"Über die Einsteinsche Theorie des Fernparallelismus," Zeit. Phys. 66 (1930), 572-576.

## **On Einstein's theory of teleparallelism**

By Raschko Zaycoff in Sofia

Translated by D. H. Delphenich

(Received on 24 October 1930)

A new modification of the theory of teleparallelism can be presented in a symmetric form. It will be shown that this theory is not a direct generalization of the 1916 theory of gravitation, but rather its physical content seems to be connected with recent concepts. All notations are adopted from the author's previous treatises (\*).

**Historical overview.** – Supported by the theory of orthogonal line congruences in a Riemannian  $R_n$  that Ricci and Levi-Civita founded, A. Einstein constructed the theory of teleparallelism, which rests upon the possibility of a parallel orientation of the local bein-system in **Riemannian**  $R_n$ . Not only the metric in  $R_n$ , but also the bein-lattice that is embedded in it, are described by the bein-components  $h_m^{\alpha}$  when one subjects the latter components to certain equations that either emerge from a variational principle (\*\*) or not (\*\*\*), whereby identities in the sense of general covariance should exist between the aforementioned equation in the last case. Whereas in the former case the difficulties were due to the fact that we are free to choose the Lagrangian function, in the other case they were based upon the mutual incompatibility of the equations that were postulated in some way. However, a more precise consideration shows that the path that is followed is not completely free of objections from a geometric and group-theoretic standpoint (\*\*\*\*), because when the transformation group leaves the metric quadratic form unchanged, one can introduce only a Euclidian connection in the **Riemannian**  $R_n$  when the parallel displacement obeys certain conservation laws for the curvature and torsion and when the vector  $\Lambda_{\mu} = \Lambda_{\mu\rho}^{\cdots\rho}$ , which one had previously endowed with an electromagnetic meaning, represents the gradient of a scalar function. Now, **A. Einstein** (<sup>†</sup>) had presented equations between the  $h_m^{\alpha}$  in the sense of that viewpoint that did not emerge from a Lagrange function, but were still compatible with each other, while the requirement of general covariance was likewise fulfilled.

<sup>(\*)</sup> Zeit. Phys. **53** (1929), pp. 719; **54**, pp. 590, 738; **56**, pp. 717, 862; **58**, pp. 143, 280.

<sup>(\*\*)</sup> **A. Einstein**, Berl. Ber. (1928), Heft 17/18; (1929), Heft 10.

<sup>(\*\*\*)</sup> **A. Einstein**, *ibidem*, (1929), Heft 1; **T. Levi-Civita**, *ibidem* (1929), Heft 9.

<sup>(\*\*\*\*)</sup> Cf., **E. Cartan**, Math. Ann. **102** (1930), pp. 698.

<sup>(&</sup>lt;sup>†</sup>) **A. Einstein**, *ibidem*, **102** (1930), pp. 685.

## Notations:

1. The symbols  $\nabla_{\mu}$ ,  $\delta_{\mu}$  denote the **Einstein** (**Riemannian**, resp.) derivatives with respect to  $x^{\mu}$ .

2. The quantity  $*S_{\alpha}$  represents the dual quantity to  $S_{\alpha\beta\gamma} = \Lambda_{\alpha\beta\gamma} + \Lambda_{\beta\gamma\alpha} + \Lambda_{\gamma\alpha\beta}$ .

3. Let  $A_{\alpha\beta}$  be an arbitrary antisymmetric quantity.  $*A_{\alpha\beta}$  will then be its dual quantity.

4. If  $A_{\alpha}$  represents any vector then we would like to denote the quantity  $\frac{\partial A_{\beta}}{\partial x^{\alpha}} - \frac{\partial A_{\alpha}}{\partial x^{\beta}}$  by  $A_{\alpha\beta}$ .

5. If A is an arbitrary quantity then A' will be the corresponding transformed quantity.  $\overline{A}$ ,  $\overline{\overline{A}}$ , etc., mean the first (second, etc., resp.) approximations to the quantity A.

**§ 1.** – We would like to give the aforementioned modification of the theory of teleparallelism a symmetric form, in which we might fix the dimension of the universe to be four from the outset. In that way, we will be content to give only final formulas.

We set:

$$\begin{aligned} G_{\alpha\beta} &\equiv \nabla_{\mu} \Lambda_{\alpha \cdot \beta}^{\ \mu} + \Lambda_{\alpha}^{\ \epsilon \rho \rho} \Lambda_{\kappa \rho \beta}, \\ L_{\alpha} &\equiv \Lambda_{\alpha} - \frac{\partial \psi}{\partial x^{\alpha}}, \\ M_{\alpha} &\equiv *S_{\alpha} \cdot e^{\psi} - \frac{\partial \omega}{\partial x^{\alpha}}. \end{aligned}$$

$$(1)$$

It follows from this that:

$$\nabla_{\mu}G^{\alpha\mu} + \nabla_{\mu}L^{\alpha\mu} + \Lambda^{\alpha\kappa\rho} L_{\kappa\rho} \equiv 0, \qquad \{G_{\alpha\beta} - G_{\beta\alpha} + L_{\alpha\beta} + S_{\alpha\beta\mu} L^{\mu}\}e^{\psi} + *M_{\alpha\beta} \equiv 0, \qquad \}$$

$$(2)$$

which collectively represent ten identities between the 24 quantities  $G_{\alpha\beta}$ ,  $L_{\alpha}$ ,  $M_{\alpha}$ . It further follows from (1) that:

$$L_{\alpha\beta} \equiv -\nabla_{\mu} \Lambda^{\dots \mu}_{\alpha\beta}, \qquad (3)$$

$$G_{\alpha\beta} + G_{\beta\alpha} - \delta_{\alpha} L_{\beta} - \delta_{\beta} L_{\alpha} + 2R_{\alpha\beta} - 2\delta_{\alpha} \left(\frac{\partial \psi}{\partial x^{\beta}}\right) + \{\Lambda_{\alpha\mu\kappa} (\Lambda_{\beta}^{\mu\kappa} + \Lambda_{\beta}^{\kappa\mu}) - \Lambda_{\alpha}^{\mu\kappa} \Lambda_{\mu\kappa\beta} - \Lambda_{\beta}^{\mu\kappa} \Lambda_{\mu\kappa\alpha} - \frac{1}{2}\Lambda_{\mu\kappa\alpha} \Lambda_{\mu\kappa\beta}^{\mu\kappa}\} \equiv 0, \qquad (4)$$

$$G^{\mu}_{\mu} + \delta_{\mu} L^{\mu} + L^{\mu} \left\{ L_{\mu} + 2 \frac{\partial \psi}{\partial x^{\mu}} \right\} + \Box \psi + \frac{\partial \psi}{\partial x^{\kappa}} \frac{\partial \psi}{\partial x^{\rho}} g^{\kappa\rho} + \Lambda_{\mu\kappa\rho} \Lambda^{\mu\rho\kappa} \equiv 0 .$$
 (4')

We now set:

$$G_{\alpha\beta} = 0, \qquad L_{\alpha} = 0, \qquad M_{\alpha} = 0, \tag{5}$$

which serve as 24 equations for determining the 18 quantities  $h_m^{\alpha}$ ,  $\psi$ ,  $\omega$ , and the ten identities (2) exist between them.

In the first approximation, we have:

$$h_{ab} = \varepsilon_{ab} + \bar{k}_{ab} \,, \tag{6}$$

and we set (\*):

$$\overline{k}_{ab} + \overline{k}_{ba} = \overline{g}_{ab}, \qquad \overline{k}_{ab} - \overline{k}_{ba} = \overline{f}_{ab}, \qquad (7)$$

in addition. Equations (5) will then read  $(^{**})$ :

$$\frac{\partial}{\partial x_m} \left( \frac{\partial \bar{k}_{ab}}{\partial x_m} - \frac{\partial \bar{k}_{mb}}{\partial x_a} \right) = 0 , \qquad (8)$$

$$\frac{\partial \bar{k}_{am}}{\partial x_m} - \frac{\partial \bar{\kappa}}{\partial x_a} = 0 , \qquad (8')$$

$$\frac{\partial^* \bar{f}_{am}}{\partial x_m} - \frac{\partial \bar{\omega}}{\partial x_a} = 0.$$
(8")

After performing an infinitesimal coordinate transformation (without a simultaneous rotation of the bein-lattice):

$$x_a - x'_a = \overline{\xi}_a$$
,

we would like to determine the  $\overline{\xi}_a$  from the five equations:

$$\frac{\partial \bar{k}'_{ma}}{\partial x_m} = 0, \qquad \bar{\kappa} = 0, \qquad (9)$$

between which, however, an identity exists, namely (8').

(<sup>\*\*</sup>) One sets:

$$\overline{\kappa} = \overline{k}_{mm} + \overline{\psi}$$
.

<sup>(\*)</sup> If the 1916 theory of gravitation, the gravitational potentials  $g_{\alpha\beta}$ , as with all components of the electromagnetic field tensor  $f_{\alpha\beta}$ , are tensor components in the sense of general relativity, while here they are defined only as tensor components [according to (7), in the first approximation] in the sense of special relativity.

It follows from (8), (8'), (9) that:

$$\frac{\partial \bar{k}'_{am}}{\partial x_m} = 0, \qquad \frac{\partial^2 \bar{k}'_{ma}}{\partial x_m^2} = 0, \tag{10}$$

and if we ultimately set:

$$\overline{\omega} = \text{const.}$$
 (11)

then it will follow from (8'') that:

$$\overline{f}_{ab} = \frac{\partial \overline{f}_b}{\partial x_a} - \frac{\partial \overline{f}_a}{\partial x_b}.$$
(12)

§ 2. – We form the following equations by means of (4) and (5):

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + T_{\alpha\beta} = 0.$$
<sup>(13)</sup>

In the 1916 theory of gravitation,  $T_{\alpha\beta}$  played the role of the energy-stress tensor. Here, it does not, because in the first approximation, it is:

$$\overline{T}_{\alpha\beta} = \frac{1}{2} \frac{\partial^2 \overline{g}_{mm}}{\partial x_{\alpha} \partial x_{\beta}} \neq 0, \qquad (14)$$

while at the same time:

$$\overline{R} = 0, \qquad \overline{T}_{mm} = 0. \tag{15}$$

There, the condition equations read:

$$\frac{\partial \overline{g}_{\alpha m}}{\partial x_m} - \frac{1}{2} \frac{\partial \overline{g}_{mm}}{\partial x_\alpha} = 0, \qquad (16)$$

in the first approximation, but here:

$$\frac{\partial \overline{g}_{\alpha m}}{\partial x_m} = 0.$$
(17)

On that basis, the aforementioned theory of gravitation does not represent a special case of the present theory. However, one can look for solutions in the second approximation using the method of step-wise approximation. To that end, we set:

$$h_{ab} = \varepsilon_{ab} + \overline{k}_{ab} + \overline{\overline{k}}_{ab} , \qquad (18)$$

from which, it will also follow for contravariant bein-components  $h_m^{\alpha}$  that:

$$h_{ab} = \varepsilon_{ab} - \bar{k}_{ba} - \bar{\bar{k}}_{ba} + \bar{k}_{bm} \bar{k}_{ma} \quad . \tag{18'}$$

If we substitute that in (5) and consider (8), (8'), (8") then 24 equations that couple the  $\overline{k}_{ab}$  with the  $\overline{k}_{ab}$  will follow. We now have:

$$x_a - \overline{x}_a = \overline{\xi}_a + \overline{\overline{\xi}}_a$$
,

and when we observe the defining equations (9) for the  $\overline{\xi}_a$ , we can exhibit four new mutuallyindependent condition equations from which the  $\overline{\xi}_a$  can be calculated. The 24 equations in the second approximation, thus-transformed, will then yield the corresponding field laws for that approximation, and so on for the higher approximations. However, that method can be followed through consistently only when:

1. The deviations of the  $h_{am}$  from their Euclidian values  $\varepsilon_{ab}$  are very small when they are based upon a Cartesian coordinate system.

2. One allows only infinitesimal coordinate transformations from the outset, such that only then can one have any right to consider the field laws on the basis of special relativity.

It is now clear that no general validity can be ascribed to this special case and that, in fact, the arbitrary fields and coordinate systems must also once more preserve the deeper meaning that they obtained in the 1916 gravitational theory. However, if we are to stand on the ground of general relativity then in order for us to assign an unambiguous physical meaning to equations (5), we must move to more novel conceptual pictures. The laws of electromagnetism and gravity are contained in (5) in a highly simplistic form and can no longer be separated from each other. In particular, the exalted role of the gravitational equations (13) in the theory of 1916 will be lost completely, and its relationship to the laws of motion of a particle will be meaningless. Now, the geodetic principle has no rigorous validity in wave mechanics, so we can hope that the solutions of the wave-mechanical equations will also be given by the rigorous solutions to the unified field equations (5), if only implicitly. However, those are ultimately only suggestions, and only a rigorous derivation could have any power to convince.

"Über die Einsteinsche Theorie des Fernparallelismus," Zeit. Phys. 67 (1931), 135-137.

## **On Einstein's theory of teleparallelism**

Second and final notice (\*).

By Raschko Zaycoff in Sofia

Translated by D. H. Delphenich

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An extraordinary result of the equations of Einstein's theory of teleparallelism is proved that contradicts some recent assertions of **A. Einstein** (\*\*).

**§ 1.** – The first identity in [I, (1)], namely:

$$G_{\alpha\beta} \equiv \nabla_{\mu} \Lambda^{\ \mu}_{\alpha\cdot\beta} + \Lambda^{\ \cdot\,\kappa\rho}_{\alpha} \Lambda_{\kappa\rho\beta}, \qquad (1)$$

and the identity [I, (3)]:

$$L_{\alpha\beta} \equiv -\nabla_{\mu} \Lambda_{\alpha\beta}^{\dots\mu}, \qquad (2)$$

play an important role in (A) because the vanishing of their left-hand sides will lead to "overdetermined" field equations for the  $h_m^{\alpha}$  (\*\*\*).

In addition to the first identity in [I, (2)]:

$$\nabla_{\mu}G^{\alpha\mu} + \nabla_{\mu}L^{\alpha\mu} + \Lambda^{\alpha\kappa\rho} L_{\kappa\rho} \equiv 0 , \qquad (3)$$

one also finds the identity:

$$\frac{\partial L_{\beta\gamma}}{\partial x^{\alpha}} + \frac{\partial L_{\gamma\alpha}}{\partial x^{\beta}} + \frac{\partial L_{\alpha\beta}}{\partial x^{\gamma}} \equiv 0 , \qquad (4)$$

in (A), which is plausible, since we have:

$$L_{\alpha\beta} \equiv \frac{\partial \Lambda_{\beta}}{\partial x^{\alpha}} - \frac{\partial \Lambda_{\alpha}}{\partial x^{\beta}} .$$
(5)

<sup>(\*)</sup> Cf., **R. Zaycoff**, Zeit. Phys. **66** (1930), pp. 572. Cited as I in what follows.

<sup>(\*\*)</sup> **A. Einstein**, Berliner Ber. (1930), Heft 1. Cited as (A) in what follows.

<sup>(\*\*\*)</sup> In the notation of **A. Einstein**, one has  $F_{\alpha\beta} = -L_{\alpha\beta}$ .

Now, it follows from (1), in conjunction with the identities:

$$\nabla_{\mu}\nabla_{\nu}\Lambda^{\mu\nu\alpha} - \frac{1}{2}\Lambda^{\dots\rho}_{\mu\nu}\nabla_{\rho}\Lambda^{\mu\nu\alpha} \equiv 0, \qquad (6)$$

$$\nabla_{\mu}\Lambda^{\dots\alpha}_{\kappa\rho} + \nabla_{\kappa}\Lambda^{\dots\alpha}_{\rho\mu} + \Lambda^{\dots\sigma}_{\mu\kappa}\Lambda^{\dots\alpha}_{\rho\sigma} + \Lambda^{\dots\sigma}_{\kappa\rho}\Lambda^{\dots\alpha}_{\mu\sigma} + \Lambda^{\dots\sigma}_{\rho\mu}\Lambda^{\dots\alpha}_{\kappa\sigma} \equiv 0, \qquad (7)$$

that one has the following identity:

$$\nabla_{\mu}G^{\mu\alpha} + \Lambda^{\dots\alpha}_{\kappa\rho} \left(G^{\kappa\rho} + \frac{1}{2}\Lambda^{\mu\alpha\kappa}\Lambda^{\dots\rho}_{\mu\sigma}\right) \equiv 0.$$
(8)

However, in (A) one finds:

$$\nabla_{\mu}G^{\mu\alpha} + \Lambda^{\dots\alpha}_{\kappa\rho} G^{\kappa\rho} \equiv 0 , \qquad (8)$$

which is an incorrect identity that **A. Einstein** added to the identities (3), (4). However, if we consider the identity (8) instead then it would also follow from the first system of equations [I, (5)]:

$$G_{\alpha\beta} = 0 , \qquad (9)$$

that:

$$\Lambda_{\kappa\rho}^{\,\,\prime\prime\sigma}\,\Lambda^{\mu\sigma\kappa}\Lambda_{\mu\sigma}^{\,\,\prime\prime\rho}=0\,,\tag{10}$$

which do not, in fact, represent four new determining equations for the  $h_m^{\alpha}$  (\*), since (10) can, of course, inevitably exist, on the basis of equations (9) and the identities (8), so it must be considered to be a special result that makes the validity of equations (9) very questionable. **Einstein**'s "principle of over-determinacy" does not seem to be compatible with equations (9), accordingly. However, if equations (9) are not fulfilled then, from (3), the equations:

$$L_{\alpha\beta} = 0 \tag{11}$$

would not be appropriate, so from [I, (1)] and [I, (2), neither would the equations:

$$L_{\alpha} = 0, \qquad M_{\alpha} = 0. \tag{12}$$

§ 2. – It then seems that the aforementioned principle is inconsistent with the choice [I, (5)] of field equations, and if we are to avoid results like (10) then we must look for another formulation of the field laws than the one that was given by formulas [I, (1)] and [I, (5)]. I have addressed the problem of finding field equations such that:

a) the identities that exist between them do not require any relations that are similar to (10).

<sup>(\*)</sup> The relations (10) play no role in the first and second approximations, since they represent relations between quantities whose order of magnitude is three.

b) the Euclidian connection that was discussed at the beginning of I can be introduced,

but I have not arrived at any satisfactory results.

**A. Einstein** and **W. Mayer** (\*) have given rigorous solutions of the field equations in the static case (central symmetry, mirror symmetry) that can serve as support for the possibility of defining field equations that simultaneously describe gravity and electricity. Solutions of that kind are incompatible with the law of motion for electrons, which are missing completely from the theory of teleparallelism. However, we know, on the other hand, that such a law must exist, at least in the macroscopic approximation, but it is not clear how the latter can be replaced with quasi-static solutions of the field equations. Apart from that question, we would like to exhibit another aspect of **Einstein**'s field equations. Those equations are invariant under, *and only under* (\*\*), properorthogonal bein-transformations:

$$h'_{\alpha m} = \mathcal{G}_{mr} h_{\alpha r} \tag{13}$$

with constant rotational coefficients  $\mathcal{G}_{mr}$ . That is because a change in the orientation of the beinlattice, which is regarded as rigid, cannot influence material reality, on symmetry grounds. However, the hypothesis that the bein-lattice constitutes a rigid structure in the world is not at all inevitable, and we can perhaps demand of the field equations that they should be invariant under transformations of the type (13), but now the rotational coefficients  $\mathcal{G}_{mr}$  in them must represent functions of the coordinates  $x^1$ ,  $x^2$ ,  $x^3$ ,  $x^4$ , ... The bein-lattice will no longer be rigid then, but it can be twisted arbitrarily at each lattice point. Such a formulation of the field laws would mean the end of the theory of teleparallelism, at least in the realm of its physical applications. The fact that this formulation can be implemented was shown already by **H. Weyl**, **V. Fock**, the author, and **L. Rosenfeld**. In a paper that appeared recently (\*\*\*), the author has discussed that new aspect of the unification problem, and in so doing, connected it with **Dirac**'s theory of the electron. It was shown that **Kaluza**'s picture of a five-dimensional cylindrical world is better suited to not just field theory, but also for relativistic wave mechanics, than anyone has imagined up to now.

Although the 1916 theory of gravitation has proved to be very viable, it seems that the theory of teleparallelism has a largely heuristic value, unless its foundations are altered completely (\*\*\*\*).

<sup>(\*)</sup> **A. Einstein** and **W. Mayer**, Berliner Ber. (1930), Heft 6.

<sup>(\*\*)</sup> If one ignores invariance under arbitrary coordinate systems (viz., general covariance!).

<sup>&</sup>lt;sup>\*\*</sup>) **R. Zaycoff**, Ann. Phys. (Leipzig) (5) **7** (1930), 650.

<sup>(\*\*\*\*)</sup> Such as, perhaps, the affine field theories of **H. Weyl**, **A. S. Eddington**, **A. Einstein**, *et al.* 

"Bemerkungen und Zusätze zu meiner Arbeit 'Über die Einsteinsche Theorie des Fernparallelismus'," Zeit. Phys. **69** (1931), 428-430.

## **Remarks and additions to my paper:** "On Einstein's theory of teleparallelism" (\*)

By Raschko Zaycoff in Sofia

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**§ 1.** – The relations [II, (10)]:

$$\Lambda^{\dots\alpha}_{\kappa\rho}\,\Lambda^{\mu\sigma\kappa}\Lambda^{\dots\rho}_{\mu\sigma}=0\;,$$

which I considered to be an "extraordinary result" of **Einstein**'s unified field equations, are fulfilled identically, because since the tensor:

 $\Lambda^{\mu\sigma\kappa}\Lambda^{...\rho}_{\mu\sigma}$ 

is symmetric in the induces  $\kappa$  and  $\rho$ , while the tensor:

 $\Lambda_{\kappa\rho}^{\ldots\alpha}$ 

is antisymmetric in those indices, [II, (10)] must, of course, also vanish identically. Thus, my objection to the principle of over-determinacy is groundless, and in fact the identity [II, (8)] breaks down, along with the identity [II, (8')]. **Einstein** himself has committed similar mistakes that occurred in the earlier formulations (\*\*) of the unified field theory (\*\*\*). One first deals with the arbitrary assumption that there are four identities that must exist between 20 field equations (\*\*\*\*), and secondly with the inapplicable use of a limiting process (viz., lim  $\sigma = 0$ ) that makes one incorrectly regard certain quantities:

$$\frac{\partial H^*}{\partial g_{\alpha\beta}} = H^* \alpha\beta$$

<sup>(\*)</sup> **R. Zaycoff**, Zeit. Phys. **66** (1930), pp. 572; *ibidem* **67** (1931), pp. 135. Cited as I and II in what follows.

<sup>(\*\*)</sup> **A. Einstein**, Berliner Ber. (1929), Heft 1 and 10.

<sup>(\*\*\*)</sup> To which I referred, moreover, in my articles: Zeit. Phys. **56** (1929), pp. 517; **58** (1929), pp. 280 [cf., also my Corrigenda, *ibidem* **58** (1929), pp. 143], and which were unknown to **A. Einstein** himself, cf., Berliner Ber. (1930), Heft 1.

<sup>(\*\*\*\*)</sup> In addition to the four identities between 16 such equations that exist already.

as quadratic functions of the quantities:

$$S_{\alpha\beta\gamma} = \Lambda_{\alpha\beta\gamma} + \Lambda_{\beta\gamma\alpha} + \Lambda_{\gamma\alpha\beta}$$

**§ 2.** – Instead of the 22 equations [II, (1), (2)], between which the 12 identities [II, (3), (4), (8')] exist (\*), we can just as well consider the 24 equations [I, (5)], between which the 14 identities [I, (2)], [II, (8')] exist. **A. Einstein** referred to other possibilities for exhibiting compatible field equations (\*\*). In a previous article (\*\*\*), I proved the following theorem:

If:

 $X^{\alpha\beta\cdots}$ 

is any tensor that is antisymmetric in  $\alpha$  and  $\beta$  then the following identity will be true:

$$D_{\alpha} \{ D_{\beta} X^{\alpha\beta\cdots} - \frac{1}{2} \Lambda^{\cdots\alpha}_{\kappa\rho} X^{\kappa\rho\cdots} \} \equiv 0 , \qquad (1)$$

in which  $D_{\alpha}$  means the operation:

$$D_{\alpha} = \nabla_{\alpha} - \Lambda_{\alpha} \, .$$

Now, the most general linear function of the  $\Lambda_{\alpha\beta}^{\dots\mu}$  that is antisymmetric in  $\alpha$  and  $\beta$  has the following form (\*\*\*\*):

$$L^{\dots\mu}_{\alpha\beta} = \Lambda^{\dots\mu}_{\alpha\beta} + a \left(\Lambda_{\alpha} \,\varepsilon_{\beta}^{\ \mu} - \Lambda_{\beta} \,\varepsilon_{\alpha}^{\ \mu}\right) + b \,S^{\dots\mu}_{\alpha\beta} \,. \tag{2}$$

If we, with A. Einstein, set:

$$U^{\alpha\mu} = D_{\beta} L^{\alpha\beta\mu} - \frac{1}{2} \Lambda^{\dots\alpha}_{\kappa\rho} L^{\kappa\rho\mu}$$
(3)

then it will follow from (1) that:

$$D_{\alpha}U^{\alpha\mu} \equiv 0, \qquad (4)$$

and the equations:

$$U^{\alpha\mu} = 0 \tag{5}$$

can be considered to be the unified field equations, with a suitable choice of the constants a and b in formula (2).

§ 3. – At any rate, it seems to me that the unified field theory is incomplete in its present form, because, first of all, it has restricted validity, since it combines gravitation and electromagnetism under a unified roof (<sup>†</sup>), but remains powerless in regard to wave-mechanical problems. Secondly,

 $(^{****})$  a and b are arbitrary constants.

<sup>(\*)</sup> Their compatibility was proved rigorously.

<sup>(\*\*)</sup> **A. Einstein**, Berliner Ber. (1930), Heft 22.

<sup>\*\*\*\*)</sup> **R. Zaycoff**, Zeit. Phys. **54** (1929), pp. 738. Cf., formulas (9), (10), (11) there.

 $<sup>(^{\</sup>dagger})$  Which was at least shown in the static case.

however, "teleparallelism" implies distinguishing a special bein-lattice, whereas, in fact, the beinlattices that emerge from each other on a **Riemannian** space under all possible proper orthogonal substitutions with position-dependent coefficients are equivalent. As **H. Weyl** was the first to point out, since position-dependent bein-transformations are identical to the continuous transformations of the  $\psi$ -quantities that are peculiar to general-relativistic wave-mechanics, the concept of a beinlattices lose its immediate significance.

Finally, it seems impossible to find a replacement for the law of motion (\*), even in the macroscopic approximation (\*\*).

<sup>(\*)</sup> Perhaps by considering quasi-static solutions.

<sup>(\*\*)</sup> *Remark in proofreading.* All of the concepts that are associated with motion are also lacking, such as energy, impulse, current, force, action, etc. In addition, it should be mentioned that a certain ambiguity also exists in the theory of teleparallelism in regard to the physical interpretation of the "unified" field  $h_m^{\alpha}$ , since the concepts of gravitation and the electromagnetic field are defined only in the first approximation, while  $h_m^{\alpha}$  remains physically incomprehensible. For a long time now, the simplicity of a theory has not served as a measure of its correctness.