“Über eine allgemeine Form der Diracschen Gleichung,” Ann. Phys. (Leipzig) **7** (1930), 650-660.

On a general form of the Dirac equation

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A general theory of the wave-mechanical electron will be presented.

Historical overview

The equations of **Einstein**’s theory of teleparallelism are locally bein-invariant; i.e., they admit the group of proper orthogonal bein-transformations:

$$h'_{\alpha m} = \vartheta_{mr}\, h_{\alpha r}$$

with constant rotational coefficients ϑ_{mr} (1). Now, in the year 1916, **Einstein**’s theory of gravitation had a truly bein-invariant form; i.e., its equations admitted the group of bein-transformations above, but with position-dependent rotational coefficients ϑ_{mr} $(x^1, x^2, \ldots)$ (2). They will then be occasionally represented by the truly bein-invariant quantities:

$$g_{\alpha\beta} = h_{\alpha m}\, h_{\beta m}\ .$$

The **Dirac** wave equation is the relativistic generalization of **Schrödinger**’s (3). **W. Pauli** and **W. Heisenberg** (4) have developed **Dirac**’s method of second quantization upon only the basis of the **Dirac** and **Maxwell** equations. Now, **H. Weyl** (5) gave a general-relativistic two-component representation of the wave equation, but only the massless one, that is proper bein-invariant. He showed that the proper bein-transformations are equivalent to the continuous spin-transformations of the wave functions ψ. **H. Weyl** remained based in the previous theory of gravitation, but added the further requirement of gauge-invariance, which is linked with the re-gauging of ψ by:

$$\psi' = \psi\, e^{i\alpha},$$

(1) Cf., **R. Weitzenböck**, Berl. Ber. **26** (1928).
(2) Cf., **T. Levi-Civita**, *ibidem*, **9** (1929).
(3) Cf., **Dirac**’s new insights, as well, Proc. Roy. Soc. (A) **126** (1930), 360, in which the “negative” states play the important role precisely in the interaction between matter and radiation.
(4) **W. Pauli** and **W. Heisenberg**, Zeit. Phys. **56** (1929), 1; **59** (1930), 168.
(5) **H. Weyl**, Zeit. Phys. **56** (1929), 330.

in order give a basis for the existence of the electromagnetic four-potential f_a . **V. Fock** ([1]) adapted **Weyl**'s idea on the basis of the four-component **Dirac** theory. In recent times, that way of thinking made it possible for **L. Rosenfeld** ([2]) to apply the method of second quantization to the three known groups of phenomena: viz., gravitation ($h_{\alpha m}$), electromagnetism (f_α), and the matter field (ψ). Shortly after that, the author ([3]) showed the connection between the **Weyl-Fock** results and **Kaluza**'s notion of a **Riemannian** R_5 that is cylindrical in the fifth dimension. He introduced a cylindrical bein-lattice in **Kaluza**'s R_5 and considered the evolution of matter waves in it, when the five-dimensional wave-functions ω must depend upon the fifth coordinate simply-periodically, in any event. The theory that was developed there was valid only in the case of a single body. For the many-body problem, according to the method of second quantization, we must consider all field-variables to be non-commuting q-quantities, not commuting c-quantities. It is, in fact, possible to construct a more highly "quantized" theory, in which complete relativistically-symmetric commutation relations represent a continuous image of the **Heisenberg** uncertainty relations.

§ 1. – Let a **Riemannian** R_5 be given that is rigorously cylindrical with respect to x^0 ([4]). The last coordinate is only an auxiliary quantity, since it does not enter into the metric quantities, and only the wave function ω depends upon it in a simply-periodic way. Let a cylindrical grid of beins be embedded in this R_5 . If we denote the covariant bein-components by $h_{\alpha' m'}$ and the contravariant ones by $h^{\alpha'}_{m'}$ then we will have ([5]):

$$\left\{ \begin{array}{lll} h_{\alpha m} = h_{\alpha m}, & h_{\alpha 0} = -f_\alpha, & h_{00} = 1, \\ h^{\alpha}_{m} = h^{\alpha}_{m}, & h^{0}_{m} = \;\; f_m, & h^{0}_{0} = 1. \end{array} \right. \tag{1}$$

It follows that:

$$|\, h_{\alpha' m'} \,| = |\, h_{\alpha m} \,| = 1. \tag{2}$$

If we set:

$$h^{\rho'}_{m'} \frac{\partial}{\partial x^{\rho'}} = \frac{d}{ds_m} \tag{3}$$

then it will follow from (1) that:

([1]) **V. Fock**, Zeit. Phys. **57** (1929), 261; Comptes rendus **189** (1929), 25.

([2]) **L. Rosenfeld**, Ann. Phys. (Leipzig) [5] **5** (1929), 113.

([3]) **R. Zaycoff**, Zeit. Phys. **61** (1930), 395. However, that paper deviated from the present one not only in its notations, but also in its essential content.

([4]) We denote the fifth dimension by x^0. The coordinates x_1, x_2, x_3, x_0 are real, but x_4 is imaginary.

([5]) The primed (unprimed, resp.) symbols run through 1, 2, 3, 4, 0 (1, 2, 3, 4, resp.). In what follows, only the non-vanishing quantities will be given.

We can obtain the corresponding bein-components of a quantity from the usual coordinates by contracting over the Greek indices with h^{ρ}_{m}, $h_{\rho m}$, and conversely (by contracting over the Latin indices).

(3′) $$\frac{d}{ds_m} = h_m^\rho\left(\frac{\partial}{\partial x^\rho} + f_\rho\frac{\partial}{\partial x^0}\right), \qquad \frac{d}{ds_m} = \frac{\partial}{\partial x^0}.$$

For the components of the "torsion":

(4) $$\Delta_{k'\,l'\,m'} = h_{\rho'm'}\left(\frac{dh_{l'}^{\rho'}}{ds_{k'}} - \frac{dh_{k'}^{\rho'}}{ds_{l'}}\right),$$

we have ([1]):

(4′) $$\Delta_{k\,l\,m} = \Delta_{k\,l\,m}\,, \quad \Delta_{k\,l\,0} = f_{kl}\,.$$

For the quantities that are constructed from them:

(5) $$\left\{\begin{aligned} \Lambda_{k'} &= \Lambda_{k'r'r'},\\ \Pi_{k'l'm'} &= \tfrac{1}{2}\{\Lambda_{k'l'm'} + \Lambda_{k'm'l'} + \Lambda_{l'm'k'}\},\\ S_{k'l'm'} &= \Lambda_{k'l'm'} + \Lambda_{l'm'k'} + \Lambda_{k'm'l'}, \end{aligned}\right.$$

we will have:

(5′) $$\left\{\begin{aligned} &\Lambda_k = \Lambda_k\,,\\ &\Pi_{klm} = \Pi_{klm}\,, \quad \Pi_{0km} = \Pi_{k0m} = \tfrac{1}{2} f_{km}\,,\\ &S_{klm} = S_{klm}\,, \quad S_{kl0} = f_{kl}\,. \end{aligned}\right.$$

The curvature of the cylindrical **Riemannian** R_5 is further given by:

(6) $$\rho = 2\frac{d\Lambda_{r'}}{ds_{r'}} - \Lambda_{r'}\,\Lambda_{r'} - \tfrac{1}{2}\Pi_{k'\,l'\,r'}\,\Lambda_{k'\,l'\,r'},$$

or when written out:

(6′) $$\rho = R - \tfrac{1}{2}\,f_{km}\,f_{km}\,,$$

in which R means the curvature of the R_4 that is embedded in R_5 .

The following coordinate transformations:

(7) $$x^\alpha = x^\alpha(\bar{x}^1, \bar{x}^2, \bar{x}^3, \bar{x}^4)\,,$$

(7′) $$x^0 = \bar{x}^0 + \lambda(\bar{x}^1, \bar{x}^2, \bar{x}^3, \bar{x}^4)$$

are compatible with the choice (1) of bein-components. It follows from:

([1]) We have set:

$$f_{\alpha\beta} = \frac{\partial f_\beta}{\partial x^\alpha} - \frac{\partial f_\alpha}{\partial x^\beta}\,.$$

(8) $$\bar{h}_{\alpha' m'} = \frac{\partial x^{\rho'}}{\partial \bar{x}^{\alpha'}} \quad \text{or} \qquad h_{m'}^{\alpha'} = \frac{\partial \bar{x}^{\alpha'}}{\partial x^{\rho'}}\, h_{m'}^{\rho'},$$

in connection with (1) and (7), (7′):

(8′) $$\bar{h}_{\alpha m} = \frac{\partial x^{\rho}}{\partial \bar{x}^{\alpha}}\, h_{\rho m}, \qquad f_\alpha = \frac{\partial x^{\rho}}{\partial \bar{x}^{\alpha}} \left(f_\rho - \frac{\partial \lambda}{\partial x^{\rho}} \right).$$

Furthermore, with the choice (1) of bein-components, the following proper bein-transformations:

(9) $$\begin{cases} h'_{\alpha' m'} = \vartheta_{m' r'}\, h_{\alpha' r'} \quad \text{or} \quad h'^{\alpha'}_{m'} = \vartheta_{m' r'}\, h^{\alpha'}_{m'}, \\ \vartheta_{m' r'}\, \vartheta_{n' r'} = \vartheta_{r' m'}\, \vartheta_{r' n'} = \varepsilon_{m' n'} \end{cases}$$

will be compatible with the conditions for the rotational coefficients ([1]):

(9′) $$\begin{cases} \vartheta_{mr}\, \vartheta_{nr} = \vartheta_{rm}\, \vartheta_{rn} = \varepsilon_{mn}, \\ \vartheta_{m0} = \vartheta_{0m} = 0, \quad \vartheta_{00} = 1, \end{cases}$$

or, when written out ([2]):

(9″) $$h'_{\alpha m} = \vartheta_{m\,r}\, h_{\alpha r}, \qquad f'_\alpha = f_\alpha .$$

§ 2. – We now choose the quantities (which are four in number):

(10) $$\begin{cases} \omega = \psi(x^1, x^2, x^3, x^4)\, e^{i a x^0}, \\ \tilde{\omega} = \tilde{\psi}(x^1, x^2, x^3, x^4)\, e^{-i a x^0} \end{cases}$$

to be our wave-functions, in which:

(11) $$a = -\frac{\pi c}{h_0 \sqrt{k_0}} (\pm e_0).$$

The ψ-functions have the dimensions $[l^{-3/2}]$. In this, h_0 is **Planck**'s action constant and $\pm e_0$ is the elementary charge of the proton (electron, resp.).

We have written h_0, e_0 in order to avoid confusion with the quantities $|h_{\alpha m}| = h$ (the basis for the natural logarithm, resp.).

([1]) $\vartheta_{m\,r}$ are functions of x^1, x^2, x^3, x^4.

([2]) If we base things on a rational system of units then we would like to make the following physical identifications:

$g_{\alpha\beta} = 2 / c^2$: gravitational potential (indeed, the latter has the dimensions $[l^2\, t^{-2}]$.

$f_\alpha = (2 / c^2) \sqrt{k_0} \cdot \varphi_\alpha$, where φ_α means the "electromagnetic potential," c means the speed of light *in vacuo*, and k_0 means Newton's gravitational constant.

$\psi, \tilde{\psi}$ are invariant under the x^α-transformations (7). By contrast, $\psi, \tilde{\psi}$ experience the following transformations:

$$\bar{\psi} = \psi e^{i a \lambda}, \quad \bar{\tilde{\psi}} = \tilde{\psi}\, e^{-i a \lambda}, \tag{12}$$

under x^0-transformations (7′), such that it will follow from (7), (7′), (10), (12) that:

$$\begin{cases} \bar{\omega} = \omega, \\ \bar{\tilde{\omega}} = \tilde{\omega}; \end{cases} \tag{13}$$

i.e., $\omega, \tilde{\omega}$ behave like five-dimensional invariants.

$\omega, \tilde{\omega}$ experience the continuous spin transformations ([1]):

$$\omega' = P\,\omega, \qquad \tilde{\omega}' = \tilde{\omega} P^{+}, \qquad |\,P\,(s, t)\,| = 1 \tag{14}$$

under the proper bein-transformations (9). If the spin matrices are, say ([2]):

$$\begin{cases} \gamma_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \gamma_2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix}, \quad \gamma_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \\ \gamma_4 = i\,\varepsilon = \begin{bmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{bmatrix}, \quad \gamma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \end{cases}, \tag{15}$$

then $P\,(s, t)$ will have the following form:

([1]) $P^{+}\,(s, t) = \tilde{P}(s, t)$.

([2]) We can define the matrix:

$$\gamma = \begin{bmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{bmatrix}$$

from the relation:

$$\tfrac{1}{2}\,\gamma_1\,\gamma_2\,\gamma_3\,\gamma_4 = \gamma.$$

(16)
$$\left\{\quad P(s,t)=\begin{bmatrix}\alpha & -\tilde{\beta} & \gamma & \bar{\delta}\\ \beta & \tilde{\alpha} & \delta & -\tilde{\gamma}\\ \gamma & \bar{\delta} & \alpha & -\bar{\beta}\\ \delta & -\hat{\gamma} & \beta & \tilde{\alpha}\end{bmatrix},\right.$$

with the conditions:

(16′)
$$\left\{\begin{array}{l}\tilde{\alpha}\alpha+\tilde{\beta}\beta-\tilde{\gamma}\gamma-\tilde{\delta}\delta=1,\\ \tilde{\alpha}\gamma-\tilde{\gamma}\alpha+\tilde{\beta}\delta-\tilde{\delta}\beta=0,\end{array}\right.$$

and we can then represent the coefficients ϑ_{mr} are quadratic functions of the α, $\tilde{\alpha}$, β, $\tilde{\beta}$, γ, $\tilde{\gamma}$, δ, $\tilde{\delta}$. It follows from somecalculation that:

(17)
$$P^{+}\gamma_{m'}P=\vartheta_{m'r'}\gamma_{r'},$$

or when written out:

(17′)
$$P^{+}\gamma_{m}P=\vartheta_{mr}\gamma_{r}, \qquad P^{+}\gamma_{0}P=\gamma_{0}.$$

One has, in addition:

(17″)
$$P^{+}\gamma P=\gamma.$$

With that, the four-vector:

$$J_m=\tilde{\psi}\gamma_m\psi$$

will transform as follows:

(18)
$$J'_m=\vartheta_{mr}J_r$$

under the proper bein-transformations, and the quantities:

$$J_0=\tilde{\psi}\gamma_0\psi, \quad J=\tilde{\psi}\gamma\psi$$

will be proper bein-invariants. One will have:

(19)
$$J_m J_m=-J_0^2-J^2$$

identically.

The identities also follow:

(20)
$$\left\{\begin{array}{c}\gamma_{m'}^{+}\gamma_{n'}+\gamma_{n'}^{+}\gamma_{m'}\equiv\gamma_{m'}\gamma_{n'}^{+}+\gamma_{n'}\gamma_{m'}^{+}\equiv 2\varepsilon_{m'n'}\cdot\varepsilon,\\ \gamma_{m'}^{+}\gamma+\gamma\gamma_{m'}\equiv\gamma_{m'}\gamma+\gamma\gamma_{m'}^{+}\equiv 0, \quad \gamma^2\equiv\varepsilon.\end{array}\right.$$

In addition to the proper bein-invariant vector J_m and the proper bein-invariant J_0, J, we also have the following proper bein-invariant tensors:

$$J_{klm} = i\tilde{\psi}\,\gamma_k\,\gamma_l^+\,\gamma_m\,\psi \qquad (k \neq l \neq m),$$

$$J_{kl} = i\tilde{\psi}\,\gamma_k\,\gamma_l^+\,\gamma_0\,\psi \qquad (k \neq l),$$

which are antisymmetric in all indices.

§ 3. – We define the components of the Riemann derivatives of the quantities $J_{m'}$ in the bein-directions:

(21) $$D_{l'}\,J_{m'} = \frac{dJ_{m'}}{ds_{l'}} - \Pi_{k'l'm'}\;J_{k'},$$

and it will then follow with some calculation that:

(22) $$\begin{cases} D_{l'}\,\omega = \dfrac{d\omega}{ds_{l'}} + \frac{1}{4}\Pi_{k'l'm'}\,\gamma_{k'}^+\,\gamma_{r'}\,\omega, \\[2ex] D_{l'}\,\tilde{\omega} = \dfrac{d\tilde{\omega}}{ds_{l'}} - \frac{1}{4}\tilde{\omega}\,\Pi_{k'l'm'}\,\gamma_{k'}\,\gamma_{r'}^+ . \end{cases}$$

We define the divergence $D_{l'}J_{l'}$ from (22) ([1]):

(23) $$D_{l'}\,J_{l'} = \frac{d\bar{J}_{l'}}{ds_{l'}} - \Lambda_{l'}\,J_{l'} = \delta_\rho\,J^\rho.$$

We set ([2]):

(24) $$M = -\,\{i\,\tilde{\omega}\,\gamma_{l'}\,D_{l'}\,\omega + b\,J_0\}.$$

It follows from:

$$J_{l'} = \tilde{\omega}\,\gamma_{l'}\,\omega$$

and (23), (24) that:

(25) $$M - \tilde{M} = -\,i\,\delta_\rho\,J^\rho.$$

([1]) δ_ρ means the **Riemann** derivative with respect to x^ρ.

([2]) The constant b is equal to $b = a - \mu$, where a is determined from (11) and:

$$\mu = \frac{2\pi\,m_0\,c}{h_0}.$$

m_0 is the rest mass of the particle.

The functions ρ and M are not only ordinary invariants, but they are also gauge-invariant and proper bein-invariant.

Some calculation will initially yield:

$$M = -\, i\,\tilde{\omega}\,\gamma_{l'}\frac{d\omega}{ds_{l'}} + \frac{i}{2}\Delta_{r'}\,\tilde{\omega}\,\gamma_{r'}\,\omega + \frac{i}{24}S_{k'l'r'}\,\tilde{\omega}\,\gamma_{k'}\,\gamma_{l'}^{+}\,\gamma_{r'}\,\omega - b\,\tilde{\omega}\,\gamma_0\,\omega \tag{24′}$$

and further:

$$M = -\, i\,\tilde{\psi}\,\gamma^{\rho}\frac{d\psi}{dx^{\rho}} + \frac{i}{2}\Delta_m\,J_m + \frac{1}{24}S_{klr}\,J_{klm} - a\,f_m\,J_m + \frac{1}{8}f_{km}\,J_{km} + \mu\,J_0\,. \tag{24″}$$

Finally, let the following auxiliary formulas be given:

$$\left\{\begin{array}{ll} \delta_{\alpha}\,h_{\beta m} = \Pi_{\beta\alpha m}, & \delta_{\rho}\,h_m^{\rho} = -\Lambda_m, \\ \delta_{\alpha}\psi = \dfrac{\partial\psi}{\partial x^{\alpha}}, & \delta_{\alpha}\tilde{\psi} = \dfrac{\partial\tilde{\psi}}{\partial x^{\alpha}}, \\ \Pi_{klm} = -\Pi_{mlk}. & \end{array}\right. \tag{26}$$

§ 4. – We now choose the density:

$$H = k\,M\,h \tag{27}$$

to be the Lagrange function, where:

$$k = -\frac{h_0\,c}{2\pi}\,. \tag{28}$$

The variation of ψ and $\tilde{\psi}$ yields the wave equations (1):

$$\left\{\begin{aligned} \frac{1}{k}X &= i\left(\frac{\partial}{\partial x^{\rho}} - i\,a\,f_{\rho}\right)\tilde{\psi}\,\gamma^{\rho} - \frac{i}{2}\Lambda_m\,\tilde{\psi}\,\gamma_m \\ &\quad + \frac{i}{24}S_{klm}\,\tilde{\psi}\,\gamma_k\,\gamma_l^{+}\,\gamma_m + \frac{i}{8}f_{km}\,\tilde{\psi}\,\gamma_k\,\gamma_m^{+}\,\gamma_0 + \mu\,\tilde{\psi}\,\gamma_0 = 0, \end{aligned}\right. \tag{29′}$$

$$\left\{\begin{aligned} \frac{1}{k}\tilde{X} &= -i\,\gamma^{\rho}\left(\frac{\partial}{\partial x^{\rho}} + i\,a\,f_{\rho}\right)\psi + \frac{i}{2}\Lambda_m\,\gamma_m\,\psi \\ &\quad + \frac{i}{24}S_{klm}\,\gamma_k\,\gamma_l^{+}\,\gamma_m\,\psi + \frac{i}{8}f_{km}\,\gamma_k\,\gamma_m^{+}\,\gamma_0\,\psi + \mu\,\gamma_0\,\psi = 0. \end{aligned}\right. \tag{29″}$$

(1) $\delta\psi$ and $\delta\tilde{\psi}$ vanish on the boundary of the domain of integration.

Under an infinitesimal shift in the space-time continuum ([1]):

$$\delta x^a = \xi^a,$$

we will have:

$$(30)\qquad \begin{cases} \delta h_{\alpha m} = -h_{\rho m}\{\delta_a \xi^\rho + \Pi_{\alpha\kappa}^{\cdot\cdot\rho}\xi^\kappa\}, \\ \delta f_\alpha = f_{\alpha\kappa}\xi^\kappa - \dfrac{\partial}{\partial x^a}(f_\kappa \xi^\kappa), \\ \delta\psi = -\dfrac{\partial\psi}{\partial x^\kappa}\xi^\kappa, \quad \delta\tilde{\psi} = -\dfrac{\partial\tilde{\psi}}{\partial x^\kappa}\xi^\kappa. \end{cases}$$

By contrast, under an infinitesimal gauge-transformation:

$$(31)\qquad \begin{cases} \delta h_{\alpha m} = 0, \qquad \delta f_a = \dfrac{\partial\lambda}{\partial x^a}, \\ \delta\psi = -i\,a\lambda\,\psi, \quad \delta\tilde{\psi} = i\,a\,\lambda\tilde{\psi}, \end{cases}$$

Finally, under an infinitesimal proper bein-transformation ([2]):

$$(32)\qquad \begin{cases} \delta h_{\alpha m} = -\omega_{mr}\,h_{\alpha r}, \quad \omega_{mr} = -\omega_{rm}, \\ \delta f_\alpha = 0, \quad \delta\psi = -p\,\psi, \quad \delta\tilde{\psi} = -\tilde{\psi}\,p^+, \\ \text{where} \\ p = \tfrac{1}{4}\omega_{kr}\,\gamma_k^+\,\gamma_r, \quad p^+ = -\tfrac{1}{4}\omega_{kr}\,\gamma_k\,\gamma_r^+. \end{cases}$$

Since we have:

$$(33)\qquad \int\left\{X_{,m}^{\rho}\,\delta h_{\rho m} + X^\rho\,\delta f_\rho + X\,\delta\psi + \delta\tilde{\psi}\,\tilde{X}\right\}h\,dx \equiv 0,$$

regardless of whether we substitute the variations (30), (31), or (32) in (33), the identities will follow:

$$(34)\qquad \delta_\rho X_{,\alpha}^{\rho} - X^{\rho\kappa}\Pi_{\rho\alpha\kappa} - f_{\alpha\rho}X^\rho + f_\alpha\,\delta_\rho X^\rho - X\,\frac{\partial\psi}{\partial x^\alpha} - \frac{\partial\tilde{\psi}}{\partial x^\alpha}\tilde{X} \equiv 0,$$

$$(35)\qquad \delta_\rho X^\rho + i\,a\,\{X\,\psi - \tilde{\psi}\,\tilde{X}\} \equiv 0,$$

$$(36)\qquad X_{\alpha\beta} - X_{\beta\alpha} - \tfrac{1}{4}\left\{X(\gamma_\alpha^+\gamma_\beta - \gamma_\beta^+\gamma_\alpha)\psi - \tilde{\psi}(\gamma_\alpha\gamma_\beta^+ - \gamma_\beta\gamma_\alpha^+)\tilde{X}\right\} \equiv 0.$$

([1]) ξ^a are the components of the infinitesimal shift vector here.

([2]) The following identity relations are fulfilled:

$$p^+\gamma_m + \gamma_m\,p = \omega_{mr}\,\gamma_r,$$
$$p^+\gamma_0 + \gamma_0\,p = 0,$$
$$p^+\gamma + \gamma p = 0.$$

One has:

$$(37)\quad \left\{\begin{aligned} X_{\alpha\beta} = -k\cdot\Big[&-\frac{i}{2}\Big\{\tilde{\psi}\,\gamma_m\Big(\frac{\partial}{\partial x^\rho}+ia\,f_\rho\Big)\psi-\Big(\frac{\partial}{\partial x^\rho}-ia\,f_\rho\Big)\tilde{\psi}\,\gamma_m\,\psi\Big\}\cdot\{h_{\alpha m}\varepsilon_\beta^{\cdot\rho}-h_m^\rho\,g_{\alpha\beta}\} \\ &+\tfrac{1}{4}\Big\{\delta_\rho J_{\alpha\cdot\beta}^{\;\rho}+J_\alpha^{\cdot\rho\mu}\left(\Lambda_{\beta\rho\mu}-\tfrac{1}{2}\Lambda_{\rho\mu\beta}\right)-\tfrac{1}{6}\,g_{\alpha\beta}J^{\kappa\rho\mu}S_{\kappa\rho\mu}\Big\} \\ &+\tfrac{1}{4}\Big\{J_\alpha^{\cdot\kappa}\,f_{\beta\kappa}-\tfrac{1}{2}\,g_{\alpha\beta}\,J^{\kappa\rho}\,f_{\kappa\rho}\Big\}-\mu g_{\alpha\beta}J_0\Big] \end{aligned}\right.$$

and

$$(38)\qquad X^\alpha = k\cdot[a\,J^\alpha+\delta_\rho\,J^{\alpha\rho}].$$

§ 5. – From (29′) and (29″), one has:

$$(29)\qquad X = 0, \qquad \tilde{X} = 0.$$

It first follows from (29) and (36) that:

$$(39)\qquad X_{\alpha\beta} = X_{\beta\alpha}\,,$$

such that the 16 components $X_{\alpha\beta}$ will be reduced to only ten components on the basis of equations (29).

It will then follow from (29) and (35) that:

$$(40)\qquad \delta_\rho X^\rho = 0,$$

and if one recalls (38) then that equation will imply that:

$$(40')\qquad \delta_\rho\,J^{\,\rho} = 0.$$

Since we know that:

$$(41)\qquad X\,y+\tilde{\psi}\,\tilde{X}\;\equiv k\cdot[2M+i\,\delta_\rho\,J^{\,\rho}],$$

it will follow from (29), (40), (41) that:

$$(42)\qquad M = 0.$$

Finally, it will follow from (26), (29), (34), (39), (40) that:

$$(43)\qquad \delta_\rho X_\alpha^{\cdot\rho}-f_{\alpha\rho}\,X^{\,\rho} = 0.$$

Summary

We have derived the following laws of matter by means of nothing but identity relations on the basis of the general **Dirac** equation: The symmetry of the energy-stress tensor $X_{\alpha\beta}$ [equation (39)]. The conservation of the current vector X^{α} [equation (40) or (40′)]. The equations of motion (43).

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