then one must set:

$$A = B$$

$$a = b \ ,$$

and when one writes the value $\cos\varphi$ for $\mu_|$, it will follow that the *density* is:

$$z = A\left[1 - \frac{3a^2}{c^2}\cos\varphi + \frac{5}{2}\frac{a^3}{c^3}\left(1 - 3\cos^2\varphi\right)\right] \ .$$

Furthermore, that *density* implies the *outward-pointing electric pressure perpendicular to the surface of the ball* at the location in question from the known law that was proved by Poisson in the cited treatise, according to which, the pressure is proportional to the square of the density, or more precisely, it is equal to the square of the density z^2 multiplied by the number 2π:

$$2\pi \cdot z^2 \ .$$

If one then decomposes that *pressure* in the direction of the extended line c and a direction that is perpendicular to it, then one will get that the *component parallel to the extended line c* is equal to:

$$-2\pi z^2 \cdot \cos\varphi \ .$$

Finally, if one substitutes the value of z above in this [expression], then one will get the decomposed *pressure for two equal elements of the surface of the ball whose connecting line is parallel to the line c*, for which the value of φ between them is then extended to π, and *collected along the direction* of the extended line c from:

$$24\frac{\pi a^2}{c^2}A^2\left[1 + \frac{5}{2}\frac{a^3}{c^3}\left(1 - 3\cos^2\varphi\right)\right]\cos^2\varphi \ ,$$

from which, one will find the *force of pressure* that is parallel to the extended line c, *first of all*, for the two zones of width $ad\varphi$, which both include the elements of the surface of the ball that belong to the values of φ, extended to π, upon multiplying by the area $2\pi a^2 \sin\varphi d\varphi$:

$$48\frac{\pi^2 a^4}{c^2}A^2\left[1 + \frac{5}{2}\frac{a^3}{c^3}\left(1 - 3\cos^2\varphi\right)\right]\cos^2\varphi \sin\varphi d\varphi \ ,$$

and *secondly*, for the total surface of the ball, by integration:

$$48\frac{\pi^2 a^4}{c^2}A^2\int_0^{\pi/2}\left[1 + \frac{5}{2}\frac{a^3}{c^3}\left(1 - 3\cos^2\varphi\right)\right]\cos^2\varphi \sin\varphi d\varphi$$

$$= \ 16\frac{\pi^2 a^4}{c^2}\left(1 - 2\frac{a^3}{c^3}\right)A^2 \ ,$$

in which A is the *mean* density of the electricity on the surface of each of the two balls of radius a, and as a result:

$$4\pi a^2 \cdot A$$

will represent the *amount of electricity* that is distributed on the surface of each ball.

However, the *desired amount of electricity* that is distributed on both surfaces collectively (whose force of repulsion should exert a unit of rotational moment on the torsion balance) was denoted by ε above; as a result, one has:

$$\frac{1}{2}\varepsilon = 4\pi a^2 \cdot A \ ,$$

from which:

$$A = \frac{\varepsilon}{8\pi a^2} \ .$$

If one substitutes this value of A, one will get the *force of pressure* parallel to the direction of the extended line c; i.e., the *force of pressure on the two balls*:

$$\frac{1}{4}\left(1 - 2\frac{a^3}{c^3}\right)\frac{\varepsilon^2}{c^2} \ ,$$

or when one substitutes the aforementioned values for a and c in this [expression], namely:

$$a = 11.567 \ ,$$

$$c = 112.05 \ ,$$

one will get:[28]

$$\frac{\varepsilon^2}{50331} \ .$$

Finally, the product of the force of repulsion between the two balls in the direction from the rotational axis to the direction of that force — i.e., along the perpendicular dropped from the line c — gives the value of the *rotational moment* that this force of repulsion exerts upon the torsion balance, which should be equal to 1.

However, since the line c that connects the centers of both balls defines a right triangle at the rotational axis with the horizontals that are drawn from both centers to the rotational axis, the perpendicular that is dropped from the rotational axis to the hypotenuse of the rectangular triangle c will be equal to the product of the two catheti[29] divided by the hypotenuse, or since the two catheti are 93.53 and 61.7 millimeters long, and $c = 112.05$ millimeters, that expression will be equal to:

$$\frac{61.7 \times 93.53}{112.05} = 51.5025 \text{ millimeters} \ .$$

Now, it follows from this that the *rotational moment* that is exerted by the electric force of repulsion on the two balls of the torsion balance will be equal to:

$$51.5025 \cdot \frac{\varepsilon^2}{50331} = \frac{\varepsilon^2}{977} \ .$$

---

[28][Note by KW:] That implies that, due to its non-uniform distribution on the outer surface, the electricity that is contained in each ball cannot be thought of as concentrated at the *center* of the ball. — However, one has:

$$\frac{\varepsilon^2}{50331} = \frac{1}{4} \cdot \frac{\varepsilon^2}{112.17743^2} \ ,$$

which then implies that the force of repulsion between the two balls is the same as if the two halves of the total amount of electricity that is contained in them were concentrated at two points that are separated by 112.1734 millimeters, that is, since that distance is 0.1234 millimeters greater than the distance between the centers, at two points that lie at a distance of 0.0617 millimeters from the two centers.

[29][Note by DHD:] Viz., shorter sides.

The requirement that the *rotational moment* that originates in the electric force of repulsion on the two balls should be equal to 1 will be satisfied in such a way that the *amount of electricity* that is contained in the two balls collectively will be:

$$\varepsilon = \sqrt{977} = 31.25 \ .$$

This determination of ε bases the unit for the amount of electricity as the amount that will make two equal amounts of electricity exert a unit force of repulsion when they are at a unit of distance and in a state of relative rest.

10 Section 10. Calculating the Torsion ϑ that the Wire from which the Coulomb Torsion Balance Hangs Must Possess in Order to Exert One Unit of Rotational Moment on the Torsion Balance by Its Force of Torsion

The *rotational moment* that is exerted upon the torsion balance by the torsion in the wire to which it [that is, the balance] hangs is known to be proportional to the *torsion* and the *torsion coefficient* of the wire — or more precisely — it is equal to the *product of the torsion angle, expressed in units of radii, with the directing force*[30] *that the wire exerts upon the torsion balance.* One therefore needs only to determine that *directing force* in order to infer from it the torsion angle ϑ for which the rotational moment that is exerted upon the torsion balance is equal to *one unit.*

From the known laws of the elasticity of solid bodies, the magnitude of the directing force that is exerted upon the wire is independent of the size and weight of the body that hangs from the wire, and other bodies, instead of the torsion balance, can therefore be hung from the wire and observed in order to determine the *directing force* of the wire.

First of all, one might hang a circular brass plate horizontally at its center from the wire, instead of the torsion balance. That brass plate has:

<div align="center">a mass of 191 112.4 milligrams,</div>

<div align="center">a radius of 63.95 millimeters.</div>

A small vertical cylinder with:

---

[30][Note by AKTA:] *Direktionskraft* in the original.

a *mass* of 2626.0 milligrams,

a *radius* of 3.25 millimeters

will serve to connect the wire with the disc. The *period t* of the torsion oscillations of the plate was then observed and found to be:

$$t = 47.139 \text{ seconds.}$$

However, from the foregoing data, the *moment of inertia of the oscillating plate* was:

$$K_1 = \frac{1}{2} \cdot 63.95^2 \cdot 191112.4 = 390\ 790\ 000 \ ,$$

and the *moment of inertia* of the small cylinder was:

$$K_2 = \frac{1}{2} \cdot 3.25^2 \cdot 2626 = 13\ 868 \ ,$$

so when they are combined:

$$K = K_1 + K_2 = 390\ 603\ 868 \ .$$

Now, from the known laws of such oscillations, one will get the value of the *directing force D* from that *moment of inertia K* and the observed *period of oscillation t*:

$$D = \frac{\pi^2 K}{t^2} = 1\ 735\ 800 \ .$$

Secondly, a brass cylinder was hung horizontally by its center from the same wire. That cylinder had:

a *mass* of 58897.1 milligrams,

a *length* of 269.7 millimeters,

a *radius* of 2.865 millimeters.

That same small vertical cylinder served to connect it with the wire, as it did in the foregoing experiment. The *period t'* of the torsional oscillation of that rod was then observed and found to be:

$$t' = 44.9537 \text{ seconds.}$$

From the foregoing data, the *moment of inertia of the oscillating rod* was:

$$K_1' = \frac{1}{12} \left(269.7^2 + 3 \cdot 2.865^2 \right) 58897.1 = 357\ 130\ 000 \ ,$$

and then the *total moment of inertia*, including the small vertical cylinder was:

$$K' = 357\ 143\ 868 \ .$$

Those observations then yielded the value of the *directing force D* as:

$$D = \frac{\pi^2 K'}{t'^2} = 1\ 744\ 200 \ .$$

As a result, the mean of the two series of observations was:

$$D = 1\ 740\ 000 \ .$$

Now, should the product of this value of D with the torsion angle, expressed in units of the radius — i.e., the *rotational moment* that the wire exerts upon the torsion balance — be equal to 1, then that would imply that the value of the angle of rotation or the desired *torsion in the wire* ϑ would be equal to the angle whose arc is equal to 1/1740000 of the radius, or:

$$\vartheta = 0.0019757 \text{ arcminutes} \ .$$

11 Section 11. Calculating the Amounts of Electricity E' and e in the Observations that were Described in Section 7

In the experiments that were described in Section 7, the following values were found for the torsion angle in the Coulomb torsion balance when it was in equilibrium, where the various experiments are distinguished by numbers:

No.	Torsion angle in minutes
1.	175.3
2.	237.1
3.	332.9
4.	265.1
5.	332.4

However, the equilibrium of the torsion balance shows that the rotational moment that is exerted on the torsion balance by the wire is equal and opposite to the rotational moment of the force of repulsion between the two balls. — Nonetheless, the *first* rotational moment was found by dividing the observed torsion angle by the angle $\vartheta = 0.0019757$ arc minutes that was determined in the previous Section, which was the angle through which the wire would have to be rotated in order to exert *one unit of rotational moment* on the torsion balance. One then gets the *rotational moment* that the wire exerts on the torsion balance in the experiments that were described.

No.	Rotational moment of the wire
1.	88 728
2.	120 010
3.	168 500
4.	134 180
5.	168 240

From Section 9, the *last* of the rotational moments that originates in the electric repulsive forces between the two balls is:

$$\frac{e^2}{\varepsilon^2} = \frac{e^2}{977} \ ,$$

where e denotes the *amount of electricity* with which the two balls of the torsion balance are collectively charged, which one can then calculate in the five cited experiments from the *equality* of the two rotational moments, which is done in the following table. In addition, the values of E' that are calculated from the proportion:

$$E' : e = 3876 : 1$$

that was found in Section 8 are entered the last column of that table.

No.	e	E'
1.	9 310	36 086 000
2.	10 828	41 970 000
3.	12 830	49 730 000
4.	11 450	44 379 000
5.	12 821	49 593 000

12 Section 12. Calculating the Correction that is Required by the Loss of Electricity and the Residue in the Leyden Jar During the Transfer of Electricity up to the Elapsed Time when the Jar is Discharged, which Equals $E' - E$

The amount of electricity E' that remains in the Leyden jar after the charging of the large ball will experience a small change during the time interval of

three seconds up to its discharge, partly by loss to the air and partly by the formation of residue. The amount E that is still present in the jar can then be determined from E' in the following way:

In Poggendorf's Annalen 91 (1854), one will find a method given for determining the formation of the residue in a Leyden jar.[31] In accordance with it, if Q is an amount of electricity that is suddenly transferred to the jar, Q_t of which is lost to the air in t seconds, then a residue of r_t will have formed at time t whose equation is:

$$r_t = p\left(Q_t - Qe^{-\frac{b}{m+1}\cdot t^{m+1}}\right) . \qquad (I)$$

From the previous investigation, the constants of the jar that is used have the values:

$$p = 0.04494 , \qquad b = 0.1834 ,$$

while $m+1$ possesses a magnitude equal to 0.4255, which is the same for all jars.

If those constants are determined for a jar then the constant α that refers to the electricity lost to air can also be easily found. One suddenly transfers an unknown charge Q from the jar to the Earth and at the times:

$$t_1, t_2, ..., t_n ,$$

one observes the available charges:

$$L_{t_1}, L_{t_2}, ..., L_{t_n}$$

with the *sine electrometer*. Now, if ν_t denotes the amount of electricity that has leaked to the air up to time t then:

$$L_t = Q - r_t - \nu_t . \qquad (II)$$

However, for small values of t, one can set:

$$\nu_t = \alpha \cdot t \frac{Q + L_t}{2} ,$$

and if $Q - \nu_t$ is written for Q_t in equation (I), in addition, then one will get:

$$L_t = Q(1 - \rho_t) - \alpha(1 - p)t\frac{Q + L_t}{2} ,$$

---

[31][Note by AKTA:] [Koh54a] and [Koh54b].

in which $p\left(1 - e^{-\frac{b}{m+1}\cdot t^{m+1}}\right)$ has been replaced with ρ_t.

Now, that equation shall suffice for all observations. If one calculates ρ_t for the times of the first and last observations and substitutes those values in the equation, along with the observed values of L_t and t, then one will get two equations in the two unknown quantities Q and α.

Now, once a charge was suddenly imparted to the Leyden jar in the location where the previous experiments were carried out, the following results for the determination of α would be obtained from the observations:

t	L_t	ρ_t
23	0.6676	0.03619
65	0.6576	0.04142
128	0.6483	0.04344
226	0.6389	0.04435

One has $L_t = \sqrt{\sin\varphi}$ in this, and φ is the deflection that is observed in the *sine electrometer*. However, ρ_t is calculated from t and the constants of the jar. — Upon combining the first and last observations, one finds that:

$$Q = 0.6956 , \qquad \alpha = 0.000\ 179\ 35 .$$

Equation (III) then yields the following associated values for t and L_t with those values:

t	L_t
23	0.6676
65	0.6592
128	0.6506
226	0.6389

which deviate from the observed values so slightly that the values that were found for α can employed precisely in order to find the correction to E'. In *three seconds* then, the *loss of electricity to the air* will amount to:

$$0.000538$$

times the total charge E'.

The *residue* that is created in the same time will be found in the following way:

Immediately before contact with the large ball, which results t seconds after the jar is charged, the latter will have an available charge of L_t and residue r_t that cannot be discharged. If one writes $Q - \nu_t$ in place of Q_t in equation (I), sets ν_t equal to its value of $\alpha \cdot t \frac{Q+L_t}{2}$, and sets Q equal to the

value that is implied by equation (III) then one will get the residue at the time t, expressed in terms of the available charge that is present at that time:

$$r_t = \frac{\rho_t - \alpha t \left(p - \frac{1}{2}\rho_t\right)}{1 - \rho_t - \frac{1}{2}\alpha t(1 - p)} \cdot L_t = \beta L_t \ . \qquad (IV)$$

After the ball has been charged, only an available charge of L_t/n will remain in the jar (Section 6), so an amount of electricity $\left(\frac{1}{n} + \beta\right) L_t$. Now, the form that the ratio of the residue will take after that partial discharge will depend upon whether the residue that forms βL_t is less than, equal to, or greater than the limiting value:

$$p\left(\frac{1}{n} + \beta\right) L_t$$

of the residue for the charge that is still present in the jar, which will, in turn, depend upon whether n is less than, equal to, or greater than $p/[\beta(1 - p)]$, respectively.

In the present experiments, t was close to 60 seconds, in the mean. If one substitutes that value in equation (IV) then that will imply that:

$$\beta = 0.04286 \ , \qquad \frac{p}{\beta(1 - p)} = 1.0978 \ .$$

Since it was found in Section 6 that $n = 1.03276$, so it is less than $p/[\beta(1-p)]$, it emerges that the residue will continue to increase. However, its growth will be slower than before the partial discharge, since the present limiting value of the residue that has already formed lies closer than it did before, and indeed the further formation will proceed as if the residue that is present βL_t were generated by the present charge $(1/n + \beta)L_t$. However, that would have required a time that follows from the equation:[32]

$$r_t = \beta L_t = \left(\frac{1}{n} + \beta\right) L_t \cdot p \left(1 - e^{-\frac{b}{m+1}t^{m+1}}\right) \ ,$$

from which, it will follow that:

$$\log t = \frac{1}{m + 1} \log \left[-\frac{m + 1}{b} \ln \left(1 - \frac{\beta}{\left(\frac{1}{n} + \beta\right)p}\right)\right] \ ,$$

which yields 85.9 seconds.

---

[32][Note by KW:] That equation is formed according to the residue equation (I), in which one must now set $Q = (1/n + \beta)$ in place of Q_t.

From the charge $E' = L_t/n$ that is present the moment after contact with the large ball, the resulting growth in the residue will then get lost in the *three seconds* up to the discharge of the jar, which is determined from:

$$\left[\left(\frac{1}{n} + \beta\right) p \left(1 - e^{-\frac{b}{m+1}88.9^{m+1}}\right) - \beta\right] L_t = 0.00010 L_t \ ,$$

or since $L_t = nE'$:

$$0.000\ 103 \cdot E' \ .$$

That finally gives the desired correction:

$$E' - E = (0.000\ 538 + 0.000\ 103)E' = 0.000\ 641 E' \ ,$$

and one will then get the corrected values E for the values of E' that were given in the previous Section, which will give the amount of electricity that is actually discharged to the multiplier, as follows:

No.	E
1.	36 060 000
2.	41 940 000
3.	49 700 000
4.	44 350 000
5.	49 660 000

13 Section 13. Calculating the Time Duration that a Current with the Normal Strength that was Described in Section 4 Must Have in Order to Produce the Deflections of the Tangent Galvanometer that were Observed in Section 7

The *deflections of the tangent galvanometer* that were cited in Section 7 were observed in *scale divisions*. One will obtain those deflections in *arc values for a radius of* 1 by dividing them by the radius (or twice the distance from the mirror to the scale), expressed in scale divisions, which equals 12875.

No.	Deflection in scale divisions	Deflection in arc values for radius = 1 φ
1.	73.5	0.005 708 7
2.	80.0	0.006 213 6
3.	96.5	0.007 495 2
4.	91.1	0.007 075 7
5.	97.8	0.007 596 2

In "Elektrodynamischen Maasbestimmungen," II, p. 363,[33,34] it was proved that a current of strength 1 that goes through a winding of a multiplier whose radius is a will exert a *force F* on a particle of the *North magnetic* fluid $+\mu$ or a particle of the *South magnetic* fluid $-\mu$ that is found at a distance of b from the plane of the multiplier winding, and whose projection onto that plane lies at a distance of x from the center, that is perpendicular to the plane of the winding of the multiplier:

$$F = \pm \frac{2\pi a^2 \mu}{(a^2 + b^2 + x^2)^{3/2}} \cdot \left\{ 1 + \frac{3}{4} \left(3a^2 - 2b^2 - 2x^2 \right) \frac{x^2}{(a^2 + b^2 + x^2)^2} + ... \right\} ,$$

from which, it will follow that the same current will exert a *rotational moment* D on a *needle* that contains the particles $+\mu$ and $-\mu$ at a very small distance of 2ε apart that is parallel to the plane of the multiplier:

$$D = \frac{4\pi a^2 \mu\varepsilon}{(a^2 + b^2 + x^2)^{3/2}} \cdot \left\{ 1 + \frac{3}{4} \left(3a^2 - 2b^2 - 2x^2 \right) \frac{x^2}{(a^2 + b^2 + x^2)^2} + ... \right\} ,$$

where $2\mu\varepsilon$ denotes the *magnetic moment* of the needle or the *needle magnetism*.

Now, three different applications can be made of this equation: *First of all*, to the *normal conditions* that were assumed for the magnetic effects in the Section 1, *next*, to the *tangent galvanometer with a single multiplier loop*, and *finally*, to the *tangent galvanometer with multiple multiplier loops* that was used in the present experiments. The first two applications show only that, as was pointed out before in *loc. cit.* in relation to the current strengths, this equation is actually the basis for the current intensity unit that is derived from magnetic effects. The last application leads to the calculation of the *desired time interval τ*.

If one applies this equation *first of all* to the *normal conditions* that were assumed for the magnetic effects of a current in Section 1, then one will

---

[33][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 454.

[34][Note by AKTA:] [Web52, p. 454 of Weber's *Werke*].

have $\pi a^2 = 1$, $b = 0$, $2\mu\varepsilon = 1$, $x = R$, and that a/R is a vanishingly-small fraction. *The equation above* will then yield the rotational moment D (without the sign, which depends upon the direction of the current):

$$D = \frac{1}{R^3} \quad \text{or} \quad R^3 D = 1 \ ,$$

which then agrees with the magnetic current effect that was established for a current intensity of 1 in Section 1. It follows from this that the equation above is the basis for the unit of current intensity that was derived from magnetic effects in Section 1.

Secondly, if one applies that equation to a *tangent galvanometer with a single multiplier loop* of radius R, where a small magnetic needle is in the center of loop, parallel to the plane of the loop, pointing to the magnetic meridian, then $a = R$, $b = 0$, $x = 0$. *The equation above* then yields the rotational moment that the current exerts on the needle when it is found *along the magnetic meridian*:

$$D = \frac{4\pi\mu\varepsilon}{R} \ .$$

For a *deflection* of the needle from the magnetic meridian that equals φ, that will go to:

$$D \cos\varphi = \frac{4\pi\mu\varepsilon}{R} \cdot \cos\varphi \ .$$

If T denotes the horizontal component of the Earth magnetism, then $-2\pi\varepsilon T \sin\varphi$ will be the rotational moment that *the Earth* exerts upon the needle. The sum of these two moments is equal to 0 when the needle persists at rest for the *deflection* φ; as a result:

$$\frac{2\pi}{R} = T \tan\varphi \quad \text{or} \quad \varphi = \arctan\frac{2\pi}{RT} \ .$$

However, this deflection is the same as the [deflection which a] *normal current* that was described in Section 4 should produce in a tangent galvanometer with a single loop.

Third, and finally, that same equation shall be applied to the *tangent galvanometer with multiple multiplier loops* that is used in the present experiment, and the rotational moment shall be determined that the aforementioned *normal current* that was described in Section 4 exerts upon the needle when it goes through all windings of the multiplier.

We next consider *one* winding of the multiplier that has radius a and whose plane is separated from the meridian plane of the needle by b. The

rotational moment D' that this winding exerts upon the needle will be determined from the equation above:

$$D' = \frac{4\pi a^2 \mu \varepsilon}{(a^2 + b^2 + x^2)^{3/2}} \cdot \left\{ 1 + \frac{3}{4} \left(3a^2 - 2b^2 - 2x^2 \right) \frac{x^2}{(a^2 + b^2 + x^2)^2} + ... \right\} \ ,$$

in which one can set $x = 0$, as in the previous application, if the length of the needle is a very small fraction of the diameter of the multiplier winding. Now, the length of the needle in our *tangent galvanometer* was, in fact, merely 60 millimeters, while the mean diameter of the multiplier windings amounted to 267 millimeters, which was, however, still not enough to be able to neglect x entirely. However, it sufficed to set x equal to an approximate value that suggested itself when one understood the $+\mu$ and $-\mu$ in the needle magnetism, [that is, in the magnetic moment of the needle,] which is equal to $2\mu\varepsilon$, to mean the combination of the north-magnetic and south-magnetic fluids that are distributed on the surface of the needle according to the *ideal distribution*, and accordingly determined 2ε, which then meant the distance from the center of mass of the north-magnetic fluid to that of the south-magnetic fluid, such that one would set $x = \varepsilon$. From the length and nature of the needle that was used, 2ε could not be very far from 40 millimeters, and one could then set:

$$x = \varepsilon = 20 \text{ millimeters}$$

with sufficient accuracy.

If one then lets a' and a'' denote the inner and outer radii of the multiplier ring and lets $2b'$ denote its width, then the cross-section of the entire ring will be equal to:

$$2(a'' - a')b' \ .$$

If one further denotes the part of the cross-section that the multiplier winding in question occupies (whose radius was equal to a, and whose plane was separated from the common center of the needle and the multiplier ring by b) by $da \cdot db$ then the product of those elements of the cross-section in the multiplier winding under consideration with the rotational moment that is exerted upon the galvanometer will be equal to:

$$\frac{4\pi a^2 \mu \varepsilon}{(a^2 + b^2 + \varepsilon^2)^{3/2}} \cdot da \, db \left\{ 1 + \frac{3}{4} \left(3a^2 - 2b^2 - 2\varepsilon^2 \right) \frac{\varepsilon^2}{(a^2 + b^2 + \varepsilon^2)^2} + ... \right\} \ ,$$

or since the terms that include the fourth and higher powers of the fraction ε/a can be neglected, due to the smallness of that fraction:

$$\frac{4\pi a^2 \mu\varepsilon}{(a^2+b^2)^{3/2}} \cdot dadb \left\{ 1 + \frac{3}{4}\frac{a^2-4b^2}{(a^2+b^2)^2} \cdot \varepsilon^2 \right\} \ .$$

It then follows from this that the sum of the products of the cross-section of each winding with the rotational moment that is exerted upon it will be:

$$4\pi\mu\varepsilon \int_{a'}^{a''} a^2 da \int_{-b'}^{+b'} \frac{db}{(a^2+b^2)^{3/2}} \cdot \left\{ 1 + \frac{3}{4}\frac{a^2-4b^2}{(a^2+b^2)^2} \cdot \varepsilon^2 \right\}$$

$$= \ 8\pi\mu\varepsilon b' \left\{ \log\frac{a''+\sqrt{a''^2+b'^2}}{a'+\sqrt{a'^2+b'^2}} + \frac{1}{4}\left(\frac{a''^3}{(a''^2+b'^2)^{3/2}} - \frac{a'^3}{(a'^2+b'^2)^{3/2}} \right) \cdot \frac{\varepsilon^2}{b'^2} \right\} \ .$$

Upon dividing this value by the cross-section of the entire ring, which is equal to $2(a''-a')b'$, one will get the rotational moment that is exerted upon the needle in the center of *one* multiplier winding, from which, after multiplying by the number of windings n, one will get the total *rotational moment* that the multiplier exerts upon the needle due to the *normal current* that flows through it, namely:

$$D = \frac{4\pi n\mu\varepsilon}{a''-a'} \left\{ \log\frac{a''+\sqrt{a''^2+b'^2}}{a'+\sqrt{a'^2+b'^2}} + \frac{1}{4}\left(\frac{a''^3}{(a''^2+b'^2)^{3/2}} - \frac{a'^3}{(a'^2+b'^2)^{3/2}} \right) \cdot \frac{\varepsilon^2}{b'^2} \right\} \ .$$

That *rotational moment D*, when divided by the *moment of inertia of the needle K*, which is then equal to:

$$\frac{D}{K} \ ,$$

will give the angular acceleration of the needle in terms of the given *normal current*, and when that acceleration is multiplied by the *duration of the current τ*, which is very brief in comparison to the period oscillation, which equals t, will give the *angular velocity* that is given to the needle by the *normal current* during its brief duration, which equals:

$$\frac{D\tau}{K} \ .$$

Finally, the *deflection* — i.e., the *initial elongation width φ* — of the needle that is set into oscillation can be calculated from that angular velocity that is suddenly given to the needle at rest by known rules (see "Elektrodynamische

Maassbestimmungen" II, p. 348),[35,36] namely, when the decrease in the arc of oscillation of the needle is given by the ratio of two successive oscillation arcs $e^\lambda : 1$:

$$\varphi = \frac{D\tau}{K} \cdot \frac{t}{\pi} \cdot \frac{e^{-\frac{\lambda}{\pi}\arctan\frac{\pi}{\lambda}}}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} \ .$$

In order to not have to determine the value of the moment of inertia of the needle K and its magnetic moment $2\mu\varepsilon$ from special observations, one can eliminate both of them by consulting the known equation for the period of oscillation, but in which one must account for the force of torsion of the wire. If $1 : \vartheta$ denotes the ratio of the geomagnetic directing force that acts upon the needle, which equals $2\mu\varepsilon T$, to the one that is exerted by the wire, then the equations for the period of oscillation t will be:

$$\frac{2\mu\varepsilon \cdot T}{K} = \frac{\pi^2}{t^2} \cdot \frac{1 + \frac{\lambda^2}{\pi^2}}{1 + \vartheta} \ ,$$

and a result, if one sets:

$$d = \frac{D}{2\mu\varepsilon} = \frac{2\pi n}{a'' - a'} \left\{ \log \frac{a'' + \sqrt{a''^2 + b'^2}}{a' + \sqrt{a'^2 + b'^2}} \right.$$

$$\left. + \frac{1}{4}\left(\frac{a''^3}{(a''^2 + b'^2)^{3/2}} - \frac{a'^3}{(a'^2 + b'^2)^{3/2}} \right) \cdot \frac{\varepsilon^2}{b'^2} \right\}$$

and multiplies the foregoing equation by $\frac{D}{2\mu\varepsilon \cdot T} = \frac{d}{T}$ then:

$$\frac{D}{K} = \frac{d}{T} \cdot \frac{\pi^2}{t^2} \cdot \frac{1 + \frac{\lambda^2}{\pi^2}}{1 + \vartheta} \ .$$

If one substitutes that value in the equation for φ then one will get:

$$\varphi = \pi \frac{d}{T} \cdot \frac{\tau}{t} \cdot \frac{\sqrt{1 + \frac{\lambda^2}{\pi^2}}}{1 + \vartheta} \cdot e^{-\frac{\lambda}{\pi}\arctan\frac{\pi}{\lambda}} \ ,$$

and that will give the desired *duration of the normal current*:

$$\tau = t \cdot \frac{\varphi}{\pi} \cdot \frac{T}{d} \cdot \frac{1 + \vartheta}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} \cdot e^{\frac{\lambda}{\pi}\arctan\frac{\pi}{\lambda}} \ .$$

---

[35][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 440.
[36][Note by AKTA:] [Web52, p. 440 of Weber's *Werke*].

However, it was determined by measurement that for the multiplier of the tangent galvanometer that was used here:

$$2\pi a' = 709.0 \text{ millimeters},$$
$$2\pi a'' = 965.35 \text{ millimeters},$$
$$2b' = 72.04 \text{ millimeters},$$
$$n = 5635,$$

from which, with the aforementioned value of $\varepsilon = 20$ millimeters, one will get the value of d:

$$d = 262.1 \;.$$

If the value of ε also has an uncertainty of 1 millimeter, then that will imply the uncertainty in d, which only amounts to 0.4, out of 262, however (i.e., only 1/657), which is not worth considering.

In addition, the period of oscillation of the needle t, the horizontal component of the Earth's magnetism at the location of the tangent galvanometer T, the logarithmic decrement in the decrease of the arc of oscillation λ, and the ratio ϑ of the directing force of the wire to the one that is due to geomagnetism T were found the usual way:

$$t = 9.244 \text{ seconds} \;,$$

$$T = 1.7983 \text{ seconds} \;,$$

$$\lambda = 0.070 \text{ seconds} \;,$$

$$\vartheta = \frac{1}{691} \;.$$

If one substitutes these values in the equation for τ then one will get:

$$\tau = 0.020\ 921 \cdot \varphi \;.$$

The values of φ that were obtained from the five experiments that were described in Section 7 were collected at the beginning of this Section. If one substitutes them in the equation for τ then one will get the following five results for the cited experiments:

No.	τ
1.	0.000 119 4
2.	0.000 130 0
3.	0.000 156 8
4.	0.000 148 0
5.	0.000 158 9

14 Section 14. Calculating the Quantity $\frac{1}{2\tau} \cdot E$

Finally, it still remains for us to calculate the value of $\frac{1}{2\tau} \cdot E$ from the values of E and τ that were found. Namely, if we summarize the corresponding values of E and τ from the previous two Sections in the following table:

No.	E	τ
1.	36 060 000	0.000 119 4
2.	41 940 000	0.000 130 0
3.	49 700 000	0.000 156 8
4.	44 350 000	0.000 148 0
5.	49 660 000	0.000 158 9

then that will yield the following five values of $\frac{1}{2\tau} \cdot E$ that result from the five measurements that were described in Section 7:

No.	$(1/2\tau) \cdot E$
1.	$151\ 000 \cdot 10^6$
2.	$161\ 300 \cdot 10^6$
3.	$158\ 500 \cdot 10^6$
4.	$149\ 800 \cdot 10^6$
5.	$156\ 250 \cdot 10^6$

All of the measurement collectively then give the mean value:

$$\frac{1}{2\tau} \cdot E = 155\ 370 \cdot 10^6 \ .$$

However, from Section 5:

$$\frac{1}{2\tau} \cdot E : 1$$

denotes the ratio of the *amount of positive electricity* that passes through the cross-section of the conductor in one second for a constant current that is composed of equally-large masses of positive and negative electricity that flow in opposite directions and whose intensity is equal to the *magnetic* current intensity, to the amount that would exert a force at a distance of one millimeter that would impart a velocity of one millimeter per second to a mass of one milligram during one second, if an equal amount of electricity were concentrated into a point. That ratio was determined in the *problem* in Section 4 that remains to be solved, which shall now be done.

15 Section 15. Reducing the Magnetic, Electrodynamic, and Electrolytic Units of the Current Intensity to Mechanical Units

However, the solution of the problem that was posed in Section 4 shall now be used to *reduce the magnetic, electrodynamic, and electrolytic units of the current intensity to mechanical units.*

From Section 2, for a constant current that is composed of equally-large masses of positive and negative electricity that flow in opposite directions whose intensity is equal to the *mechanical* unit of current intensity, the *amount of positive electricity* that passes through the cross-section of the conductor in one second shall be equal to one; i.e., it is equal to the amount of electricity concentrated into a point that would exert a force at another equal amount of electricity concentrated into another point at a distance of one millimeter that would impart a velocity of one millimeter per second to a mass of one milligram in one second. However, from the foregoing Section, that *unit* amount of positive electricity has a ratio with the *amount of positive electricity* that passes through the cross-section in one second for a current whose intensity is given by the *magnetic* current unit of:

$$155\ 370 \cdot 10^6 : 1 \ .$$

Now, since the current intensities are proportional to the amounts of electricity that pass through the cross-section in equal time intervals, that will immediately imply the *reduction of the magnetic unit for current intensity to the mechanical unit*, since the *magnetic* current unit of the amount of electricity that passes through the cross-section in the same time interval will then be:

$$155\ 370 \cdot 10^6$$

times greater than the amount in the *mechanical* unit of current. As a result, from the cited proportion, the *magnetic unit of the current intensity will itself also be* $155\ 370 \cdot 10^6$ *times larger than the mechanical unit.*

Furthermore, since, from Section 1, p. 223,[37,38] the *magnetic unit* of current intensity has a ratio of $\sqrt{2} : 1$ with the *electrodynamic* one, the *electrodynamic unit of current intensity will be* $109\ 860 \cdot 10^6 (= 155\ 370 \cdot 10^6 \cdot \sqrt{\frac{1}{2}})$ *times greater than the mechanical unit.*

---

[37][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 613.

[38][Note by AKTA:] Page 8 of this work, or [KW57, p. 613 of Weber's *Werke*].

Finally, since, from Section 1, p. 224,[39,40] the *mechanical unit* of current intensity has a ratio of $1 : 106\frac{2}{3}$ with the *electrolytic* one, *the electrolytic unit of current intensity will be* $16\ 573 \cdot 10^9 (= 106\frac{2}{3} \cdot 155\ 370 \cdot 10^6)$ *greater than the mechanical unit.*

The problem in this treatise, as it was expressed in Section 2, of reducing those three units of current intensity to the mechanical unit, is then solved, and all that remains is to discuss the *applications* that can be made of the result that was found.

Applications

16 Section 16. Determining the Amount of Electricity that is Required to Liberate 1 Milligram of Hydrogen from 9 Milligrams of Water

The first application that we shall make of the results that were found is to the precise determination of the amount of electricity that is required to liberate 1 milligram of hydrogen from 9 milligrams of water, over which the determination that Buff found with the help of his tangent galvanometer and a long conducting wire and published in *Annalen der Chemie und Physik*, Vol. 86, p. 33 was referred to already in the footnote to Section 3, p. 226.[41,42,43]

According to Buff, that amount of electricity was sufficient to charge a battery of 45480 Leyden jars, each of which were 480 millimeters high and 160 millimeters in diameter, up to a spark gap of 100 millimeters. That determination that Buff made lacked only more precise data on the amount of electricity that a Leyden jar contained when it had been charged as described.

Now, the results that were found in the present treatise imply that the amount of electricity that is required to liberate 1/9 milligram of hydrogen from 1 milligram of water is equal to the *amount of positive electricity* that passes through the cross-section of the conductor in one second for a constant current whose intensity has the *electrolytic* unit. However, the latter is, in proportion to the current intensities that correspond to the *electrolytic* and

---

[39][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 614.

[40][Note by AKTA:] Page 8 of this work, or [KW57, p. 614 of Weber's *Werke*].

[41][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 616.

[42][Note by AKTA:] Page 10 of this work, or [KW57, p. 616 of Weber's *Werke*].

[43][Note by AKTA:] As a matter of fact, this paper was published in the *Annalen der Chemie und Pharmacie* and not in the *Annalen der Chemie und Physik*: [Buf53, p. 33].

magnetic current units (see Section 1, p. 224),[44,45] $106\frac{2}{3}$ times greater than the *amount of positive electricity* that passes through the cross-section in one second for a constant current whose intensity has the *magnetic* current unit, and from Section 14, that is:

$$155\ 370 \cdot 10^6$$

times greater than the *unit* amount of electricity concentrated into a point that would exert a force at a distance of one millimeter that would impart a velocity of one millimeter per second on a mass of one milligram during one second if an equal amount were concentrated into a point.

It follows form this that:

$9 \cdot 106\frac{2}{3} \cdot 155\ 370 \cdot 10^6 = 149\ 157 \cdot 10^9$ *units, as it was just determined, are required to liberate 1 milligram of hydrogen from 9 milligrams of water.*

If such an amount of positive electricity were concentrated into a cloud and an equal amount of negative electricity were concentrated on the surface of the Earth at the location that is directly below it, then that would yield an attraction of the cloud to the Earth that would be equal to a weight of 45 000 hundredweights (= 2 268 000 kilograms) if they were at a distance of 1000 meters from each other.

If one divides that number of units by the number of Leyden jars in the battery that Buff described (viz., 45 480), then one will get the precise data for the amount of electricity that is contained in the charge in one Leyden jar as described by Buff, namely:

$$3280 \cdot 10^6 \text{ units}.$$

However, from Buff's description, the charged surface of such a jar has an area of:

$$480 \cdot 160 \cdot \pi = 241\ 300 \text{ square millimeters}$$

and as a result, each square millimeter will be charged with:

$$13\ 600 \text{ units},$$

from which, one can determine the compression or condensation of electricity in the jar that is required for a spark gap of 100 millimeters.

---

[44][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 616.

[45][Note by AKTA:] Page 8 of this work, or [KW57, p. 614 of Weber's *Werke*].

17   Section 17. Determining the Constant c

From the fundamental law of electrical action that was established in the first treatise on electrodynamic measurements,[46] which encompassed *electrostatics*, *electrodynamics*, and *induction*, the force that an amount of electricity e exerts upon an amount of electricity e' at a distance of r with a relative velocity of dr/dt and an acceleration of d^2r/dt^2 is expressed by:

$$\frac{ee'}{r^2}\left[1 - \frac{1}{c^2}\left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2}\right)\right] \ .$$

That force splits into two parts, the first of which, which is equal to ee'/r^2, can be called the *electrostatic* force, and the second of which, which is equal to $-(ee'/c^2r^2)(dr^2/dt^2 - 2rd^2r/dt^2)$, can be called the *electrodynamic* force. The ratio of those two forces is determined from the *constant c. c* means the value of relative velocity (assumed uniform) at which the *electrostatic* force would cancel the *electrodynamic* force. That *constant c* will now be determined in the following way:

In Section 14, the ratio $\frac{1}{2\tau}E : 1$ (that is, the *ratio of the magnetic unit of current intensity to the mechanical one*) was found to be:

$$155\ 370 \cdot 10^6 : 1 \ .$$

In Section 26, p. 261 of the second treatise on electrodynamic measurements,[47,48] the ratio of the *magnetic* unit of current intensity to the *electrodynamic* one was given as:

$$\sqrt{2} : 1 \ ,$$

and in Section 27, p. 269,[49,50] the ratio of the *electrodynamic* unit for the current intensity to the *mechanical* one was given as:

$$c : 4 \ ,$$

from which, the *ratio of the magnetic unit of current intensity to the mechanical one* would follow:

$$c\sqrt{2} : 4 \ .$$

---

[46][Note by AKTA:] [Web46] and [Web87] and [Web07].
[47][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 360.
[48][Note by AKTA:] [Web52, p. 360 of Weber's *Werke*].
[49][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 367.
[50][Note by AKTA:] [Web52, p. 367 of Weber's *Werke*].

Setting this ratio equal to the one that was found in Section 14 of the treatise will then give:

$$c = 4 \cdot 155 \ 370 \cdot 10^6 \cdot \sqrt{\frac{1}{2}} = 439 \ 450 \cdot 10^6 \ .$$

From this determination of the *constant c*, one then sees that two electrical masses must move with a very large velocity with respect to each other if the *electrodynamic* force were to cancel the *electrostatic* one, namely, with a velocity of 439 million meters or 59320 miles per second, which exceeds the speed of light significantly.

However, the speed of light is not the speed of motion of a body, but of a wave, while all of the speeds of actual motions of bodies that are known to us, even those of the celestial bodies, constitute only very small fractions of it. Now, if one observes that the ratio of the *electrodynamic* force to the *electrostatic* one corresponds to the square of that fraction, then that will imply that the electrodynamic force can always be considered to be vanishingly small in comparison to the electrostatic one. Indeed, we still have no knowledge of the speeds at which electric fluids move in metallic conductors. However, in various situations, one can assume that the amount of neutral electricity that is contained in those conductors is exceptionally large. Nonetheless, the greater the latter gets, the less the speed of the actual motion will be, which is then implied by the *unit of current intensity* that is present. The speed of those motions probably defines only a very small fraction of the speed *c* then.

Furthermore, the large value of the *constant c* that was found implies the interesting consequence that such a dynamical part could also be attached to the *gravitational force on ponderable bodies* (which would exhibit a great analogy between the interactions of *ponderable* and *imponderable* bodies) without that dynamical part of the force having the slightest observable influence of the motions of the celestial bodies.

The fact that the effect of the *electrodynamic* force does not always vanish for electricity, but can emerge very apparently for galvanic currents, has its basis in merely the *complete cancellation of all electrostatic forces* that takes place during the *neutralization* of positive and negative electricity, against which those [electrostatic] forces would disappear. Wherever no such neutralization takes place, but free electricity is present, only the *electrostatic* force would come under consideration in the effect of free electricity. That explains why not all experiments that were intended to establish the fundamental laws of electrical action could be performed with merely *two* masses of free electricity, but some experiments had to be performed with *two pairs*

of electrical masses (viz., current elements) that were *neutralized electrostatically.*

For *ponderable* masses, for which the law of indifferent attraction is true, one can speak of *no neutralization of the masses.*

Remark. — At the beginning of this Section, the following equation for the determination of the *constant c* was presented:

$$c = \frac{E}{\tau} \cdot \sqrt{2} \ ,$$

in which $\frac{1}{2\tau} \cdot E : 1$ denoted the ratio that was found in Section 14 of the amount of *positive electricity* that passes through the cross-section of a conductor in one second for a constant current whose intensity is measured *magnetically* to the amount of electricity concentrated into a point that would exert at an equal amount of electricity concentrated into a point a force at a distance of one millimeter that would impart a velocity of one millimeter per second on a mass of one milligram in one second. — The second treatise on electrodynamic measurements[51] was referred to in order to prove that equation. However, the validity of that equation can also be inferred directly from the *fundamental law of electrical action* and the *definition of the magnetic current measure.* To that end, one merely needs to consider the interaction of two equal current elements α and α of a current flowing along a straight line separated by a distance of r, about which, as it was already mentioned in the footnote on p. 224[52,53] that they repel each other with a force equal to:

$$\frac{\alpha^2}{r^2} i^2 \ ,$$

if i is expressed in terms of the *magnetic* current unit. As is known, that follows from *Ampère's fundamental law* and the relationship between *electromagnetism* and *electrodynamics* that it gives.

Assuming that, one proposes that the rectilinear conductor of our current should contain one unit of positive and negative electricity in each piece of it that is one millimeter long. (From Section 14), $\frac{1}{2\tau} \cdot E$ then denotes the number of millimeters that both electrical currents must traverse in the opposite directions in order to make:

$$i = 1 \ .$$

---

[51][Note by AKTA:] [Web52].

[52][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 614.

[53][Note by AKTA:] Page 8 of this work, or [KW57, p. 614 of Weber's *Werke*].

Those simple relationships give not only the amounts of electricity in the two current elements α and α, whose distance and the remaining relationship depends upon their force of repulsion (according to the fundamental law of electrical action), but also the *magnitude of that force of repulsion* itself; namely, since $i = 1$:

$$\frac{\alpha^2}{r^2} \ .$$

That merely depends upon the fact that this force of repulsion, which is known already, can be derived from the fundamental law of electrical action, so since c is contained in that fundamental law, it will contain an expression for that force that depends upon c, and one needs only to set [the repulsive force] equal to the value that is known already in order to find c. However, the force of repulsion between the two current elements α and α can be derived very easily from the fundamental law of electrical action with the simple relationships that were described. We then decompose the total force that is given by the fundamental law into two parts, namely, into the *electrostatic* and *electrodynamic* forces. That will shed light upon the fact that the sum of the electrostatic forces between the two current elements is zero (due to the electrostatic neutralization that is present in both current elements). It will likewise illuminate the fact that no acceleration exists between the electrical masses in both current elements, so $d^2r/dt^2 = 0$. With that, the general expression for the electrical action:

$$\frac{ee'}{r^2}\left[1 - \frac{1}{c^2}\left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2}\right)\right]$$

will reduce to:

$$-\frac{1}{c^2}\frac{ee'}{r^2}\frac{dr^2}{dt^2}$$

in our case. Now, when that expression is applied:

1) to the two positive masses in the two current elements $e = +\alpha$ and $e' = +\alpha$, it will give a force of repulsion that is equal to zero, since the relative velocity of the masses $dr/dt = 0$ (because both of them move in the same direction with equal velocities).

2) The same thing will be true for two negative masses $e = -\alpha$ and $e' = -\alpha$.

3) However, when the same expression is applied to a positive mass $e = +\alpha$ and a negative one $e' = -\alpha$, it will give a force of repulsion that is equal to $+\frac{1}{c^2}\frac{ee'}{r^2} \cdot \frac{1}{\tau^2} \cdot E^2$, since the relative velocity of those masses is $dr/dt = E/\tau$ (because they both move in opposite directions with the velocity $\frac{1}{2\tau} \cdot E$).

4) The same thing will be true for a negative mass $e = -\alpha$ and a positive one $e' = +\alpha$.

It then follows from this that the sum of all forces of repulsion between the electrical masses that are contained in the two current elements is equal to:

$$2 \cdot \frac{1}{c^2} \frac{\alpha^2}{r^2} \cdot \frac{1}{\tau^2} \cdot E^2 \ ,$$

and if that sum is set equal to its value α^2/r^2 that is known already, then that will imply the following equation for the determination of c:

$$\frac{\alpha^2}{r^2} = 2 \cdot \frac{1}{c^2} \cdot \frac{\alpha^2}{r^2} \cdot \frac{1}{\tau^2} \cdot E^2 \ ,$$

or

$$c = \frac{E}{\tau} \cdot \sqrt{2} \ ,$$

which was to be proved.

18 Section 18. The Electrical Laws, with the Numerical Determination of Their Constants

The electrical laws that were developed in the first and second treatise on electrodynamic measurements are the following:

1) *The fundamental law of electrical action.* — According to it, the force the electrical mass e exerts upon the electrical mass e' at a distance of r with a relative velocity of dr/dt and an acceleration of d^2r/dt^2 is expressed by:

$$\frac{ee'}{r^2} \left[1 - \frac{1}{c^2} \left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2} \right) \right] \ .$$

2) *The fundamental law of electrodynamics.* — According it, the force that an unchanging and motionless current element of length α and current intensity i will exert upon an equal current element of length α' and current intensity i' at a distance of r when α makes an angle of ϑ with r, α' makes an angle of ϑ' with the extension of r, and α makes an angle of ε with α' is expressed by:

$$\frac{\alpha\alpha'}{r^2} ii' (3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon) \ .$$

3) *The law of voltaic induction for an unchanging current element that moves with respect to a conductor.* — According to it, the electromotive force that a current element of length α and current intensity i exerts upon an element of a conductor of length α' that moves with a velocity u at a distance of r when α makes an angle of ϑ with r, α' makes an angle of φ with r, u makes an angle of ϑ' with the extension of r, and α makes an angle of ε with u is expressed by:

$$\frac{2\sqrt{2}}{c} \cdot \frac{\alpha\alpha'}{r^2} \cdot ui \cos\varphi (3\cos\vartheta \cos\vartheta' - 2\cos\varepsilon) \ .$$

4) *The law of voltaic induction for a variable current element that does not move with respect to a conductor.* — According to it, the electromotive force that a current element of length α whose current intensity grows uniformly by i in a time interval t exerts upon a conductor element of length α at a distance of r when α makes an angle of ϑ with r and α' makes an angle of ϑ' with the extension of r is expressed by:

$$-\frac{2\sqrt{2}}{c} \cdot \frac{\alpha\alpha'}{r} \cdot \frac{i}{t} \cos\vartheta \cos\vartheta' \ .$$

5) *The law of voltaic induction for a location where there is sliding.* — According to it, the electromotive force that a current of intensity i and sliding velocity v that goes through the sliding location exerts upon a conducting element of length α' at a distance of r when v makes an angle of ϑ with r, and α' makes an angle of ϑ' with the extension of r is expressed by:

$$-\frac{2\sqrt{2}}{c} \cdot \frac{\alpha'}{r} vi \cos\vartheta \cos\vartheta' \ .$$

A positive value in the expressions (1) and (2) means a force of repulsion, while a negative value means a force of attraction. The numerical values of our measurements give the magnitudes of the forces as ratios with the force that would impart a velocity of one millimeter per second on a mass of one milligram during one second. In the expression (2), as well as in all of the following ones, the current intensities i and i' are assumed to be measured in *magnetic* units, which can always be easily done with the *tangent galvanometer.* If one lets ε' denotes the *electrical capacity* of the conductor α' — i.e., the ratio of the amount of positive electricity that it contains (which is equal to that of the negative) to its length, — then for $\varepsilon' = 1$ the expressions (3), (4), (5) will give the difference between the two forces that act in the direction of α' on the amounts of positive and negative electricity that are contained in α', and in fact, they will give that *force difference* as a ratio

with the force that would impart a velocity of one millimeter per second on a mass of one milligram during one second. — If ε' is not equal to 1, then the expressions (3), (4), (5) must be multiplied by ε' in order to get the given *force difference.*

A complete determination of all forces by means of the given laws requires that the *constant c* must be set equal to the numerical value that was found in the previous Section in all of the expressions above. One will then get:

$$\frac{ee'}{r^2}\left[1 - \frac{1}{c^2}\left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2}\right)\right]$$

$$= \frac{ee'}{r^2}\left[1 - \frac{1}{193\,120 \cdot 10^{18}}\left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2}\right)\right] , \qquad (1.)$$

$$\frac{\alpha\alpha'}{r^2}ii'(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon) , \qquad (2.)$$

$$\frac{2\sqrt{2}}{c} \cdot \frac{\alpha\alpha'}{r^2} \cdot ui\cos\varphi(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon)$$

$$= \frac{1}{155\,370 \cdot 10^6} \cdot \frac{\alpha\alpha'}{r^2} \cdot ui\cos\varphi(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon) , \qquad (3.)$$

$$-\frac{2\sqrt{2}}{c} \cdot \frac{\alpha\alpha'}{r} \cdot \frac{i}{t}\cos\vartheta\cos\vartheta' = -\frac{1}{155\,370 \cdot 10^6} \cdot \frac{\alpha\alpha'}{r} \cdot \frac{i}{t}\cos\vartheta\cos\vartheta' , \qquad (4.)$$

$$-\frac{2\sqrt{2}}{c} \cdot \frac{\alpha'}{r} \cdot vi\cos\vartheta\cos\vartheta' = -\frac{1}{155\,370 \cdot 10^6} \cdot \frac{\alpha'}{r} \cdot vi\cos\vartheta\cos\vartheta' . \qquad (5.)$$

When all constants have been determined numerically, the law of electricity, in the last form, will satisfy all requirements *in practice*. However, for *theoretical* investigations, it can be necessary in many cases to substitute the values of i and i' that are derived from the *causes* of the current intensities (see Section 2) in the expressions above, instead of the current intensities i and i' that are measured in *magnetic* units. Namely, if $+\alpha\varepsilon$ and $-\alpha\varepsilon$ denote the amounts of positive and negative electricity, respectively, that are contained in the conductor α, and $+u$ and $-u$, respectively, are the velocities with which they move in the conductor, and if $+\alpha'\varepsilon'$, $-\alpha'\varepsilon'$, $+u'$, and $-u'$, respectively, denote the same things for the conductor α', then εu and $\varepsilon'u'$,

respectively, will be the values of the current intensities, when determined in *mechanical* units, and from the relationships that were found in Section 15, those values must be divided by $155\ 370 \cdot 10^6$ in order to obtain the values of the same current intensities, when expressed in *magnetic* units. As a result, one will have:

$$i = \frac{\varepsilon u}{155\ 370 \cdot 10^6} \ , \qquad i' = \frac{\varepsilon' u'}{155\ 370 \cdot 10^6}$$

in the expressions above, and those values can be substituted for i and i' in the expressions above, if that should be necessary.

19 Section 19. Application to Electrolysis

All electrical forces that are determined by means of the laws that were cited in the foregoing Section are forces that act directly upon only electrical masses. *However, all forces that act directly upon only electrical masses will also act indirectly upon the ponderable carriers of those electrical masses.* In that way, the application of electrical laws to the investigation of ponderable bodies opens up a broad field, since electricity will, in that way, become an instrument for us, with whose help we can make known forces act upon ponderable bodies by means of relationships for which no other known forces act.

When electrical masses are coupled with their ponderable carriers, the law above explains why the electrical masses cannot move without their carriers. However, even in metallic conductors, in which the electricity can move, while their ponderable carrier (the metal) remains at rest, so the electrical masses go from one metallic particle to another, one still finds a coupling between the electrical masses and the metallic particles that must be resolved before the electrical mass can go from one metallic particle to another. As long as that coupling exists, all forces that act upon only the electrical masses will, however, carry over directly to the metallic particles that they are coupled with, and only those forces that act upon the electrical masses, once they have been liberated from the metallic particles, will no longer carry over to those metallic particles, but will impart a certain velocity on those electrical masses until they arrive at the next metallic particle, but due to the coupling between those electrical masses and the next metallic particle, it will again be cancelled, which would have the same effect as if the electrical forces that produced that velocity were carried over to that next metallic particle. One calls all of those forces that emerge from the coupling of electrical masses with individual metallic particles *forces of resistance*, by which the metal opposes

the motion of electricity in its interior, from which *Ohm's law* follows, that the electricity in the metallic conductor can persist in a uniform motion only when it is driven forward continually by an equally-large force, and that current will momentarily vanish as soon as the driving force ceases. — It will then follow from this that, even in conductors, all forces that act upon the electricity in the conductor directly, will be transferred indirectly to the conductor itself due to the *resistance* of the conductor.

In *electrolysis*, one does not deal with a metallic conductor that remains at rest while the electrical fluid moves in it, but with a body (e.g., water) that is composed of various kinds of ponderable particles, of which, the one kind (viz., hydrogen particles) follows the motion of the *positive* electricity, while the other (viz., oxygen particles) follows the *negative* electricity. That then raises the question: What is the origin of the forces that produce the various motions of the two components of the water? The laws of electrolysis show that these motions must be an indirect effect of the electrical forces, if not also a direct one. Now, if the electrical forces act directly upon only the electrical masses that are bound to the hydrogen and oxygen particles, then the fact that the hydrogen particles follow the motion of the positive electricity and the oxygen particles follow the motion of the negative electricity shows that the one must be bound to positive electricity in water, and the other, with negative electricity, so it will remain in the water, regardless of whether it contains a quantity of neutral fluid, in addition to the free electricity. The strength of that coupling of the hydrogen particles with free positive electricity and the oxygen particles with the negative electricity might also go unmentioned, such as whether it is so strong that they cannot be separated at all, so the electricity will only move with its ponderable carrier under electrolysis, or if it behaves as it does in metallic conductors, such that the electricity will take on a motion that is independent of the motion of the ponderable carrier, in addition to the latter motion. However, in the latter case, the law that the decomposition of the different combined bodies that is due to that current will be proportional to the chemical equivalent will not be strictly valid, which has been shown by the most recent investigations of that case.

Now, if the electrical forces, which only seek to separate the electrical fluid directly, are transmitted to the components of the water by whatever bond that couples the fluid to the components, then one can achieve a closer determination of the *chemical separating forces*[54] that produce the separation of the ponderable components from a more precise knowledge of the electrical separating forces, and that is the reason for the special interest that

---

[54][Note by AKTA:] *Chemischen Scheidungskräfte* in the original.

electrolysis enjoys in comparison to the other methods of chemical separation. Namely, electricity can be used as an instrument by which we link each hydrogen and oxygen particle in the water by a thread and we can stretch both threads in opposite directions with known forces until the hydrogen and oxygen particles are torn from each other.

In order to employ that instrument, and in that way to actually determine the forces that are required to separate the chemically bound parts in terms of known measurements, we must give *the electrical law, along with the numerical determination of its constants*. Once that has been done, we would also like to attempt to apply that to the known results.

The forces that put the electrical fluid into current motion will be called *electromotive forces*. That special terminology (which will be used to distinguish *that type of force*, and not merely its effects) is merely based upon the fact that up to now those forces cannot be measured with known units, but can be determined only indirectly by the effects of the currents that they produce (e.g., thermal, chemical, and magnetic effects), by which they can indeed be compared to each other, but absolutely cannot be expressed in terms of known units, and therefore they also cannot be compared with other known forces. That argument breaks down when one determines those forces from the laws that were given in the foregoing Section, by which they will be expressed in terms of known units. One can also express the forces that one cannot calculate directly from the laws above in terms of known units by comparison them with the ones that can. — Finally, since one can determine the resistance in a closed circuit precisely, and for a constant current the electromotive force and resistance must always have the same ratio to each other, according to Ohm's law, one also learns how the electromotive forces are distributed over the various parts of the circuit. Thus, if a voltmeter is introduced into a circuit, then the electrical separating forces that act in the water can be ascertained precisely.

However, with water, one encounters the special circumstance that it defines a very bad conductor in its pure state and is very difficult to decompose. All electrolytic measurements then relate to water that has been mixed with sulfuric acid or other chemicals: One obtains different results in regard to decomposability for different mixtures. It is necessary to initially restrict oneself to a particular mixture then, and here we shall choose a mixture of water and sulfuric acid with a specific gravity of 1.25, following the investigations that Horsford published in Poggendorf's *Annalen*, Vol. 70 (1847), p. 238, which is the easiest of all mixtures of water and sulfuric acid to decompose.[55]

---

[55][Note by AKTA:] [Hor47].

For equal lengths and cross-sections, the *resistance* by which that mixture opposes the current that Horsford gave was found to be:

$$696\ 700$$

times larger than the resistance of silver, or when one sets the ratio of the resistance of silver to that of copper equal to 1 : 0.7417, following Lenz (Poggendorf's *Annalen*, Vol. 34, p. 418, Vol. 45, p. 105):[56]

$$516\ 750$$

times larger than the resistance of the copper that Lenz used. — From the measurements that were communicated in the *Abhandlungen der K. Gesellschaft der Wissenschaften in Göttingen*, Vol. 5 ("Über die Anwendung der magnetischen Induktion auf Messung der Inklination mit dem Magnetometer"),[57] the resistance of a copper wire of length one millimeter and a mass of one milligram (= 1/8.427 square millimeters of cross-section) was found to be equal to:[58]

$$2\ 310\ 000$$

in absolute units of the *magnetic* system; i.e., for a copper wire of length one millimeter and a cross-section of 1 square millimeter, it will be equal to:

$$274\ 100.$$

That yields the resistance of the mixture above when it is one millimeter long and one square millimeter in cross-section as being:

$$141\ 640 \cdot 10^6$$

in *magnetic* resistance units. However, that mixture contained about nine parts water to one part sulfuric acid by volume, and the pure water would then amount to only 9/10 of the total cross-section. If one assumes that the total current goes merely through the water (because if a part of the current were conducted by the sulfuric acid then that would define an auxiliary current, which would have to be excluded from any consideration of the decomposition of water) then the resistance would refer to just the water, and one would have to set it equal to:

$$127\ 476 \cdot 10^6$$

for one millimeter of length and one square millimeter of cross-section.

---

[56][Note by AKTA:] [Len35] and [Len38].

[57][Note by AKTA:] [Web53c], [Web53a] and [Web53b].

[58][Note by KW:] In the cited place, [Wilhelm Weber's *Werke*, Vol. II, p. 319] one finds the resistances given for various types of copper, among which, one finds the one above, which corresponds to the copper that Jacobi used for his standard resistance (*Widerstands-Etalon*), which is the largest of them. That value was chosen because Lenz often referred to the same papers as Jacobi, so he probably appealed to the same types of copper as Jacobi in his experiments.

Now, should this *resistance* to the current intensity in *magnetic* units be equal to $106\frac{2}{3}$ — namely, strong enough that, from Section 1, p. 224,[59],[60] one milligram of water would decompose in one second — then the electromotive force for each millimeter in *magnetic* units would have to amount to:

$$106\frac{2}{3} \cdot 127\ 476 \cdot 10^6 \ ,$$

which must be multiplied by $\frac{2\sqrt{2}}{c} = \frac{1}{155\ 370 \cdot 10^6}$ in order to obtain its expression in *mechanical* units.

However, from the foregoing Section, that number means the *difference between the forces* that act in each direction of the current on *each unit* of free positive electricity (in the hydrogen particles) in a column of water that is one millimeter long and on *each unit* of the free negative electricity (in the sulfuric acid that is found in it), and that number must then be multiplied by n in order to obtain the *total force that acts*, if n is the number of units of free positive or free negative electricity that is contained in the hydrogen or oxygen particles, respectively, in a water column that is one millimeter long.

However, the hydrogen in one milligram of decomposed water gives up its free positive electricity to the electrode where it develops, which will then flow through the electrode (or, what amounts to the same thing, in effect, it will be neutralized by the supply of negative electricity in it) and will flow through the cross-section in one second. However, since the current intensity in *electrolytic* units is equal to 1, and from Section 15, with that current intensity, $106\frac{2}{3} \cdot 155\ 370 \cdot 10^6$ units of positive electricity and just as much negative electricity will go through the cross-section in one second (when one-half of the free positive electricity that is on the electrode flows through the electrode, while the other half is neutralized by the negative electricity that the electrode supplies), which will yield:

$$\frac{1}{2}n = 106\frac{2}{3} \cdot 155\ 370 \cdot 10^6 \ ,$$

If one then multiplies that number by:

$$\frac{2\sqrt{2}}{c} \cdot n = 2 \cdot 106\frac{2}{3}$$

then the product

$$2 \cdot \left(106\frac{2}{3}\right)^2 \cdot 127\ 476 \cdot 10^6$$

---

[59][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 614.

[60][Note by AKTA:] Page 8 of this work, or [KW57, p. 614 of Weber's *Werke*].

will give the *difference between the forces* that must act in the direction of the current on the hydrogen particles in one milligram of water that defines a column that is one millimeter long, which contain free positive electricity, and on the negative electricity that is contained in the oxygen particles (under the influence of the neighboring sulfuric acid) if the decomposition of the water is to result with a velocity of one millimeter per second, and indeed that *difference in forces* is determined from the number above as a ratio to the force that would impart a velocity of one millimeter per second on a mass of one milligram during one second.

The weight of one milligram is a force that will impart a velocity of 9811 millimeters per second on a mass of one milligram in one second. Therefore, if one divides the given number by 9811, then one will get that *force difference*, as expressed in milligram weights:

$$\frac{2}{9811} \cdot \left(106\frac{2}{3}\right)^2 \cdot 127\ 476 \cdot 10^6 = 2 \cdot 147\ 830 \cdot 10^6 \ .$$

One can express that result in the following way: *If all of the hydrogen particles in one milligram of water in a column one millimeter long were coupled by one thread and all of the oxygen particles were coupled with another thread, then both threads would each have to be tensed in opposite directions with a weight of:*

<div align="center">

147 830 *kilograms,*

</div>

or about 2956 hundredweights, in order to produce a decomposition of the water with such a rate that one milligram of water would decompose in one second. The tension would remain the same for columns of different cross-sections but would increase in proportion to the length of the column.

Should the water decompose at a small rate under the same conditions — e.g., with a rate of one milligram per 2956 seconds — then the tension above would have to be proportionally smaller; viz., only one hundredweight. Above all, the tension could then be arbitrarily small, and decomposition would always result, but only at a lower rate as the tension become smaller. However, that is true only under the assumption that the *force of resistance* by which the water opposes its decomposition (the motion of the hydrogen and the oxygen in opposite directions), which is analogous to the *force of resistance* that opposes the motion of positive and negative electricity inside of a metallic conductor according to Ohm's law, is proportional to the rate of decomposition.[61] However, for metallic conductors, it is very likely that

---

[61][Note by KW:] From Ohm's law, the ratio of the force of resistance by which a conductor opposes the motion of the electricity inside of it to the velocity of that motion is a *constant* that is called the *resistance* of the conductor.

Ohm's law does not correspond to reality precisely, but that, strictly speaking, the force of resistance consists of two parts, one of which is proportional to the rate, while the other is constant, since it is only in that way that the better conductors (e.g., metals) can be included in the same law as the worse ones (e.g., insulators). The same thing is also probably true for the force of resistance by which the water opposes the motion of the hydrogen and oxygen in opposite directions in its interior. The *resistance* (viz., the force of resistance divided by the current velocity) will then be represented by the sum of a constant w and a part k/i that is inversely proportional to the current velocity. Now, if one substitutes that sum for the *resistance* in Ohm's law then one will get the current intensity i, expressed in terms of the electromotive force E and the given sum, in the following way:

$$i = \frac{E}{w + k/i} \ ,$$

or

$$E = k + wi \ .$$

For metallic conductors, k is very small compared to the value of wi that comes from the measurements; for insulators, wi vanishes in comparison to k.

Now, no precise experiments involving water exist from which the value of the constant k could be measured. However, there do exist experiments in which it was shown that this constant does not vanish completely, although it is still very small. Namely, if one conducts magnetically induced currents through water, then one can infer from the measurable current effects that this induction would decompose more or less water according to whether it happened faster or slower, respectively, which would not be the case if one had $k = 0$. — For electrolytic measurements, wi is typically so large that k will not come under consideration in comparison to it.

One refers to the forces that define the resistance to the decomposition of the hydrogen and oxygen in water as *forces of chemical affinity*, which one is not, however, in a position to express in known units. In this Section, it will be shown in an example how the results of the foregoing investigation can actually be employed to implement such a determination. In that way, the path to a more detailed exploration of the *laws of forces of chemical affinity* will be blazed, but numerous measurements of those forces would be necessary for that, of which, only one measurement shall be given as an example.

20 Section 20. Electricity Content in a Conductor

The intensity of the current that goes through a conductor is proportional to the velocity with which the positive and negative electricity flows through the *cross-section of the conductor* and therefore depends upon two factors:

1. The amount of electricity that is contained in each *element of length* of the conductor (which can be called the *capacity* of the conductor).

2. The velocity with which that amount of electricity (viz., positive and negative moving in opposite directions) advances in the conductor.

The intensity of the current that flows *through the cross-section of the conductor* — that is, the amount of positive and negative electricity — can also be measured in known units, so either the amount of electricity that is contained in an *element of length* in the conductor or the *velocity* with which it advances in that conductor can be determined individually: That could happen only in those cases where the one kind of electricity does not move by itself, but the particles of the conductor in which it is contained move with it.

Now, whether that case comes about when the electricity jumps from one conductor to another (through a layer of air), whereby small particles of the one conductor break away and go over to the other conductor, has not, in fact, been ascertained experimentally, and it also cannot be ascertained completely and with certainty. However, it seems that under certain conditions, it can be established factually that small particles can break away from only the positively charged conductor and go over to the negative conductor. There is also no doubt that these small particles that break away are charged with free positive electricity and that the transfer of a well-defined amount of electricity from one conductor to another will be mediated by them. However, whether the transfer of only part of the positive electricity or all of it from one conductor to the other will be mediated in that way, and furthermore whether those small breakaway particles contain merely free positive electricity or also a well-defined amount of negative electricity, in addition, and finally, how the negative electricity on the other conductor behaves during the process, has not been subjected to a more detailed discussion up to now.

As far as the behavior of the electricity on the negatively-charged conductor is concerned, of which, no particles will break away and move to the positive conductor under the aforementioned conditions, it would seem to emerge from this that the negative charge on the conductor suffers some sort

of deceleration under those conditions and therefore that before that charge has attained the strength that is required for the liberation of small particles, the particles that break away from the positively-charged conductor have already arrived at the negative one and hinder the growth in negative charge by transmitting their positive charge. Hence, no electricity at all would go from the negatively charged conductor to the positively charged one under those conditions.

As far as the other question is concerned, of whether the liberated particles contain merely positive electricity or whether they carry a well-defined quantity of neutral fluid with them, in addition, a definite opinion on that could only be based upon some fact about the liberated particles at the highest level of detail.

Namely, it is known that when a larger ball is separated from a smaller one after contact, the free electricity that is contained in both of them will split between them in a well-defined ratio, and indeed in such a way that the mean densities of the layers of electricity that are found on the surface of each ball will not be equal, but the mean density that is found on the surface of the smaller ball will be larger than the density that is found on the surface of the larger ball, and in fact that ratio will approach:

$$1.6449 : 1$$

as the two balls become the more unequal.

Now, a particle that breaks away can be considered to be only an extremely-small ball, and therefore when one denotes the density of amount of electricity that is present on the surface of the positively-charged conductor by ε, the density of liberated particles that are present on the surface will be set equal to $1.6449 \cdot \varepsilon$. Now, it is known that whereas ε vanishes in comparison to the radius of curvature of the surface of the positively-charged conductor, $1.6449 \cdot \varepsilon$ will also vanish in comparison to the radius of the smallest liberated particle, but in contrast, due to the extreme smallness of that particle, one must assume that its radius is smaller than $1.6449 \cdot \varepsilon$, or at least no larger than it. However, it would then follow that this layer of positive electricity would fill up the entire positive particle, and therefore no space would be left in that layer that might contain a well-defined amount of neutral fluid. The small liberated particle would then contain merely free positive electricity. Finally, in regard to the question of whether the free electricity goes from the positively-charged conductor to the negative conductor only by means of the liberated particles or if another quantity of positive electricity without ponderable carriers finds a path to the negatively-charged conductor by itself, as well, one can only assert that given the lack of any physical basis upon which it would depend, under exactly the same conditions, the one part of the electricity should move independently of its ponderable carrier, while

the other part must move with its ponderable carrier. Since that would then actually establish that part of the transferred electricity was drawn along by its ponderable carrier, that must be assumed of all the transferred electricity until the contrary has been proved.

The case of a current for which the conducting particles, which contain only positive electricity, would advance would then actually exist. The amount of advancing electricity that goes from the one conductor to the other can now be determined precisely from the measurements that are obtained (by measuring the current intensity). As a result, all that remains is to measure precisely the amount of ponderable mass that simultaneously breaks away from the positive conductor and lands on the negative conductor. Although that ponderable mass might also be so small, nonetheless, it can still be clearly observed, and from that, one can assume that its weight can also be determined with the most accurate balance that we possess.

In any event, that implies that even for very large amounts of electricity that go from the positively-charged conductor to the negatively-charged one, the ponderable mass of the conducting particles that break away is very small, and as a result, the amount of electricity that is contained in each *element of length* in the conductor is exceptionally large. However, the larger that amount of electricity gets, the smaller that the velocity with which that amount of electricity advances in the conductor will be, and that smaller velocity with which the electrical fluid moves in its conductor can then by no means be confused with the extremely large velocity with which the perturbation of the equilibrium in the electrical fluid propagates through the metallic conductor, to which the well-known experiment of Wheatstone referred.[62]

The facts that the amount of electricity that is contained in one element of length in a *metallic conductor* is very large and that the velocity with which the amount of electricity moves in the conductor is very small for all currents that are presented in reality, could have been expected beforehand by analogy with the results that were found in Section 15 for a *wet conductor* (e.g., water), because it was found there that for a current whose intensity is equal to 1 in *electrolytic* units, an amount of positive electricity of $106\frac{2}{3} \cdot 155\,370 \cdot 10^6$ units, together with 1/3 milligram of hydrogen, will move in one direction, while an equally-large amount of negative electricity that is bound to 8/9 milligram of oxygen will move in the opposite direction through the cross-section of the conductor in one second, from which, it would follow that $106\frac{2}{3} \cdot 155\,370 \cdot 10^6$ units of positive electricity and equally-much negative electricity must be contained in one milligram of water, but they (together with their ponderable carriers) advance only with the very small velocity of

---

[62][Note by AKTA:] [Whe34].

1/2 millimeter in one second when the area of the cross-section of the wet conductor is only 1 square millimeter. If the cross-section were larger, then the velocity would be proportionally smaller.

21   Section 21. Applying This to Units

The units that are useful in physics are divided into the *fundamental units* and the *derived units.* In general mechanics, where all forces are considered to be given individually, all units can be reduced to the known fundamental units of *space*, *time*, and *mass.* — In all of those branches of physics where the *law of gravitation* must be assumed to apply, all units can be reduced to merely the two fundamental units of *space* and *time*, since the units of *mass* can also be derived with the help of the law of gravitation. Namely, one can take the unit of mass to be the mass that would, if it were concentrated at a point, exert a force upon another mass at a unit distance that would impart a velocity to the latter that would equal one unit length per unit time according to the law of gravitation.

Now, it is interesting to note that this system of units is capable of being simplified even further, and that it is possible to derive all of the units that are used in physics *from the single basic unit of space* when one assumes two fundamental laws of nature to that end, namely, in addition to the *law of gravitation of ponderable masses*, one assumes *the fundamental law of electrical action,* since one can also *derive the unit of time from the unit of space* with the help of the latter. Namely, one can take that unit of time to be the time during which two electrical masses that move with uniform relative velocity must move towards or away from each other if they are to have no influence on each other according to that law.

If one chooses the *millimeter* to be the spatial unit, then the unit of time could be derived from it under the assumption of the fundamental law of electrical action, and it would be the:

<div align="center">439 450 millionth part of a second,</div>

since when two electrical masses that move with uniform relative velocity approach or move apart from each other [the distance] of 1 millimeter in that small time interval, they will exert no effect on each other according to the fundamental law of electrical action.

Once the time unit has been derived from the spatial unit in that way, the unit of mass can also be derived from those two units under the assumption of the law of gravitation. Namely, from the law of gravitation, the Earth is a mass that, if it were concentrated into a point, would impart an acceleration equal to 9811 upon another mass at a distance equal to the Earth radius if

the millimeter were used as the spatial unit and the second were used as the unit of time. If one were to take the unit of time that was just derived instead of the second, which is 439450 million times smaller, then the derived unit of acceleration would be 4394502 billion times larger, and the acceleration would be equal to:

$$\frac{9811}{439450^2 \cdot 10^{12}}$$

in that larger unit. Now, if one sets the radius of the Earth equal to $6370 \cdot 10^6$ (millimeter), then according to the law of gravitation, if the mass of the Earth were concentrated into a point, then it would impart an acceleration upon another mass at a unit of distance that would equal:

$$\frac{9811 \cdot 6370^2 \cdot 10^{12}}{439450^2 \cdot 10^{12}} \ ,$$

and as a result, a mass that amounts to $\frac{439450^2}{9811 \cdot 6370^2}$, or almost one-half the mass of the Earth, which is the mass that one will get as the *derived mass unit* from the law of gravitation, under the assumption that the millimeter is the spatial unit and with the help of the time unit that was derived from it before.

Finally, all of the remaining units that are used in physics can be derived from the millimeter as the spatial unit and the units of time and mass that were just derived from it in known ways.

According to this system, in which all units can be derived from the single fundamental unit of distance, the force of attraction between two masses m and m' at a distance of r will be equal to mm'/r^2, and the force of repulsion of two amounts of electricity e and e' at a distance of r will be equal to $(ee'/r^2)(1-dr^2/dt^2+2rd^2r/dt^2)$, without having to add that these expressions or the individual terms in them must be constant factors.

Appendix

A   Appendix I. Description of the Torsion Balance

In order to avoid, as much as possible, an unequal reaction of the charged balls on the moving ball of the torsion balance due to the electrostatic induction of its electrified walls, the balance is usually associated with a very large scale. The case in which the balls were hung was a parallelepiped that was 1.16 meters long, 0.87 meter wide, and 1.44 meter high. The twelve edges of the parallelepiped were constructed from square posts (with a thickness of 80 mm) of hard wood. Once the framework was established on a large stone foundation, a heavy sheet of wood was laid upon it as a lid, but the side walls were draped with a tightly-stretched oilcloth in such a way that the edges of the posts would not protrude into the interior of the space. After that draping, which left merely the upper fourth of a wall open for one to hang the apparatus, the rigidity of the case was increased appreciably by bolted struts. For the measurement itself, once the fixed ball was introduced, the opening was closed with a slide. However, in addition, the entire case was covered with multiple layers of towels and blankets that rested upon the stone in order to keep the draft off of it. Nevertheless, it was necessary to make the observations at night in an unheated room, since the opening and closing of the doors in other parts of the building and the uneven warming of the floor by the Sun would give rise to air currents that would produce an occasional oscillation of the moving ball of up to one-half of a degree. However, at night, when the outside air was not too agitated, the ball did not oscillate by even one minute.

Fig. 2.

The torsion circle T was fixed over the center of the lid, whose cross-section is denoted by D in Figure 2, whose alidade AA' allowed one to read off the individual minutes from its vernier scale and would lead to a finer

adjustment of the torsion with a Hook key[63] H or also by freely loosening it by hand. Furthermore, the definitions of the symbols in the Figure are:

- a the hard-drawn brass wire (no. 12), which is 398 mm long and fixed in the axis of the alidade;

- b a small brass cylinder with a side screw for clamping it fast to the lower end of a. Under it, is

- c a 5 mm protruding threaded spindle, in order to attach either the body whose period of oscillation is to be determined from the torsion coefficient, or the brass wire

- d, to which the 5 mm thick, 450 mm long, cylindrical rod ef of pure shellac was fused.[64]

- hi means the shellac lever for the moving ball, which was tapered on both sides up to 2.5 mm in thickness along perhaps 60 mm in length.

- fg is a wire that is immersed an inch deep in olive oil with a mirror s on it that is attached to wood. The oil has the effect of damping out not only the oscillation of the moving ball, but also the pendulum motions that arise from vibrations, in the shortest time, while on the other hand, it is no impediment that the lever follows the most imperceptible changes in torsion.

The two balls of the torsion balance consist of very thin Argentan sheet metal that were finely polished and gold-plated, and merely heat-glued to the shellac.

The long vertical shellac rod for the *fixed ball*, which was tapered below, was glued to a curved brass rod mn. A horizontal axis pq with two steel tips was solidly fixed to it, and at right angles to it, a brass rod rt with a running weight. The running weight pushed the upper end of the brass rod mn against an adjusting screw, such that precisely the same position of the torsion balance would result whenever the fixed ball was taken out or put in. If one pushed the brass rod mn forward in order to charge the moving ball until the rod tr joined up with an adjustment screw, then the charged fixed ball would be found to be near the moving one, so the former could attract and charge the latter without the latter needing to describe a large path.

[63][Note by DHD:] *Hook'schen Schlüssel* in the original.

[64][Note by KW:] The length ef, and above all, the length Tg, are too negligible in comparison to the size of the upper part of the figure to be indicated. The balls were further away from the lid.

Opposite to the mirror s, there was an opening in the wall of the torsion balance that was closed with flat glass. Outside at some distance, one found a horizontal scale whose mirror image could be observed in a telescope. The distance to the scale was chosen such that when the rotation of the level in the torsion balance amounted to one minute, the scale in the telescope would move by one scale division. At the same time, the scale was positioned such that when the centers of the two balls defined precisely a right angle with the axis of rotation, its zero-point, which was placed in the center and from which the scale was numbered on both sides outward, would appear in the crosshair of the telescope.

That was the position of the balls in which they should be observed, which could always be known with great accuracy in that way. Had the moving ball moved further from the fixed ball after they were electrified, then the observer who was found at the telescope could likewise read off how many degrees or minutes would be needed to correct the state of the moving ball by torsion. On the other hand, a disc was installed on the hook key that allowed one to see the rotation of that key in minutes of the rotation of the alidade, and the torsion-adjusting second observer could bring about the correction on command[65] without needing to look at the vernier scale. Some practice with the timely assignment and performance of that command and the excellent effect of the oil soon brought one to the point that the moving ball, which was thought to be put into a state of violent motion by the charging, could be brought to rest completely in a relatively short time in such a way that the centers of the two balls would define an angle with the axis of rotation that was larger than a right angle by only a few minutes; i.e., such that the zero-point of the scale in the telescope would be at a distance of a few tick marks on the crosshair of the telescope. The loss of electricity would then bring the ball gradually closer to the fixed ball due to the torsion on it that was present, such that the time-point at which the zero-point of the slowly-drifting scale passed the crosshair of the telescope would be determined accurately. The torsion could be read off from that.

The state in which the centers of the two balls define precisely a right angle with the rotational axis of the torsion balance is found in the following way:

---

[65][Note by KW:] If one wished to bring the lever in an uncharged torsion balance from one position to another without producing long-lasting oscillations, then one would make one-half the correction suddenly when the lever was still at rest and the other half just as suddenly at the moment when attained its greatest elongation and began to reverse. It would then become more still the less the air resistance came into consideration in comparison to its moment of inertia. One will achieve the goal approximately for the charged torsion balance in that way.

Fig. 3.
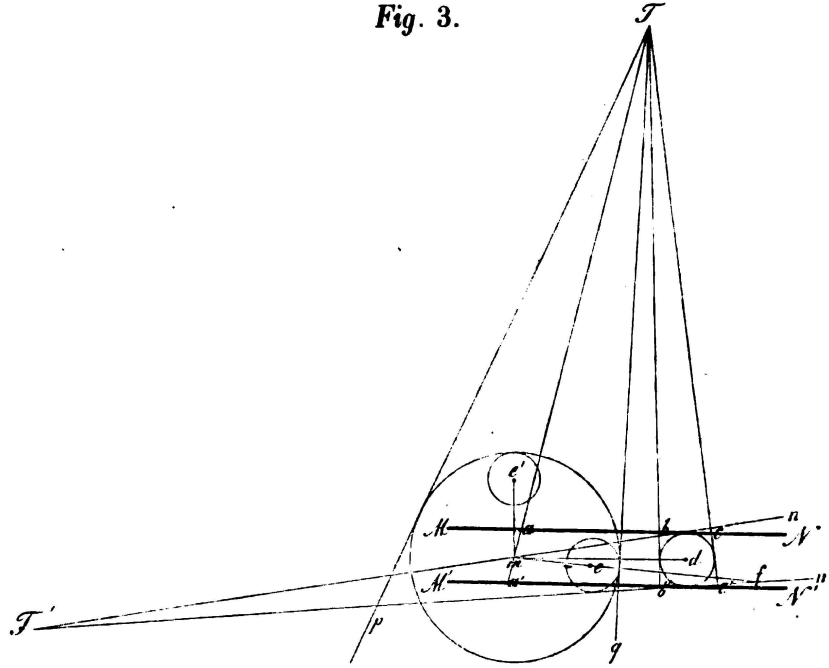
Once a fine filament that was weighted down (whose projection m represents the axis of rotation in Figure 3), was fixed to the small cylinder on the torsion wire in place of the shellac rod, a theodolite T was placed at a distance of a few meters, and the distance Tm was measured precisely. From there, an ivory yardstick that was divided into millimeters was brought into the positions MN and $M'N'$ horizontally, such that it stood parallel to md each time and was tangent to the fixed ball at one-half its height. The vertical crosshair in the telescope of the theodolite allowed one to estimate the lengths ab, ac, $a'b'$, and $a'c'$ to one tenth of a millimeter due to its higher magnification. One then had:

$$md = \frac{1}{4}(ab + ac + a'b' + a'c') \ .$$

After that, a second theodolite was placed at a point T' such that the vertical line in its telescope covered the rotational axis m and was tangent to the fixed ball. Once $T'm$ was measured, the telescope was rotated into the position $T'n$ such that the line was tangent to the other side of the fixed ball, and it then remained unperturbed.

One then hung the shellac rod with the moving ball from the torsion wire again and measured the angle pTq with the theodolite T. The moving ball, which was protected from light reflection, stood out very sharply from the white background, and the theodolite pointed to the tangent to the circle

inside of which it moved by slow rotation. The distance from the center of the moving ball to the axis of rotation was then:

$$me = Tm \sin \frac{1}{2} pTq - r' \ ,$$

in which r' is the previously measured radius of the moving ball.

The fixed ball was now taken out, and in order to avoid air currents, the case of the torsion balance was closed completely, except for two small openings in the already-known direction $T'n'$, and the moving ball was placed in such a way that it would be tangent to the direction $T'n'$ by means of the torsion wire.

It would then be necessary to rotate the moving ball through $90^{\text{o}} + dme$ in order to make its center come to the position e', in which it would describe a right angle with m and d. Now, the angle:

$$dme = mfT' + mT'f - nmd \ ,$$

while:[66]

$$mfT' = \arcsin \frac{T'm \sin mT'n' - r'}{me} \ ,$$

$$mT'f = 2 \cdot \arcsin \frac{r}{T'm + md \cos nmd} \ ,$$

$$nmd = \arcsin \frac{r}{md} \ .$$

Since everything in that has been given, dme could be easily calculated, and the rotation of the moving ball through $90^{\text{o}} + dme$ was accomplished by means of the torsion circle, so the zero-point of the observer scale was located correctly.

B Appendix II. Description of the Tangent Galvanometer

The copper wire that was employed for the multiplier was wound quite tightly with silk, and then almost 2/3 of a mile of collodion was pulled along its entire

---

[66][Note by KW:] The multiplicity of these possibilities was due to the opacity of the hanging shellac rod.

length.[67] From the large roll on which it was then found, with the help of a very uniformly tensed pulley, it would be wound around the circular ring of the tangent galvanometer with 5635 windings. That metal ring, which defined a channel of rectangular cross-section, was previously given a thick coating of heated sealing wax everywhere that the wire was laid in it. After that, a 20 pound copper weight was placed into the ring as a damper. All of the remaining procedures are known.

The main idea was to confirm one's belief that all windings of the tangent galvanometer would actually be traversed by the discharge current, and that it would not perhaps jump over some of them by a spark that occurred deep within the windings, but perhaps not visibly. Now, a small multiplier of 1000 windings that had been used often at Marburg was on hand, and it could be predicted from the dimensions of the two instruments that they would have roughly the same sensitivity to the discharge of a Leyden jar. Both multipliers were coupled in such a way that the same discharge from a larger Leyden jar, when retarded by a column of water, would have to flow through the windings in both of them. Now, since not only the predicted behavior of the sensitivity occurred, but upon raising the charge, the data from both galvanometers remained proportional to each other, as well as the data that corresponded to a sine electrometer, which allowed one to compare the charge on the Leyden jar in isolation when coupled to it, one could convince oneself that the large tangent galvanometer would serve its purpose. For all discharges that would be regulated by a specially-constructed pendulum, the knob on the jar remained coupled with the multiplier for the same time (and in fact, only 2/3 of a second) in order to allow only a very small (and in fact proportional) part of the residue to appear again. The results are as follows:

---

[67][Note by KW:] Experiments concerned with whether the degree of insulation would actually increase in that way have not been performed, but one should, nonetheless, assume that is so. In any event, in that way, one will arrange that the silk not only adheres to the wire very firmly, but also that it does not become slightly rough on the surface. The process is simple: One leads the wire from the original roll to a small fixed roll with a horizontal axis, and from there, to a larger roll at a greater distance, around which it will be temporarily wound. The small fixed roll is immersed halfway in a container of collodion.

No.	a. Deflection φ of the sine electrometer	b. $\sqrt{\sin \varphi}$	c. Small multiplier. Elongation in scale divisions	d. Tangent-galvanometer. Elongation in scale divisions	d/c	d/b
1.	9°31′	0.4078	41.75	170.40	4.1060	417.85
2.	19°59′	0.5845	59.50	244.85	4.1151	418.91
3.	34°57′	0.7569	76.95	316.10	4.1078	417.62
4.	49°54′	0.8746	88.97	365.45	4.1076	417.85

Each of the numbers under c and d is the mean of 2 to 3 measurements that differed from each other by at most one scale division. The desired proportionality then emerged from this completely. Now, the distance from the mirror to the scale was 1633 for the small multiplier and 6437.6 scale divisions for the large one, and their sensitivities then had roughly the ratio that was required above, namely, 1 : 1.0423.

Those measurements, the second of which could obviously be assumed to include an observation error in the tangent galvanometer, showed an extraordinary accuracy in the comparison of the available charge in a Leyden jar for all three instruments.

References

[Buf53]   H. Buff. Tangentenbussole mit langem Multiplicatordrahte. *Annalen der Chemie und Pharmacie*, 86:1–34, 1853.

[Cas43]   W. T. Casselmann. *Ueber die galvanische Kohlenzinkkette und einige mit derselben angestellte Beobachtungen*. PhD thesis, Marburg, 1843. Inaugural-Dissertation, philosophischen Facultät zu Marburg.

[Hor47]   E. N. Horsford. Ueber den elektrischen Leitungswiderstand der Flüssigkeiten. *Annalen der Physik und Chemie*, 70:238–243, 1847.

[Koh53]   R. Kohlrausch. Das Sinus-Elektrometer. *Annalen der Physik und Chemie*, 88:497–518, 1853.

[Koh54a] R. Kohlrausch. Theorie des elektrischen Rückstandes in der Leidener Flasche. *Annalen der Physik und Chemie*, 91:56–82, 1854.

[Koh54b] R. Kohlrausch. Theorie des elektrischen Rückstandes in der Leidener Flasche (Schluss). *Annalen der Physik und Chemie*, 91:179–214, 1854.

[KW57]   R. Kohlrausch and W. Weber. Elektrodynamische Maassbestimmungen insbesondere Zurückführung der Stromintensitäts-Messungen auf mechanisches Maass. *Abhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften, mathematisch-physische Klasse*, 5:221–290, 1857. Reprinted in Wilhelm Weber's *Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 609-676.

[Len35]   E. Lenz. Ueber die Leitungsfähigkeit der Metalle für die Elektricität bei verschiedenen Temperaturen. *Annalen der Physik und Chemie*, 34:418–438, 1835.

[Len38]   E. Lenz. Ueber die Leitungsfähigkeit des Goldes, Bleis und Zinns für die Elektricität bei verschiedenen Temperaturen. *Annalen der Physik und Chemie*, 45:105–121, 1838.

[Pla45]   J. Plana. Mémoire sur la distribution de l'électricité à la surface de deux sphères conductrices complètement isolées. *Memorie della Reale Accademia delle Scienze di Torino*, 7:71–401, 1845.

[Pla54]   J. Plana. Mémoire sur la distribution de l'électricité à la surface intérieure et sphérique d'une sphère creuse de métal, et à la surface

d'une autre sphère conductrice électrisée que l'on tient isolée dans la cavité. *Memorie della Reale Accademia delle Scienze di Torino*, 16:57–95, 1854.

[Poi12a] Poisson. Extrait d'un mémoire sur la distribution de l'électricité à la surface des corps conducteurs. *Journal de Physique, de Chimie, d'Histoire Naturelle et des Arts*, 75:229–237, 1812.

[Poi12b] Poisson. Mémoire sur la distribution de l'électricité à la surface des corps conducteurs. *Mémoires de la Classe des Sciences Mathématiques et Physiques*, pages 1–92, 1812. Lu les 9 mai et 3 août 1812.

[Poi13]   Poisson. Second mémoire sur la distribution de l'électricité a la surface des corps conducteurs. *Journal de Physique, de Chimie, d'Histoire Naturelle et des Arts*, 77:380–386, 1813.

[Web41]   W. Weber. Ueber das elektrochemische Aequivalent des Wassers. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1840*, volume V, chapter IX, pages 91–98. Weidmannschen Buchhandlung, Leipzig, 1841. Reprinted in Wilhelm Weber's *Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 13-18.

[Web42]   W. Weber. Ueber das elektrochemische Aequivalent des Wassers. *Annalen der Physik und Chemie*, 55:181–189, 1842. Reprinted in Wilhelm Weber's *Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 13-18.

[Web46]   W. Weber. Elektrodynamische Maassbestimmungen — Über ein allgemeines Grundgesetz der elektrischen Wirkung. *Abhandlungen bei Begründung der Königl. Sächs. Gesellschaft der Wissenschaften am Tage der zweihundertjährigen Geburtstagfeier Leibnizen's herausgegeben von der Fürstl. Jablonowskischen Gesellschaft (Leipzig)*, pages 211–378, 1846. Reprinted in Wilhelm Weber's *Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 25-214.

[Web52]   W. Weber. Elektrodynamische Maassbestimmungen insbesondere Widerstandsmessungen. *Abhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften, mathematisch-physische Klasse*, 1:199–381, 1852. Reprinted in Wilhelm Weber's *Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 301-471.

[Web53a] W. Weber. Die Anwendung der magnetischen Induction auf Messung der Inclination mit dem Magnetometer. *Annalen der Physik und Chemie*, 90:209–247, 1853.

[Web53b] W. Weber. Ueber die Anwendung der magnetischen Induktion zur Messung der Inklination mit dem Magnetometer. *Göttinger Nachrichten*, 2:17–24, 1853. Reprinted in Wilhelm Weber's *Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 328-332.

[Web53c] W. E. Weber. Ueber die Anwendung der magnetischen Induktion auf Messung der Inklination mit dem Magnetometer. *Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen*, 5, 1853. Reprinted in Wilhelm Weber's *Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 277-327.

[Web87] W. Weber. Mesures électrodynamiques. In J. Joubert, editor, *Collection de Mémoires relatifs a la Physique,* Vol. III: *Mémoires sur l'Électrodynamique*, pages 289–402. Gauthier-Villars, Paris, 1887.

[Web07] W. Weber, 2007. Determinations of electrodynamic measure: concerning a universal law of electrical action, 21st Century Science & Technology, posted March 2007, translated by S. P. Johnson, edited by L. Hecht and A. K. T. Assis. Available at `http://21sci-tech.com/translation.html` and `www.ifi.unicamp.br/~assis`.

[Whe34] C. Wheatstone. An account of some experiments to measure the velocity of electricity and the duration of electric light. *Philosophical Transactions of the Royal Society of London*, 124:583–591, 1834.