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On the possibility of an electromagnetic basis for mechanics

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H. A. Lorentz ([1]) has recently sought to reduce gravitation to the electrostatic attraction between elements that consist of ions. To that end, he made the assumption that the attraction between positive and negative electricity exceeds the repulsion between the electricity of the same kind. I was induced by that to publish considerations on the same topic that I had already begun a long time ago in which I nonetheless went beyond the **Lorentz**ian standpoint.

Undoubtedly, one of the most important problems in theoretical physics is to couple the initially completely-isolated domains of mechanical and electromagnetic phenomena with each other and to derive differential equations that are valid for each of them from a common foundation. **Maxwell** and **Thomson**, and subsequently **Boltzmann** and **Hertz**, have embarked upon a path that certainly seems natural at first, which chooses mechanics as the foundation and derives **Maxwell**'s equations from that. Numerous analogies that exist between electrodynamical and hydrodynamical, as well as elastic, processes increasingly seem to point to that path. **Hertz**'s mechanics seems to me to be designed, by its very nature, for the purpose of encompassing not only mechanical, but also electromagnetic, phenomena. It is known that **Maxwell** himself showed that a mechanical derivation of **Maxwell**'s electrodynamics is possible.

Those investigations have doubtless performed a great service by showing that both domains must have something in common and that the present separation is not in the nature of things. On the other hand, it seems to me that it emerges with certainty from those considerations that our system of mechanics up to now is unsuited for the representation of electromagnetic processes.

Nowhere will one acknowledge that the complicated mechanical models that one constructs for machines, which are devised for specialized engineering purposes, are an ultimately satisfactory picture for the internal composition of the ether.

Whether **Hertz**'s mechanics, whose structure is, in fact, especially suited for the representation of very general kinematical connections, proves to be more appropriate remains to be seen. For the time being, it has also not been able to represent even the simplest processes that lie beyond the scope of kinematics.

([1]) **H. A. Lorentz**, Konikl. Akad. v. Wetensch. te Amsterdam, 31 March 1900.

It seems to me that a much more promising foundation for the further theoretical work is to try to do the opposite thing, namely, to regard the basic equations of electromagnetism as the more general ones from which the mechanical equations can be deduced.

The actual foundations would be defined by the electrical and magnetic polarization in the free ether, which are connected with each other by **Maxwell**'s differential equations. How those equations can best be derived from the facts is a question that we shall not address here.

If we let X, Y, Z denote the components of the electric polarization, and let L, M, N denote those of the magnetic polarization, while A denotes the inverse speed of light, and x, y, z are the rectangular coordinates then we will have:

$$(1) \qquad \left\{ \begin{aligned} A\frac{\partial X}{\partial t} &= \frac{\partial M}{\partial z} - \frac{\partial N}{\partial y}, & A\frac{\partial L}{\partial t} &= \frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z}, \\ A\frac{\partial Y}{\partial t} &= \frac{\partial N}{\partial x} - \frac{\partial L}{\partial z}, & A\frac{\partial M}{\partial t} &= \frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x}, \\ A\frac{\partial Z}{\partial t} &= \frac{\partial L}{\partial y} - \frac{\partial M}{\partial x}, & A\frac{\partial N}{\partial t} &= \frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y}. \end{aligned} \right.$$

The electric and magnetic quanta will give the integration constants from that when we differentiate equations (1) with respect to x, y, z, respectively, and add them. Namely, we will then have:

$$\frac{\partial}{\partial t}\left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z}\right) = 0 , \qquad \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z}\right) = 0 ,$$

so

$$(2) \qquad \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} = -4\pi\varsigma , \qquad \frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z} = -4\pi m ,$$

in which ς and m are independent of time, so they are temporal and variable quanta.

If one multiplies the first series of equations (1) by $X / 4\pi$, $Y / 4\pi$, $Z / 4\pi$, resp., and the second one by $L / 4\pi$, $M / 4\pi$, $N / 4\pi$, resp., then after one partially integrates over a closed space whose surface and surface element might be n and dS, resp., one will get the theorem:

$$(3) \qquad \left\{ \begin{aligned} &\frac{1}{8\pi}\frac{d}{dt}\iiint dx\,dy\,dz\,(X^2 + Y^2 + Z^2 + L^2 + M^2 + N^2) \\ &= \iint dS\,[(Y\,N - Z\,M)\cos(n\,x) + (Z\,L - X\,N)\cos(n\,y) + (X\,M - Y\,L)\cos(n\,z)]. \end{aligned} \right.$$

If either X, Y, Z or L, M, N vanish on the surface then we will have:

$$(4) \qquad \frac{1}{8\pi}\iiint dx\,dy\,dz\,(X^2 + Y^2 + Z^2 + L^2 + M^2 + N^2) = \text{const.}$$

We call the expression on the left-hand side, which always stays constant when it is summed over a sufficiently-large space, the *electromagnetic energy*.

We now make the assumption that the mechanical processes are also of an electromagnetic nature, so they can also be developed from the foundations that we just considered.

We shall first assume that the substrate that we shall refer to as "matter" is composed of positive and negative electrical quanta, and indeed as quanta that we have to regard as simply the points where electrical lines of force converge.

Meanwhile, we must attribute a certain extension to such an elementary quantum, since otherwise the energy supply that it represents would become infinitely large compared to that of the quantum itself. Since all of matter is supposed to be constructed from such quanta, they must be assumed to be so small that the atomic weight must be a whole number multiple of it. The elementary positive quantum is further regarded as separated from the negative one by a certain small segment.

The statement that matter is also atomistically constructed from electricity is equivalent to the viewpoint that we shall assume here.

The ether itself is regarded as being at rest in the Lorentz process. Changes of position can occur only for the electrical quanta, so to speak of a motion of the ether would make no sense according to the basic law that will be pursued here.

All forces are reducible to known electromagnetic ones, in the **Maxwell** sense, so to stresses in the ether, although the concept that one infers from the theory of elasticity is hardly meaningful.

For small velocities of the moving quanta, it is the electrostatic forces that act between the quanta.

The question of whether it is possible to reduce molecular forces to such forces must initially remain open. It is only clear that one can obtain very complicated actions from various groupings of positive and negative quanta at various distances. With that assumption, one can lessen the difficulty that the **Michelson** interference experiment has introduced into the theory of a rest ether up to now.

H. A. Lorentz ([1]) has drawn attention to the fact that the length of a body in the direction of motion of the Earth will be shortened by the velocity v of that motion by the ratio $\sqrt{1-A^2 v^2}$ when the molecular forces can be replaced with electrostatic forces.

That will explain **Michelson**'s result when one distances oneself from the molecular motions. The extent to which it would then apply would have to be be indicated by gas-theoretic investigations.

In order to explain gravitation, like **Lorentz**, we must assume that two different types of electric polarization are juxtaposed. Each of them satisfies **Maxwell**'s equations. In addition, for a static field, one also has:

$$X = -\frac{\partial \Phi}{\partial x}, \qquad Y = -\frac{\partial \Phi}{\partial y}, \qquad Z = -\frac{\partial \Phi}{\partial z},$$

and the energy will be:

([1]) **H. A. Lorentz**, *Versuch einer Theorie der elektromagnetischen Erschienungen in bewegten Körpern*, Leiden, 1895.

$$\frac{1}{8\pi}\iiint dx\,dy\,dz\left[\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2+\left(\frac{\partial\Phi}{\partial z}\right)^2\right] = \frac{1}{8\pi}\iint dS\,\frac{d\Phi}{dn}-\frac{1}{8\pi}\iiint dx\,dy\,dz\,\Phi\,\Delta\Phi\,.$$

If Φ or $\partial\Phi / \partial n$ vanishes on the surface of the space then the energy is:

$$= -\frac{1}{8\pi}\iiint dx\,dy\,dz\,\Phi\,\Delta\Phi\;.$$

Now, from (2), one has:

$$\Delta\Phi = -4\pi\varsigma\,, \qquad \Phi = \iiint \frac{\varsigma\,dx\,dy\,dz}{r}\,,$$

so the integral is:

$$= \tfrac{1}{2}\iiint \frac{\varsigma\,dx\,dy\,dz}{r}\iiint \varsigma'\,dx'dy'\,dz'$$
$$= \iiint\iiint \frac{\varsigma\,\varsigma'\,dx\,dy\,dz\,dx'dy'\,dz'}{r}\,.$$

If two quanta with the same type are found at a distance of r from each other:

$$e = \varsigma\,dx\,dy\,dz\,,$$
$$e' = \varsigma'\,dx'\,dy'\,dz'$$

then the energy will be:

$$\frac{e\,e'}{r} = -\int_\infty^r \frac{e\,e'}{r^2}\,dr\,. \tag{5}$$

That energy is created by the work done against an attractive force with a magnitude of:

$$-\frac{e\,e'}{r^2} \tag{6}$$

that acts between the quanta.

The force that acts between two quanta is defined by that.

That law must be valid for both of the types of polarization.

If positive and negative quanta interact with each other then the **Lorentz** assumption will be that the attractive force that will then appear is greater by a certain ratio than the repulsive force between quanta of the same type. At large distances, the dipole will act as if the positive and negative quanta are at the same location. Thus, one will obtain an excess of attraction by the total effect of the negative and positive quanta on a second dipole.

That explanation for gravitation has the immediate consequence that its perturbations propagate with the speed of light and that it must itself experience a modification by the motion of the attracting bodies. **Lorentz** has investigated whether that modification of gravitation can explain the anomalies in the motion of Mercury, and nonetheless found a negative result. Some astronomers believe that one must assume a greater velocity than the speed of light. Meanwhile, one cannot speak of a speed of propagation of gravitation itself, since it is a static force.

That would be sensible only if gravitation become stronger or weaker, and one could then observe the speed of propagation of the perturbations that are produced in that way.

However, since gravitation always acts in an unvarying way, only exceptionally-small changes can come into question, and as **Lorentz** has shown, they would be produced by the motion at second order.

The inertia of matter, which gives a second independent definition of mass, along with gravitation, can be deduced from the concept of electromagnetic inertia that has already been used repeatedly with no further hypotheses.

We imagine that the elementary quantum of electricity is an electrified point. The forces and polarizations that emanate from such moving points were derived by **Heaviside** ([1]).

Since equally-large positive and negative quanta always move together, the forces that emanate from them will cancel out at a distance that is large compared to their separation if one ignores the aforementioned gravitation and polarizations. However, we shall assume in what follows that the extension of the quanta themselves is so small compared to their separation distance that the energy of each individual one is large enough that it is as if the second one were not present.

From a calculation of **Searle** ([2]), those polarizations start from an ellipsoid that moved in the direction of its *a* axis with velocity *v*, and whose other two axes are $a/\sqrt{1-A^2v^2}$, and it carries its charge on its surface. The ratio of the axes then depends upon the velocity.

From **Searle**, the energy of such an ellipsoid is:

$$E = \frac{e^2}{2a}\left(1+\tfrac{1}{3}A^2v^2\right) .$$

The ellipsoid with the same axes has an energy of:

$$\mathfrak{E} = \frac{e^2}{2a}\frac{\sqrt{1-A^2v^2}}{Av}\arcsin Av$$

in the rest state.

Naturally, $\mathfrak{E}$, the energy of the ellipsoid at rest, cannot include the velocity *v*.

Hence, since *e* is invariable and *a* is variable:

$$2a = \frac{e^2\arcsin Av}{\mathfrak{E}}\frac{\sqrt{1-A^2v^2}}{Av} ,$$

$$E = \mathfrak{E}\frac{Av\left(1+\tfrac{1}{2}A^2v^2\right)}{\sqrt{1-A^2v^2}\arcsin Av} ,$$

(1) **O. Heaviside**, *Electrical Papers*, 2.
(2) **G. F. C. Searle**, Phil. Mag. **44** (1897), pp. 340.

or by the series development:

(7) $$E = \mathfrak{E}(1 + \tfrac{2}{3} A^2 v^2 + \tfrac{16}{45} A^4 v^4 + \cdots) .$$

The increase in energy that is produced by the motion is then:

$$\tfrac{2}{3}\,\mathfrak{E} A^2 v^2 = \tfrac{1}{2} m v^2 ,$$

in the first approximation, so the inertial mass is $m = \tfrac{4}{3}\,\mathfrak{E} A^2$.

Thus, the mass that is defined by inertia is constant only for small velocities and would increase as the velocity becomes larger. Since the inertia is proportional to the number of quanta that comprise a body, and likewise the gravitation that emanates from that body, it will follow that the mass that is defined by inertia must be proportional to the mass that is defined by gravitation. If we let a body whose mass is $m = \tfrac{4}{3}\,\mathfrak{E} A^2$ be attracted to a body of mass M at a distance r then the electromagnetic energy supply of gravitation will be diminished by an amount $\varepsilon \tfrac{4}{3}\,\mathfrak{E} A^2 M / r$, in which ε denotes the gravitational constant.

That energy is converted into energy of motion by the production of the velocity v. We then have:

$$\tfrac{2}{3}\,\mathfrak{E} A^2 v^2 (1 + \tfrac{8}{15} A^2 v^2 + \cdots) = \varepsilon \frac{\tfrac{4}{3}\,\mathfrak{E} A^2 M}{r} ,$$

or since $v = dr / dt$:

(8) $$\tfrac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{\varepsilon M}{r}\left[1 - \tfrac{8}{15} A^2 \left(\frac{dr}{dt}\right)^2\right] .$$

We write that as:

(9) $$\tfrac{1}{2}\left(\frac{dr}{dt}\right)^2 \left(1 + \tfrac{16}{15} A^2 \frac{\varepsilon M}{r}\right) = \frac{\varepsilon M}{r} .$$

If the masses M and m are attracted according to **Weber**'s law then one will have:

$$m\,\frac{d^2 r}{dt^2} = -\frac{\varepsilon\, m M}{r^2}\left[1 - \tfrac{1}{2} A^2 \left(\frac{dr}{dt}\right)^2 + r A^2 \frac{d^2 r}{dt^2}\right] .$$

If we multiply by dr / dt and integrate then we will have:

$$\tfrac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{\varepsilon M}{r}\left[1 - \tfrac{1}{2} A^2 \left(\frac{dr}{dt}\right)^2\right] ,$$

in which the integration constant is determined in such a way that body will be at rest at an infinite distance.

If we write that equation as:

$$\text{(10)} \qquad \tfrac{1}{2}\left(\frac{dr}{dt}\right)^2\left(1+A^2\frac{\varepsilon M}{r}\right) = \frac{\varepsilon M}{r}$$

then that will coincide with equation (9), up to the factor $\frac{16}{15}$, rather than 1. By considering the inertia in the second approximation, we will then get approximately the same action between the two masses as when the masses themselves are unvarying, except that **Weber**'s law would be valid for them, instead of **Newton**'s.

It is known that **Weber**'s law has been applied to the theory of the motion of Mercury with some success.

A precise test of these investigations and their extension by applying them to other fast-moving celestial bodies with strongly-eccentric orbits would lead us to a comparison of our results with experiments. However, in so doing, we must consider the fact that new terms of the same order will be added by motion along curved paths. The calculation would then need to be extended for a body that moves along an elliptic path.

We have velocities that are large enough for the square of the velocity that multiplies the reciprocal of the speed of light to not be too small only for cathode rays.

The fastest rays that have been produced up to now have 1/3 the speed of light. The apparent increase in the mass would then be about 7 percent in that case. The smallest velocity is 1/30 the speed of light ([1]), so the corresponding increase in mass would amount to only 0.07 percent then. An increase in mass compared to the electric charge for cathode rays of greater velocity is, in fact, included in **Lenard**'s observation ([2]). However, the difference that **Lenard** found is much too large to be explainable by electromagnetic inertia.

Meanwhile, those quantitative measurements are still not regarded as definitive.

If we restrict ourselves to small velocities then we will have the same expression for the energy of motion that mechanics exhibits for the *vis viva*. However, the magnitude of the acceleration cannot be derived from it with no further assumptions.

Acceleration assumes that the velocity varies. However, the expressions for the electromagnetic energy were derived only under the assumption that the value of the velocity was independent of time.

For variable velocity, the problem of a moving electric quantum has not been rigorously solved up to now.

Nonetheless, we can use **Maxwell**'s equations in order to arrive at a criterion for the magnitude of the error that we will introduce when employ the expression for energy for variable velocity, as well.

The electric and magnetic polarizations in our case when the motion proceeds in the *x*-direction are:

([1]) Cf., **P. Lenard**, Sitzungsber. d. kön. Akad. Wiss. Wien **108** (1899), pp. 1649.
([2]) **P. Lenard**, Wied. Ann. **64** (1898), pp. 287, and *loc. cit.*

$$X = \frac{\partial U}{\partial x}(1 - A^2 v^2), \quad Y = \frac{\partial U}{\partial y}, \qquad Z = \frac{\partial U}{\partial z},$$

$$M = -\ A v \frac{\partial U}{\partial z}, \qquad N = \ A v \frac{\partial U}{\partial y}, \quad L = 0,$$

$$U = \frac{e}{\sqrt{r^2 - A^2 v^2 \varsigma^2}}, \qquad \rho^2 \ = \ x^2 + y^2 .$$

The coordinate system is rigidly fixed in the moving point in that way.

Those expressions will satisfy **Maxwell**'s equations when one has:

$$\frac{d}{dt} = -\ v \frac{\partial}{\partial x},$$

and that will lead to the equation:

$$(1 - A^2 v^2) \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0 .$$

Should our value for x be true in general then we would also need to have that:

$$\frac{\partial X}{\partial t} \quad \text{is small compared to} \quad v \frac{\partial X}{\partial x} .$$

Now, we have:

$$\frac{\partial X}{\partial t} = \frac{\partial^2}{\partial x \, \partial t} U (1 - A^2 v^2) ,$$

so we must have that:

$$\frac{\partial}{\partial t}\left[\frac{\partial U}{\partial x}(1 - A^2 v^2)\right] \quad \text{is small compared to} \quad v \frac{\partial^2 U}{\partial x^2}(1 - A^2 v^2),$$

or that:

$$A^2 x \frac{\partial v}{\partial t} \qquad \text{is small compared to} \quad 1 - A^2 v^2 .$$

Likewise, the values of Y, Z and M, N imply that:

$$[2x^2 - (1 - A^2 v^2)\rho^2] A^2 \frac{\partial v}{\partial t} \quad \text{is small compared to} \quad 3x(1 - A^2 v^2),$$

and

$$(1 - A^2 v^2)\left[(x^2 + (1 - A^2 v^2)\rho^2\right] \frac{\partial v}{\partial t} - \left[2x^2 + (1 - A^2 v^2)\rho^2\right] A^2 v^2 \frac{\partial v}{\partial t}$$

must be small compared to $3x\,(1 - A^2\,v^2)\,v^2$.

That condition will be fulfilled when the dimensions of the space in which the energy essentially comes under consideration are sufficiently small. That is because the terms that are neglected include all of the linear dimensions to a higher power. However, $dv\,/\,dt$ cannot be too large, and the absolute velocity v cannot be too small.

When neglecting those terms is permissible, we can set the change in the energy of motion equal to:

$$\frac{d}{dt}\left(\tfrac{1}{2}\,m v^2\right) = m v\,\frac{dv}{dt} = K\,\frac{dr}{dt}\,dt = m\,\frac{dr}{dt}\,\frac{d^2 r}{dt^2}\,,$$

when K denotes the electric force. In that way, we have obtained **Newton**'s first and second laws of motion.

That is because when no external forces act, the law of inertia is simply the law of conservation of electromagnetic energy and **Newton**'s second law says that the work done by the force during dt is equal to the corresponding change in electromagnetic energy.

Newton's third law, which asserts the equality of action and reaction, is true for all electromagnetic forces between electric quanta. The mechanical forces must be identified with such forces from out standpoint. Since we make the assumption of an ether at rest, the law will not be valid for the general electromagnetic forces.

The parallelogram law of forces is included in our foundations insofar as it is true for electric polarizations and the forces that act between two electric quanta.

Finally, as far as the rigid constraints that can exist between several electric masses are concerned, they would not exist from our standpoint, strictly speaking. For example, when a pendulum swings, the force of gravity will stretch the string of the pendulum until the elastic forces that it produces are equally large. When no work is done, such forces are introduced in the well-known **Lagrangian** form.

One can refer to the foundation for mechanics that was sketched out here as being diametrically opposite to that of **Hertz**. The rigid constraints that belong **Hertz**'s assumption prove to be an effect of complicated isolated forces here. Likewise, the law of inertia is a relatively belated consequence of the electromagnetic assumptions. Whereas **Hertz**'s mechanics obviously has the goal of producing the electromagnetic equations as consequences, the relationship is precisely the opposite here. Naturally, in relation to its logical structure, a theory of mechanics that is based upon electromagnetism cannot measure up to that of **Hertz**, already because the system of **Maxwell**'s differential equations has still not found any precise critical revision, but it seems to me that it has the very significant advantage that it does, in fact, go beyond ordinary mechanics, which then relates to it as only a first approximation, as was shown. In that way, it is then possible to decide for or against it by experiments.

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