From Carl Friedrich Gauss Werke, Königlichen Gesellschaft der Wissenschaften zu Göttingen, 1867, Bd. V, pp. 637640.

## Remarks

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Translated by D. H. Delphenich

The fragmentary investigations that are recorded here are combined into 21 numbers belong to rather widely separated points in time. Nos. 1 and 2 are found in a diary that was concerned with the protocols of observations that were made in March, June, and July of 1833 of the galvanic currents that are induced by magnets. Nos. 6, 12, 13, 21 came from particular letters, and except for 18 , which bears a date, suggest no particular time determination. The remaining numbers, with exclusion of the letter [20], are again presented here in the same sequence in which they were found in a handbook, but it was one in which numerous entirely heterogeneous developments were included. The last of those numbers, with the proof of AMPERE's fundamental law, was first recorded after 1843, while the others seem to belong to the time intervals from 1833 to 1836.

The different forms that are assumed here for the law of interaction between galvanic current elements are implied by the principle of the interchangeability of magnetism with galvanic currents, which is emphasized in no. 20 especially. The investigations of WILHELM WEBER that are cited in that letter define the preliminary work for the presentation (completed in the year 1846 in the first treatise on electrodynamical measurements) of a theory according to which the total interaction between two electrical particles ( $e, e^{\prime}$, provided with corresponding signs) at a mutual distance of $r$ is measured by:

$$
e e^{\prime}\left[\frac{1}{r^{2}}-\left(\frac{1}{c r} \frac{d r}{d t}\right)^{2}+\frac{2}{c^{2} r} \frac{d^{2} r}{d t^{2}}\right],
$$

in which a positive value of that quantity means a repulsion, while a negative one means an attraction. In that expression, $t$ means the time, and $c$ means a velocity that KOHLRAUSCH and WEBER have found to be equal to $439450 \times 10^{6}$ millimeters / second by reducing the current intensity measurements to mechanical units.

The proposition in no. 4 is presented in purely-geometric clothing. Due to its importance for the theory of galvanic currents, I believe that it deserves some mention. When $d s^{\prime}$ is replaced with all of the closed galvanic currents that are present in space, the integral by which one determines the number of times that the closed curve $s$ links the system of closed curves $s^{\prime}$ is, in fact, given by the algebraic sum of the intensities of those currents that pierce a surface that is bounded by $s$, but is otherwise assumed to be arbitrary. However, with that interpretation of $d s^{\prime}$, the integral itself is equal to $\frac{1}{4 \pi} \int \frac{d V}{d s} d s$, when $V$ denotes the potential function for the magnetic effects of the current
$s^{\prime}$, as in no. 9. The theorem then defines the analogue of one that GAUSS presented in the treatise on the attraction of ellipsoids that determined the mass that is found inside of a closed surface ( $\omega$ ) from the normal forces of its attraction $\left(\frac{d V}{d N}\right)$ on the surface, pointing inward, namely, $\frac{1}{4 \pi} \int \frac{d V}{d s}$ $d \omega$.

One can ascertain the cited value of the integral above $\frac{1}{4 \pi} \int \frac{d V}{d s} d s$, e.g., when one converts the integral expression for the derivative of $V$ with respect to the coordinates into an integral that extends over an arbitrarily-chosen surface $\omega$ that is bounded by the isolated current conductor $s^{\prime}$. From a theorem that was cited in art. 39 on the general theory of geomagnetism that gave the difference between the value of the potential function for a surface that is placed on both sides with opposite magnetic fluids in a certain way and the value of the corresponding locations on both sides of the surface, the form for the derivative of $V$ with respect to $s$ that is obtained in that way can be recognized immediately from the fact that the desired integral is equal to the algebraic sum of the intensities of the currents that move in the boundary lines of surfaces $\omega^{\prime}$ that are pierced by the curve $s$.

The conversion of the integral that extends over a closed curve into one that refers to a surface that is bounded by $s$ can be performed with the help of the theorem that for any rectangular rectilinear or curvilinear coordinates $\xi, \eta, \zeta$ that then generally represent the square of the length element by an expression of the form:

$$
\xi^{\prime 2} d \xi^{2}+\eta^{\prime 2} d \eta^{2}+\zeta^{\prime 2} d \zeta^{2}
$$

and for arbitrary functions $\lambda, \mu, v$ of the coordinates $\xi, \eta, \zeta$ that vary continuously, along with their derivatives, at the points of the surface $\omega$, one will always have:

$$
\int\left(\lambda \frac{d \xi}{d s}+\mu \frac{d \eta}{d s}+v \frac{d \zeta}{d s}\right) d s=\int\left\{\left(\frac{\partial \mu}{\partial \zeta}-\frac{\partial v}{\partial \eta}\right) \frac{\eta^{\prime}}{\zeta^{\prime} \xi^{\prime}} \frac{d \eta}{d n}+\left(\frac{\partial \lambda}{\partial \eta}-\frac{\partial \mu}{\partial \xi}\right) \frac{\zeta^{\prime}}{\xi^{\prime} \eta^{\prime}} \frac{d \zeta}{d n}\right\} d \omega
$$

when $\xi, \eta, \zeta$ mean the coordinates of a point with the length element $d s$ in the first integral, and $\xi$, $\eta, \zeta$ mean the coordinates of a point with the surface element $d \omega$, and $n$ means the normal to the surface element in the second one. The positive direction of the normal is chosen so that when $d t$ denotes the first element of curve from the point of $d s$ to that normal itself, but in the surface $\omega$, the positive directions of the $\xi^{\prime} d \xi, \eta^{\prime} d \eta, \zeta^{\prime} d \zeta$ can be made to overlap with the positive directions of $d n, d s, d t$, respectively, by a continuous displacement of the position of the coordinate system.

The repeated application of that theorem will also give a proof of AMPÈRE's fundamental law in the general form that immediately reduces the potential function for the interaction between the surfaces with magnetic fluids that are placed in a certain way to the potential function for the interaction between galvanic currents whose positions and intensities are determined by those surfaces and the magnetization.

One can give the proof that is presented in no. 9 for the equality of the values of the different expressions for the potential function $V$ a symmetric form when one introduces the function:
$R=x^{2} \arctan \frac{y z}{x r}+y^{2} \arctan \frac{z x}{y r}+z^{2} \arctan \frac{x y}{z r}+2 y z i \arctan \frac{x i}{r}+2 z x i \arctan \frac{y i}{r}+2 x y i \arctan \frac{z i}{r}$,
in which $i$ appears instead of $\sqrt{-1}$, and considers the fact that one has the equations

$$
\begin{array}{cc}
\frac{\partial^{2} R}{\partial x^{2}}=2 \arctan \frac{y z}{x r}, & \frac{\partial^{2} R}{\partial y \partial z}=2 i \arctan \frac{x i}{r}, \\
\frac{\partial^{2} R}{\partial y^{2}}=2 \arctan \frac{z x}{y r}, & \frac{\partial^{2} R}{\partial z d x}=2 i \arctan \frac{y i}{r}, \\
\frac{\partial^{2} R}{\partial z^{2}}=2 \arctan \frac{x y}{z r}, & \frac{\partial^{2} R}{\partial x \partial y}=2 i \arctan \frac{z i}{r}, \\
\frac{\partial^{2} R}{\partial x^{2}}+\frac{\partial^{2} R}{\partial y^{2}}+\frac{\partial^{2} R}{\partial z^{2}}=(2 m+1) \pi, & \frac{\partial^{3} R}{\partial x \partial y \partial z}=-2 \frac{1}{r} .
\end{array}
$$

The potential function for the magnetic effect of a galvanic current of conductors that consist of linear pieces that are rectilinearly parallel to the axes of a rectangular coordinate system can also be represented in finite form by the derivatives of the function $R(x, y, z)$.

We have the note that Herr Geh. Hofrat WEBER produced in the years 1839 or 1840 to thank for the sequence of length comparisons that GAUSS applied to the determination of a subdivision of a graded yardstick into parts of the whole yardstick.

The theoretical investigations into diffraction phenomena were probably prompted by the work of F. M. SCHWERD on that topic that he published in the year 1835. The general formulas for the effect of an illuminated point $P$ on a point $p$ that are presented are not identical. The general conversion of surface integrals whose elements do not depend upon the position the plane that goes through a point of the associated surface particle $d s$ and the points $P$ and $p$ into line integrals whose elements do not depend upon the position of the plane that goes through a point of the associated particle on the boundary line $u$ and through the points $P$ and $p$, but whose differential means a change in only the angle $\theta$ that the plane subtends with a fixed plane that is laid through $P$ and $p$, is obtained from the equation:

$$
\begin{aligned}
\int Q d \theta & =\int \frac{h \cdot Q \cdot \sin v \cdot d u}{r R \sin w} \\
& =\int \frac{h}{r R} \cdot \frac{1}{2 \sin \frac{1}{2} w^{2}} \cdot \frac{d Q}{d(R-r)} \cdot \frac{d(R+r)}{d \rho} \cdot d s-\int \frac{h}{r R} \cdot \frac{1}{2 \sin \frac{1}{2} w^{2}} \cdot \frac{d Q}{d(R+r)} \cdot \frac{d(R-r)}{d \rho} \cdot d s,
\end{aligned}
$$

which is a special case of the theorem that was mentioned in the foregoing remark, namely, when $Q$ means a quantity that depends upon only $R$ and $r$, along with its partial derivatives with respect to $R+r$ and $R-r$ that varies continuously with the points of the surface $s$, and furthermore, that $w$ is determined precisely as it was in the text, such that it denotes the angle between $R$ and $r$ that the positive directions of those lines subtend with the directions of the advancing light ray, which are assumed to coincide.

## SCHERING

