"Mißt der Aberrationswinkel im Fall einer dispersion des Äthers die Wellengeschwindigkeit?" Ann. Phys. (Leipzig) 338 (1910), 1571-1576.

Does the angle of aberration measure the wave velocity in the case of a dispersive ether ([1])?

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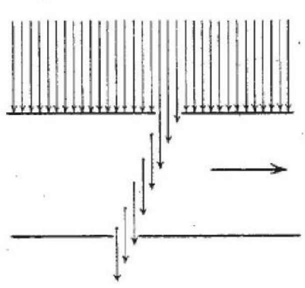
Translated by D. H. Delphenich

In the textbooks on optics, one occasionally encounters the question of whether the pure ether exhibits any noticeable dispersion. That is perhaps connected with the following statement ([2]): **Römer**'s measurement of the speed of light operates with the speed of propagation of chopped wave-trains. It correspondingly yields the *group velocity U*. By contrast, the measurement of the *angle of aberration* yields the *wave velocity V* directly. Insofar as the two results obtained in that way coincide, one can conclude that light propagates without dispersion in interplanetary space.

The statement that the angle of aberration measures the wave velocity seems to have been first expressed by Lord **Rayleigh** ([3]). At the relevant place, Lord **Rayleigh** explained (in the context of the experiments of **Young** and **Forbes** 1881) that the **Fizeau**, **Foucault**, and **Römer** methods operated with chopped wave trains, and for that reason, they yielded the group velocity *U*. By contrast, in regard to the measurement of the angle of aberration, he said:

> "The former does not depend upon observing the propagation of a peculiarity impressed upon a train of waves, and therefore has no relation to *U*. If we accept the usual theory of aberration as satisfactory the result of comparison between the coefficient found by observations and the solar parallax is *V* – the wave velocity."



By means of the following schematization of the measurement of the aberration, I would like to make it cleat it also rests "upon observing the propagation of a peculiarity impressed upon a train of waves." The figure represents two parallel plates that are provided with an opening and move with common constant velocity to the right in the direction of their extension. Monochromatic light rays might fall normally on the upper plane. We determine the angle through which the lower opening must displace with respect to the upper one in order for a (comoving) observer that is found below the

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([1]) From a talk that was given to the *St. Petersburg Physical Society* in Fall of 1908.
([2]) **R. Wood**, pp. 18; **P. Drude**, pp. 116; **O. D. Chwolson** II, pp. 246.
([3]) Lord **Rayleigh**, *Scientific Papers*, I, pp. 537, 1881.

lower opening to receive as much light as possible. If one employs the concept of "light ray" then one will see with no further analysis that the angle is determined by the velocity with which the *chopped light packet* moves backwards. If the ether possesses dispersion then *the measurement of the aberration angle will not imply the wave velocity V*.

If one attempts to operate with the wave planes in order to avoid the measurable concept of "light ray" then one will get the following picture: The upper plane (when one ignores the implications of the argument in regard to diffraction) punches a small circular disc out of each wave plane, and the circular disc moves backwards (each being displaced with respect to its predecessor. If one observes only the middle part of one such wave disc then naturally it will move with *wave velocity*, and *that* then seems to determine (in the **Rayleigh** sense) the angle through which the upper opening must be delayed in order for the wave disc to be able to slip through it as completely as possible. However, one cannot forget that dispersion will gradually alter the leading and trailing edge (in the sense of Earth motion) of any wave disc during the backward motion in ways that are difficult to comprehend, and in that way affect the most favorable position of the upper opening.

The following schematization of **Fizeau**'s toothed-wheel method shall show that the measurement of the speed of light with the help of the aberration and **Fizeau**'s toothed-wheel method are essentially identical for the question that is addressed here. Let two parallel circular discs, each of which is perforated near its periphery, be connected by a long rapidly-rotating axis. Planar light-waves fall normally on the right-hand disc. One now changes the "phase" of the left-hand side with respect to the right-hand one with a given constant angular velocity until the left-hand disc lets a maximum amount of light pass through. (Behind the left-hand disc, a convex lens will focus the light into a focal point independently of the location of the disc.) One sees the original agreement with the aberration schema that is depicted in the figure from this immediately.

What kind of velocity do all of those methods measure?

From a discussion with a group of English physicists ([1]) that granted a special position to **Foucault**'s initial method (rotating mirror), it was generally agreed that all of the methods (except the aberration method) measured the *"group" velocity U*: That is, the velocity with which the region of greatest excitation displaces when one considers the overlapping of two infinitely-long trains of sine waves with very close wavelengths λ, $\lambda + \Delta\lambda$. In that way, as is known, U will be connected with the wave velocity V as follows:

$$U = V - \lambda \frac{\Delta V}{\Delta \lambda}. \tag{1}$$

In support of that assertion, it would seem that at this point in time, one can hardly do more than add the following remarks:

1. The chopped wave-trains that all of the methods work with can be thought of as being generated by the overlapping of *infinitely many*, infinitely long trains of sine waves *of all possible wavelengths* and with suitably-chosen phases and amplitudes (i.e., the representation of an

([1]) Nature **33** (1886).

arbitrary intermittent excitation by a Fourier integral). The overlapping of at least *two* wave-trains with *very close* wavelengths λ, $\lambda + \Delta\lambda$ might perhaps give a representation of what happens in that complicated case.

2. The actually-*observed* velocity with which an arbitrarily-bounded perturbation spreads out on the surface of water coincides very well with U.

3. The absolute measurement of the speed of light in carbon disulfide (**Michelson**, with a rotating mirror) can be made to agree quite well with the measurement of the index of refraction under the assumption the U is precisely the speed of propagation ([1]).

4. For a medium whose dispersion from $\lambda = 0$ to $\lambda = \infty$ is represented by the equation:

$$V(\lambda) = a + b\,\lambda\,, \tag{2}$$

Schuster could prove *rigorously* that any arbitrarily-bounded excitation in it would move with the velocity U ([2]):

$$U = V - \lambda \frac{dV}{d\lambda} = a\,. \tag{3}$$

In all other cases besides that of **Schuster**, the rigorous treatment of the question of how a bounded perturbation would spread out encountered difficulties that have still not been overcome as soon as one proposes to go on to the quantitative treatment. In some cases (e.g., water waves), one could generally succeed in analyzing the spreading of the behavior in the *nearest neighborhood* of the original region of perturbation during its first moments. However, in the present question, one must, conversely, address the behavior at *large* distances. As long as those difficulties have not been overcome, one will then, willy-nilly, maintain the assumption that in general U represents the speed of propagation of bounded perturbations.

However, the extent to which the limits of validity of that assumption demand a closer examination is shown by those cases in which the "group" velocity U is greater than the velocity with which the front moves forwards, i.e., those surfaces *before* which the medium is certainly *exactly* at rest at the given moment.

As **W. Voigt** showed ([3]), one such case is provided by the telegraph equation:

$$\frac{\partial^2 \Phi}{\partial t^2} = c^2 \frac{\partial^2 \Phi}{\partial x^2} - \omega \frac{\partial \Phi}{\partial t}\,. \tag{4}$$

As **M. Laue** proved ([4]), it includes the medium that all dispersion theories treat when one considers wavelengths that fall within the region of selective absorption.

([1]) Cf., e.g., **R. Wood**, *loc. cit.*
([2]) **A. Schuster**, *Boltzmann-Festschrift*, pp. 569.
([3]) **W. Voigt**, Wied. Ann. **68** (1899), pp. 598; Ann. Phys. (Leipzig) **4** (1901), pp. 203.
([4]) **M. Laue**, Ann. Phys. (Leipzig) **18** (1905), pp. 523, § **6**.

In the detailed analysis of the case that he cited, **Laue** especially pointed to the fact that in it, the elementary waves, and therefore, the so-called "groups" of them, as well, propagate with a degree of absorption that is large enough that the concept of group velocity loses any physical meaning at all.

For that reason, let me refer briefly to the peculiarities that the following case exhibits:

Let a string be given whose infinitely-small longitudinal motions satisfy the equation:

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2} + \beta^2 u \,, \tag{5}$$

i.e., the ordinary string, but for which every point is pushed from its rest position by a force $\beta^2 u$.

1. When the string is at rest and undeformed, it is in (unstable) equilibrium.

2. An originally-bounded perturbation spreads out (ever increasing) from the original string at rest with the *front velocity* α ([1]).

3. Infinitely-long sine-wave trains move *without deformation* with the wave velocity:

$$V(\lambda) = \alpha \left(1 - \lambda^2 \frac{\beta^2}{4\pi^2 \alpha^2}\right)^{1/2} . \tag{6}$$

4. The "groups" that appear in the argument that involves two infinitely-long sine-wave trains of wavelengths λ, $\lambda + d\lambda$ move with group velocity:

$$U(\lambda) = \alpha \frac{1}{\left(1 - \lambda^2 \frac{\beta^2}{4\pi^2 \alpha^2}\right)^{1/2}} . \tag{7}$$

As one sees, for all values of λ that make V and U real, the group velocity U will be greater than the front velocity α.

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([1]) One will get that result when one integrates equation (5) for given initial conditions using **Riemann**'s method of integration (**Riemann-Weber**, Bd. II, § 121; **W. Voigt**, *loc. cit.*)