"Sur l'article de M. L. de Broglie: 'l'univers à cinq dimensions et la mécanique ondulatoire'," J. Phys. Radium (5) **8** (1927), 242-243.

On the article by L. de Broglie: 'The five-dimensional universe and wave mechanics'

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In an article that appeared recently in this journal ([1]), L. de Broglie gave an appraisal of my work on that subject ([2]), and on that occasion, he made a very severe criticism of the wave equation for electrified particles that I had proposed. Since that critique seems to be based upon a misunderstanding, I would like to respond with a few words. First, a remark on the history of the subject. According to de Broglie, the five-dimensional representation of Einstein's theory was developed by Kaluza and Kramers. However, the article by Kramers that he cited [de Broglie cited Proc. Amst. **28** (1922), pp. 7, which can only be the article by Kramers in Proc. Amst. **23** (1922), pp. 1052, no. 7] was only concerned with the static gravitational field in the context of the usual theory of relativity, and the coordinate of a fifth dimension did not figure in it. Furthermore, Kramers assured me that he has written nothing on the theory of five dimensions.]

The critique that de Broglie made of my work, to which he alluded in the introduction to his article is found on page 72, after he gave the five-dimensional wave equation (equation 28, pp. 72). He said: "O. Klein wrote equation (38) without the right-hand side, and he concluded that the world-lines are null-length geodesics. There seems to be no doubt that the right-hand side of (38) is necessary and that the world-lines are geodesics, but not *null length* geodesics." Although I am in perfect agreement with the second half of that phrase, I did not reach the same conclusion that de Broglie spoke of. *On the contrary, for the problem of the motion of an electrified particle, our equations differ only in form.* I say immediately that I myself prefer de Broglie's form, which is invariant under arbitrary transformations of the five coordinates. Also, I have made use of it in a paper that I hope to publish soon (before de Broglie's paper appeared)."

De Broglie's wave equation (38) is written (for the notations, see de Broglie's article):

$$\gamma^{ik}\left[\frac{\partial^2 u}{\partial x^i\,\partial x^k} - \begin{Bmatrix} i\,k \\ r \end{Bmatrix}\frac{\partial u}{\partial x^r}\right] = -\frac{4\pi^2}{h^2}\,I^2\,u,$$

with:

$$I^2 = m_0^2\,c^2 + \frac{e^2}{\alpha^2\,\gamma_{00}}$$

([1]) "L'universe à cinq dimensions et la mécanique ondulatoire," J. Phys. Radium **8** (1927), pp. 65.

([2]) Zeit. Phys. **37** (1926), pp. 895; *ibid.* **118** (1926), pp. 516.

[see equation (29)].

It follows from the formulas in my note, with my notation [see equation (34) and relations (31), (32), (4), and (5)]:

$$\sum \gamma^{ik}\left[\frac{\partial^2 u}{\partial x^i\,\partial x^k} - \sum \begin{Bmatrix} i\,k \\ r \end{Bmatrix} \frac{\partial u}{\partial x^r}\right] + \left(\frac{1}{\mu} - \frac{1}{\alpha}\right)\frac{\partial^2 u}{\partial (x^0)^2} = 0.$$

In order to arrive at the equation of propagation of an electrified particle, one must introduce Planck's constant by the hypothesis that u is a harmonic function in x^0 with period h / p_0, which is a hypothesis that de Broglie also used. That will give [see (32), (40), and (17) of my note]:

$$\left(\frac{1}{\mu} - \frac{1}{\alpha}\right)\frac{\partial^2 u}{\partial (x^0)^2} = -\,\frac{4\pi^2}{h^2}\left(m^2 c^2 - \frac{e^2}{2\kappa c}\right).$$

The constant κ and the constant of gravitation G are related by:

$$\kappa = \frac{8\pi G}{c^4}.$$

Hence:

$$\sum \gamma^{ik}\left[\frac{\partial^2 u}{\partial x^i\,\partial x^k} - \sum \begin{Bmatrix} i\,k \\ r \end{Bmatrix} \frac{\partial u}{\partial x^r}\right] - \frac{4\pi^2 c^2}{h^2}\left(m - \frac{e^2}{16\pi G}\right) u = 0.$$

If one replaces γ^{ik} with $-\gamma^{ik}$ and G with $-G$ then one will obtain de Broglie's final equation (41) precisely. The first difference in sign is due to the fact that the square of the line element ds^2 has the opposite signs in de Broglie's note and my note. Since de Broglie does not seem to have noted that difference, there is a sign error in his relation (12), which he used in order to arrive at his equation (41) from equation (38), which explains the second sign difference between our equations. However, that error will have no effect on de Broglie's results, since the term in question will disappear when one introduces the periodicity in x^0.

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